Generalized Trapezoidal Fuzzy Numbers: A New Approach to Ranking

T.LAKSHMI,

Assistant Professor, Department of Humanities and Science, Samskruti College of Engineering and Technology, Ghatkesar.

ABSTRACT

Fuzzy numbers play an essential part in decision making, optimization, forecasting, and other areas of analysis. Prior to taking action, fuzzy numbers must be rated by an executive. The ranking approach presented by Chen and Chen (Expert Systems with Applications 36 (2009) 6833-6842) is shown to be wrong in this work using various counter instances. New methods for ranking generalised trapezoidal fuzzy numbers are the focus of this study. Because the suggested technique provides the right ordering of generalised and normal trapezoidal fuzzy numbers, it is a significant benefit. According to Wang and Kerre's (Fuzzy Sets and Systems 118 (2001) 375-385), the suggested ranking function meets all the acceptable features of fuzzy quantities.

Keywords—Ranking function, Generalized trapezoidal fuzzy num- bers

INTRODUCTION

Real-world problems may be effectively addressed with the help of UZZY set theory [1]. Real numbers can be sorted by or, however fuzzy numbers do not have this form of inequality. It is difficult to tell whether one fuzzy number is greater or smaller than another since fuzzy numbers are represented by a range of possible outcomes. The employment of a ranking function is an efficient way to sort the fuzzy numbers. Real numbers are used to define the set of fuzzy numbers (F (R) R), which maps each fuzzy number to the real line in a natural order. Fuzzy set theory has grown more concerned with the specific ranking of fuzzy numbers, which is an essential process for making decisions in a fuzzy environment.

Jain was the first to come up with the idea of ranking. In [0,1], Yager [3] introduced four indices that may be used to sort fuzzy quantities. There is a method for sorting fuzzy numbers in Kaufmann and Gupta [4]. [5] Campos and Gonzalez [5] suggested a subjective method of rating fuzzy numbers. Integral value index was established by Liou and Wang [6]. Cheng [7] proposed a distance-based ranking algorithm for fuzzy integers. Kwang and Lee have a lot in common.

A ranking approach was developed by [8] based on the overall probability distributions of fuzzy numbers. Modarres and Nezhad [9] presented a ranking approach based on preference function in which the fuzzy numbers are measured point by point and the most favoured number is identified at each step in the ranking. According to Chu and Tsao [10], the region between the centroid and original point may be used to rank fuzzy integers. For sorting fuzzy numbers, