Inflation Based Fuzzy EOQ Model

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Abstract: The purpose of filling is to continue the flow of goods through the system. Implementing a tailored compensation policy is an important issue in developing the production ordering model. Inflation effects the Cost values in an inventory system. Financial state of any organization depends largely on the fluctuations in the inflation rate. Based on the above context this document provides a detailed policy of defective items for different cycle times subject to inflation and incorporating the uncertainties prevailing in the environment. The change in the cost parameters over a passage of time is included in the fuzzy sense so as bring the model close to a realistic solution. Insufficiency in the supply of the goods is partially fulfilled. Validation of the results in done through numerical example.

Keywords: Fuzzy, Octagonal Fuzzy number, time functional demand, Inventory

1. Introduction

Inflation is measured by the percentage rate on the cost of time and services increases with respect to time, and therefore leads to a decrease in the buying tendency of the money. Most models are designed to maintain consistency of costs, but this is not really the case, since costs are influenced by this percentage value over time. The influence of inflation is significant in a behavioral environment. [1] presented a model formulating inventory policies under several inflation phases. [2] has established a model where demand depends on stocks of volatile raw materials and partial accumulation of goods. [3] has introduced a favorable ordering policy for stock-dependent demand with deterioration that is weibull in nature for usable parameters under the effect of inflation.

Furthermore, deterioration is impairment, loss, decay, decrease of quality or distortion of goods or services. That means the object is likely to deteriorate over time. [4] first presented an inventory model taking into account the decrease. An integrated approach in the determination of the best possible time period for ordering inventory with delay in payment is done by [5]. Applying a fuzzified approach in analyzing the flood analysis is done by [6]. Furthermore to add in the supply chain process, deterioration that is time dependent and weibull [7] and [8] derived a model in their research work. Inflation cannot be ignored while framing any inventory model. [9] and [10] in their research work formulated an inventory problem in an inflationary environment. When it comes to formulating an inventory problem the effect of the defective goods on the environment cannot be ignored. [11-13] in their work formulated a model with the green technology of several R's. Also the unpredicted behavior of various costs values is taken into consideration in the article work done by [14]. Nevertheless, in today's competitive and dynamic business world, it is not possible to access all the required information. In consequence, the information about the inventory system is not well defined as seen in traditional models. One of the effective ways to overcome these deficiencies is to use fuzzy set theory (FST), developed by [15], which changes the definition/information about mathematical expressions used in inventory models.

In the present article, modeling is done for an inventory framework for a single retailer and item. The objective is to determine the best possible time period for refilling the inventory which also minimizes the total cost. The problem is devised keeping in mind inflation rate. Also the unpredicted behavior of the cost parameters involved are approximated by fuzzyfying them. The article proceeds with the prerequisites, then mathematical representation followed by the numerical example validating the theory.

2. Preliminaries

A fuzzy number $\tilde{F}$ is defined as an octagonal fuzzy number as $\tilde{F} = (a, b, c, d, e, f, g, h)$ where (a,b,c,d,e,f,g,h) belongs to real number set. Its membership function $\mu_F(y)$ is given as

$$\mu_F(y) = \begin{cases} 0\,, y \le a \\ k\left(\frac{y-a}{b-a}\right), a \le y \le b \\ k,\ b \le y \le c \\ k + (1-k)\left(\frac{y-c}{d-c}\right), c \le y \le d \\ 1, d \le y \le e \\ k + (1-k)\left(\frac{f-y}{f-e}\right), e \le y \le f \\ k, f \le y \le g \\ k\left(\frac{h-y}{h-g}\right), g \le y \le h \\ 0, y \ge h \end{cases}$$

With k lying between 0 and 1

Definition

Let $\tilde{F}$ be an octagonal fuzzy number, then the value of the measure of fuzzy number $\tilde{F}, M_{\pi}^{Oct}(\tilde{F})$ is determined as:

$\frac{1}{4}[(a + b + g + h)k + (c + d + e + f)(1 - k)]$

2.1 Presumptions

1. Every cost parameter is subjected to a constant rate of inflation.
2. The time lag in placing and receiving an order is zero.
3. The inventory problem without shortages.
4. Goods supply is insufficient and a fraction of demand turns into shortages L(t) from which a fraction 1 – L(t) turns into lost revenue.
5. The Demand is taken as a variable function of time.

6. Octagonal fuzzy number system is taken to fuzzify the cost parameters for appropriateness.

2.2 Symbols used

1. i = d - r = discount rate - inflation rate
2. P_u = cost per unit of purchasing
3. D_e = cost per unit of deterioration
4. H_l= cost per unit of carrying inventory
5. S_t = cost per unit of shortage
6. L_s = cost per unit of lost revenue
7. R_c = Cost per unit per order
8. $Z(t)$ = time function of demand
9. $L(t) = \frac{1}{1+\mu t}$, $\mu > 0$ *with waiting time t*

3. Mathematical Representation of the Proposed Inventory Framework

The following figure represents the inventory framework. The model begins with zero inventory and then proceeds with the SFI policy. The objective of this work is to determine the best possible time period for ordering the inventory.

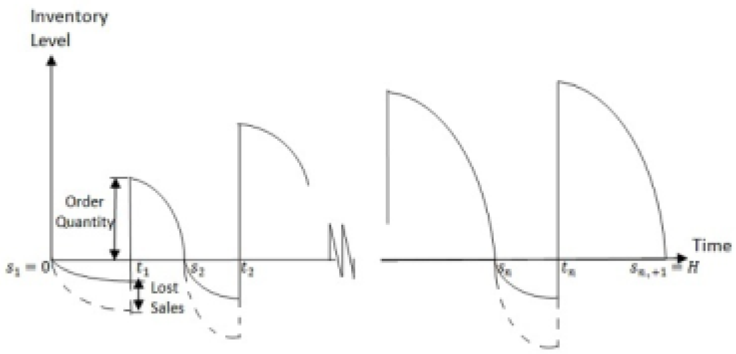


Fig 1 : Proposed Inventory Model

The amount of inventory is given as

$$\frac{dI_n(t)}{dt} + \gamma(t)I_n(t) = -Z(t), [t_i, s_{i+1}]$$

The amount of shortage is given as

$$\frac{dS(t)}{dt} + Z(t).L(t) = Z(t).\frac{1}{\mu(t_i - t)}, [s_i, t_i]$$

3.1 Crisp Inventory Framework:

Present Measure of Total cost is given by:

$$P_rTC = \sum_{i=1}^{i=n}(R_c e^{it_{i-1}} + D_e e^{it_{i-1}} * \int_{t_i}^{s_{i+1}}(\gamma * t)\left(1 + \frac{\gamma}{2}t^2\right)(t - t_i) - \frac{\gamma}{6}(t^3 - t_i^3)Z(t)dt + S_t e^{it_{i-1}} * \int_{s_i}^{t_i}((t_i - t)\frac{1}{(1+\mu(t_i - t))}Z(t)dt + L_s e^{it_{i-1}} * \int_{s_i}^{t_i}((t_i -$$

$t)\frac{1}{(1+\mu(t_i-t))}Z(t)dt + P_u e^{it_{i-1}}(\int_{t_i}^{s_{i+1}}\left(1+\frac{\gamma}{2}t^2\right)(t-t_i) - \frac{\gamma}{6}(t^3-t_i^3)Z(t)dt + \int_{s_i}^{t_i}((t_i - t)\frac{1}{(1+\mu(t_i-t))}Z(t)dt + H_l e^{it_{i-1}} * \int_{t_i}^{s_{i+1}}\left(1+\frac{\gamma}{2}t^2\right)(t-t_i) - \frac{\gamma}{6}(t^3-t_i^3)Z(t)dt$

The optimality of model is derived by the first derivative of the total cost equation.
$\frac{\partial P_rTC}{\partial t_i} = 0$ and $\frac{\partial P_rTC}{\partial s_i} = 0$

