ECONOMIC ORDER QUANTITY MODEL FOR SUSTAINABLE INVENTORY CONSIDERING DIFFERENT SOURCES OF ENVIRONMENTAL COST ALONG WITH THE NON-ENVIRONMENTAL COST

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Abstract:

Mitigating carbon emission in inventory has become an important issue for companies. Nowadays, companies are looking for solutions to reduce the carbon emission due to carbon policies imposed by various regulatory bodies. Not only for regulations, ISO certification, business competitive and customer satisfaction also encourage organizations to adopt green practices in their supply chain. In this paper, we develop a new sustainable inventory model considering carbon emission function which includes different sources of pollution. A mathematical model is to determine the optimal order quantity and total cost of this system. A numerical example is illustrated to present the proposed model.

Keywords: Inventory, carbon emission, pollution control, carbon tax, recycling, environmental sustainability.

1. Introduction

Carbon emissions is the primarily source of green house gas emissions which are the main reason for environmental pollution. Rising sea level, loss of crops and global warming are the cruel effects of emissions. Carbon can be emitted from the direct human- induced impact which includes usage of fossil fuel, deforestation, land clearing for agriculture and degradation of soils. Black carbon is a fine solid particle but it also contributes the global warming of the atmosphere. Environment, social and economic criteria are three main concept for sustainability issues in inventory models. Currently, companies are forced to follow sustainable practices in manufacturing procedure to carbon policies. Stiff competitions, waste reduction, cost benefit, customer awareness and branding also performance an important role for an organization to go Green. Most of the businesses are supply chain oriented. That means supply chain deals with the planning, control of the movement, proper storage of goods and services from point of origin to final destination (recycling or disposal). Supply chain management can lower companies total cost and enhances profit because the returned product from customer to supplier can be reused or recycled to produce the new products.

Economic Order Quantity (EOQ) is the first inventory model developed by Harris which has still being studying extensively till nowadays. Bonney and Jaber (2011) developed a simplistic model that extends the EOQ to include some environmental costs. They examined some possible environmental consequences of common activities and suggested that all functions within the product life cycle including inventory planning and control should be looked at from an environmental point of view. Arslan and Turkay (2013) explained the standard EOQ inventory model to integrate sustainability considerations that take account of environmental issues. Hua, et al (2011) examined the impact of carbon cap and carbon price on order size as a method of inducing the retailer to reduce carbon emissions that result in the increase of total costs. Green et al (2012) suggested that green supply chain management practices should include internal environmental management, green information systems, green purchasing, cooperation with customers, eco-design and investment recovery. Alfonso Angel Medina Santana and Leopoldo Eduardo Cardenas-Barron were developed an inventory model with new carbon emissions function to include different sources of pollution in the decision making process and also permits to obtain a continuous and a discrete solution for the lot size. Due to transportation and industrial emission, it increases the environmental pollution. Arindum Mukhopadhyay and A. Goswami (2013) describe a pollution quantity model. Mac, et al (2016) explained green production strategies for carbon sensitive products under carbon cap policy to regulate emission reduction. They also described low carbon economy direction to solving global warming. Emissions can be reduced to some extend by capital investment on green technology, Tapan Kumar Datta (2017). Environmental collaboration is one of the initiative responses to environmental problems, focuses on environmental protection and promotes synchronized development of economic and environment perspectives, Li. Y (2011).

In this research work, we developed a new sustainable inventory model with carbon emission function which includes different sources of pollution of the manufacturing, transportation, recycling and disposal. The objective of the Economic order quantity (EOQ) model is to minimize the total cost. The structure of the paper is as follows: Section 2 formulates a mathematical sustainable inventory model which comprehends environmental and non-environmental costs with its notations and assumptions. Section 3 solves a numerical example and Section 4 concludes the proposed inventory model.

2. Mathematical Formulation

In order to derive the mathematical model, we use the following notations and assumptions.