3.2 Fuzzy Inventory Framework

$\widetilde{PrTC} = \sum_{i=1}^{i=n}(R_c e^{it_{i-1}} + \widetilde{D_e} e^{it_{i-1}} * \int_{t_i}^{s_{i+1}}(\gamma * t)\left(1+\frac{\gamma}{2}t^2\right)(t-t_i) - \frac{\gamma}{6}(t^3-t_i^3)Z(t)dt + \widetilde{S_t} e^{it_{i-1}} * \int_{s_i}^{t_i}((t_i - t)\frac{1}{(1+\mu(t_i-t))}Z(t)dt + \widetilde{L_s} e^{it_{i-1}} * \int_{s_i}^{t_i}((t_i - t)\frac{1}{(1+\mu(t_i-t))}Z(t)dt + \widetilde{P_u} e^{it_{i-1}}(\int_{t_i}^{s_{i+1}}\left(1+\frac{\gamma}{2}t^2\right)(t-t_i) - \frac{\gamma}{6}(t^3-t_i^3)Z(t)dt + \int_{s_i}^{t_i}((t_i - t)\frac{1}{(1+\mu(t_i-t))}Z(t)dt + \widetilde{H_l} e^{it_{i-1}} * \int_{t_i}^{s_{i+1}}\left(1+\frac{\gamma}{2}t^2\right)(t-t_i) - \frac{\gamma}{6}(t^3-t_i^3)Z(t)dt$

4. Numerical Illustration

For Crisp Inventory Framework

Example 1: When the demand is quadratic

Taking demand to be quadratic Z(*t*) = $5t^2 + 10t + 25$, with partially fulfilling the scarcity of the demand with the cost values being:
H_l = 4, D_e = 4, P_u = 4, S_t = 2; L_s = 10; *i=0.07*, α = 0.001, δ = 4 and
R_c = 60. The best favorable solution to the total system cost along with the best possible time period t_i and s_i are determined and presented in table 1 and table 2 respectively.

Table 1: Present Amount of total system cost and the best possible time period for Example 1

n	*PW TC*
1	1940.72
2	1633.87
3	1306.82
4	1159.75
5	1084.30
6	1054.07
7	1054.65

Table 2 : Best possible values to time period t_i and s_i for Example 1

i	t_i^*	s_{i+1}^*
1	0.267	1.04751
2	1.23541	1.90397
3	2.02519	2.56404
4	2.65493	3.11048
5	3.18434	3.58263
6	3.64547	4.00

Example 2: When the demand is linear

Taking demand to be quadratic Z(t) = $10t + 25$, with partially fulfilling the scarcity of the demand with the cost values being:
H_l = 4, D_e = 4, P_u = 4, S_t = 2; L_s = 10; i=0.07, α = 0.001, δ = 4 and
R_c = 60. The best favorable solution to the total system cost along with the best possible time period t_i and s_i are determined and presented in table 3 and table 4 respectively.

Table 3 : Present Amount of total system cost and the best possible time period for Example 2

n	$PW\,TC$
1	1343.622
2	1119.538
3	904.949
4	838.143
5	820.494
6	832.898
7	861.388

Table 4: Best possible values to time period t_i and s_i for Example 2

i	t_i^*	s_{i+1}^*
1	0.235	1.01942
2	1.20972	1.94638
3	2.08681	2.72167
4	2.83521	3.39694
5	3.49368	4.00

For fuzzy Inventory Framework

Example 3 : Taking demand to be quadratic Z(t) = $5t^2 + 10t + 25$, with partially fulfilling the scarcity of the demand with the cost values being:
$\widetilde{H_l}$ = (0.2,0.4,0.6,0.8,1,1.2,1.4,1.6), $\widetilde{D_e}$ = (1,2,3,4,5,6,7,8), $\widetilde{P_u}$ = (1,2,3,4,5,6,7,8), $\widetilde{S_t}$ = (0.2,0.4,0.6,0.8,1,1.2,1.4,1.6), $\widetilde{L_s}$ = (7,8,9,10,11,12,13,14), d = 0:12, r = 0:05, α = 0.001, δ = 4 and R_c = 60. The best favorable solution to the total system cost along with the best possible time period t_i and s_i are determined and presented in table 5 and table 6 respectively.