2.1 Notations

D Demand rate

Q Order quantity per cycle

T Order cycle

A_c Ordering cost

P_c Purchasing cost per unit

h_c	Holding cost
f_c	Fixed cost per trip
v_c	Variable cost per unit transported per distance travelled
d_t	Distance travelled from supplier to buyer
α	Proportion of waste returned $(0 \leq \alpha \leq 1)$
ε	Unit of carbon emissions per replenishment
g	Unit variable emissions in the warehouse
β	Social cost from vehicle emission
a_v	Average velocity
w_d	Cost of disposing waste to the environment
f_d	Fixed cost of disposing waste to the environment
θ	Proportion of waste produced per lot Q $(0 \leq \theta \leq 1)$
C_e	Cost incurred due to the emissions when ordering and holding
e	Carbon emission price/carbon tax
M	Manufacturing cost for investing on green technology
M_e	Carbon emission from manufacturing
S_c	Screening cost
G	Capital amount invested on green technology
R	Recycling cost per unit
R_e	Carbon emission from recycling process
R_m	Number of returned materials that are suitable for recycle
π_0	The Pollution factor
C_π	The pollution cost
$C_{\pi c}$	The fixed capital cost of pollution control for production run
$C_{\pi 0}$	The operating and maintenance cost of pollution control per unit of production quantity

2.2 Assumptions

1. The Proportion of the demand and returned products are constant per cycle.
2. The replenishment is instantaneous.
3. The company has the opportunity to invest on Green technology in each source of emissions separately to reduce emissions.
4. The cost spent on enhancing the environment is incorporated into the conventional model.
5. Waste management focuses on source reduction, pollution prevention and disposal.

2.3 Mathematical Model

The sustainable inventory model includes environmental and non-environmental costs are taken into a total cost function. Both costs are discussed below.

Environmental costs:

The inventory system acquires an amount of carbon emissions each time there is replenishment and which acquires amount of carbon emissions related to the holding stock. The emissions (ε) per replenishment are associated with the carbon emissions during each cycle such as the usage of machines. An amount g of emissions per unit of average inventory level is acquired too. This is due to the fact that a greater amount of stock requires more illumination, continues usage of machine for preservation and air conditioning to manipulate the inventory. From these processes large amount of carbon is emitted.

$$C_1(Q) = \left(\varepsilon + \frac{gQ^2}{2D}\right) C_e$$

The transportation process is one of the most important sources of emission. Since the emissions are directly related to the time the vehicle is traveling for delivery and collection of returned items, thus the emission cost from transportation is formulated as in [3],

$$C_2(Q) = \frac{2\beta d_t}{a_v}$$

In waste disposal process, the amount $Q\theta$ is assumed that there is an amount of waste produced by the company during each cycle and the customer uses the company's transportation system to return an amount of waste $Q\alpha$ each time that units are supplied in order to meet the demand. Fixed cost f_c and a variable cost v_c are acquired because of the disposal activity. Waste disposal cost is expressed as follows

$$C_3(Q) = f_d + w_d Q(\theta + \alpha)$$

During manufacturing process, carbon is emitted by operating the machines and using hazardous chemicals etc. A green technology practice is followed to minimize the emission from the manufacturing process. Emission also occurs in recycling process. The emission cost from manufacturing and recycling processes is formulated as

$$C_4(Q) = e(M_e + R_e)$$

Environmental pollution is mainly due to transportation and industrial emissions. In order to reduces the pollution, we used the pollution control cost which is formulated as

$$C_5(Q) = C_{\pi c} + C_{\pi 0}\pi_0 Q$$

Carbon tax, carbon cap and cap and trade are the tools implemented by many countries to reduce carbon emissions. Investment on green technology helps to reduce the carbon emissions. The amount of capital invested on green technology investment is formulated as

$$C_6(Q) = G$$

Non-Environmental costs:

Traditional costs of OrderingA_c, holding h_c and purchasing P_c are considered as in the EOQ basic inventory model proposed by Harris,

$$C_7(Q) = A_c + P_c Q + \frac{h_c Q^2}{2D}$$

The transportation cost is used for delivering the products to the customer and collecting the returned waste products from the customer. The transportation cost is expressed as follows

$$C_8(Q) = 2f_c + v_c d_t Q(1 + \alpha)$$

The screening cost is used to inspect the finished product which reduces the number of returned product, is expressed as follows

$$C_9(Q) = S_c$$

Manufacturing cost is the addition of all costs involved in the process of making a product. The manufacturing cost is categorized as material cost, labour cost and manufacturing overhead. Here, manufacturing process is in green sense that is using some green technology process. The manufacturing cost is expressed as follows

$$C_{10}(Q) = M$$

Recycling process preserves the natural resources by decreasing the demand for new products. Recycling also conserves energy by lowering green house gas emissions and decreasing the demand for products made from manufacturing processes that are very energy intensive. Recycling cost can be expressed as follows