Table 5: Present Amount of total system cost and the best possible time period for Example 3

n	$PW\,TC$
1	1917.25
2	1558.64
3	1171.94
4	1087.72
5	1161.3
6	1203.98
7	1229.55

Table 6 : Best possible values to time period t_i and s_i for Example 3

i	t_i^*	s_{i+1}^*
1	0:471	1.49
2	1:73	2.56
3	2:70	3.35
4	3.44	3.99

Example 4 : Taking demand to be quadratic Z(t) = $10t + 25$, with partially fulfilling the scarcity of the demand with the cost values being:
$\widetilde{H_l}$ = (0.2,0.4,0.6,0.8,1,1.2,1.4,1.6), $\widetilde{D_e}$ = (1,2,3,4,5,6,7,8), $\widetilde{P_u}$ = (1,2,3,4,5,6,7,8), $\widetilde{S_t}$ = (0.2,0.4,0.6,0.8,1,1.2,1.4,1.6), $\widetilde{L_s}$ = (7,8,9,10,11,12,13,14), d = 0:12, r = 0:05, α = 0.001, δ = 4 and R_c = 60. The best favorable solution to the total system cost along with the best possible time period t_i and s_i are determined and presented in table 7 and table 8 respectively..

Table 7: Present Amount of total system cost and the best possible time period for Example 4

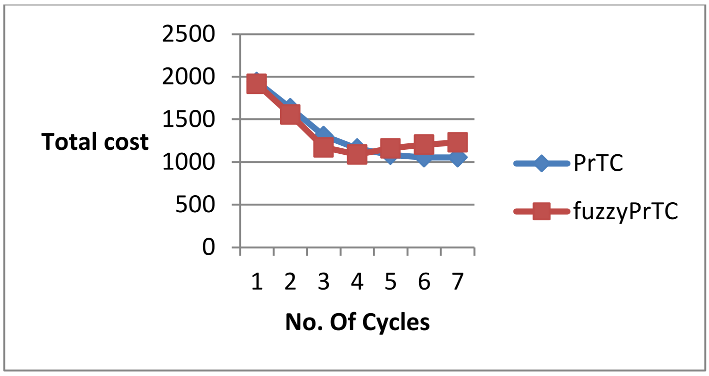
n	$PWTC$
1	1327.05
2	1061.75
3	856.83
4	794.85
5	783.33
6	801.73
7	831.81

Table 8 : Best possible values to time period t_i and s_i for Example 4

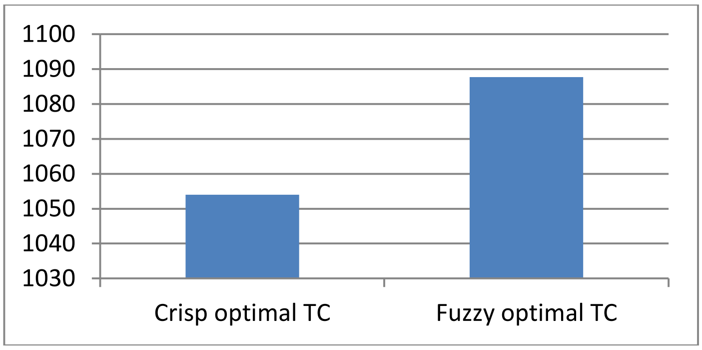
i	t_i^*	s_{i+1}^*
1	0.195	1.005
2	1.16	1.92
3	2.04	2:70
4	2.80	3:38
5	3:46	3:99

5. Comparison

The following chart shows the comparison of the total cost values in crisp and fuzzy form when the demand is quadratic in nature.



The Crisp value calculated for total cost is rs.1054 in 6th cycle whereas in fuzzy model this cost increases to Rs.1087 in 4th cycle. This implies that incorporating fuzziness in the model reduces the optimal number of cycles but shows an increase in the optimal value of total cost for quadratic form of demand. It predicts that the cost is crisp form is not including the uncertainty in the cist values whereas in the fuzzy form the cost is made more accurate to give a more realistic solution.



7. Conclusion

This article presents a comparative study in an inventory framework for the crisp and the fuzzy environment. Moreover the cost obtained in the crisp environment is less than the fuzzy cost value but that cost does not gives an accurate measure to the parameters. The crisp environment fixes every value to its parameters which does not hold in framing a model as the parameters are subjected to the uncertainties in the market. The demand is taken in quadratic and linear form. Also an inflationary valuation for the cost values gives the present measure to the cost parameters in the current time period.

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