$$C_{11}(Q) = RR_m$$

Total cost function

The total cost function per unit of time is

$$TC = A_c + P_cQ + \frac{h_cQ^2}{2D} + 2f_c + v_cd_tQ(1+\alpha) + S_c + M + RR_m + \left(\varepsilon + \frac{gQ^2}{2D}\right)C_e + \frac{2\beta d_t}{a_v} + f_d + w_dQ(\theta+\alpha) + e(M_e+R_e) + C_{\pi c} + C_{\pi 0}\pi_0Q + G$$

$$TC(Q) = \frac{D}{Q}A_c + DP_c + \frac{h_cQ}{2} + 2\frac{D}{Q}f_c + D\big(v_cd_t(1+\alpha)\big) + \frac{D}{Q}S_c + \frac{D}{Q}(M+RR_m) + \frac{D}{Q}\varepsilon C_e + \frac{gQ}{2}C_e + \frac{D}{Q}\frac{2\beta d_t}{a_v} + \frac{D}{Q}f_d + D\big(w_d(\theta+\alpha)\big) + \frac{D}{Q}\big(e(M_e+R_e)\big) + \frac{D}{Q}C_{\pi c} + DC_{\pi 0}\pi_0 + \frac{D}{Q}G$$

Where $T = \frac{Q}{D}$

$$TC(Q) = \frac{D}{Q}\left[A_c + 2f_c + S_c + M + RR_m + \varepsilon C_e + \frac{2\beta d_t}{a_v} + f_d + e(M_e+R_e) + C_{\pi c} + G\right] + \frac{Q}{2}[h_c + gC_e] + D[P_c + v_cd_t(1+\alpha) + w_d(\theta+\alpha) + C_{\pi 0}\pi_0]$$

Where $H = A_c + 2f_c + S_c + M + RR_m + \varepsilon C_e + \frac{2\beta d_t}{a_v} + f_d + e(M_e+R_e) + C_{\pi c} + G$

$$TC(Q) = \frac{D}{Q}H + \frac{Q}{2}[h_c + gC_e] + D[P_c + v_cd_t(1+\alpha) + w_d(\theta+\alpha) + C_{\pi 0}\pi_0] \quad (1)$$

The first order derivative with respect to Q is

$$\frac{dTC(Q)}{dQ} = \frac{-D}{Q^2}H + \frac{1}{2}[h_c + gC_e]$$

The second order derivative with respect to Q is

$$\frac{d^2TC(Q)}{dQ^2} = \frac{2D}{Q^3}H > 0, for\ Q > 0$$

The order quantity is obtained by putting the first derivative equal to zero and solving for Q.

$$\frac{dTC(Q)}{dQ} = 0,\ \ \frac{-D}{Q^2}H + \frac{1}{2}[h_c + gC_e] = 0$$

$$Q^2 = \frac{2DH}{h_c + gC_e}$$

$$Q^* = \sqrt{\frac{2DH}{h_c + gC_e}} \quad (2)$$

3. Numerical Example

Consider the following data to illustrate the proposed sustainable inventory model

D	10,000	A_c	\$ 100

P_c	\$ 25	e	0.2
h_c	\$ 8	θ	0.1
f_c	\$ 80	M	\$ 250
v_c	\$ 4	M_e	\$ 300
d_t	250 km	S_c	\$ 0.2
α	0.1	R	\$ 150
β	\$ 30	R_e	\$ 200
a_v	50 km/h	R_m	\$ 250
ε	200 (kg of Co_2/order)	g	300
g	3 (kg of Co_2/order)	$C_{\pi c}$	40
C_e	\$ 10	$C_{\pi 0}$	4
f_d	\$ 20	π_0	0.3
w_d	\$ 5		

Using the above data in the equation (2), we obtain the optimal order quantity as
Q^* = 4520 units
Using the equation (1), we obtain the total inventory cost as
TC = Rs. 11,443,743.2743

4. Conclusion

In this paper, a sustainable inventory model is discussed under different source of emissions. This model helps the company's to improve the profit as well as reduce the emissions and minimize the total cost through recycling process. Nowadays, the companies are looking forward to reduce the emissions by the law of carbon regulatory policies. In order to preserve our environment without pollution, everyone must minimize the usage of transport and reduces the running of heavy and old machines, because which emits more amount of carbon. This model helps to reduce the pollution and to create a healthier environment for our future generation.

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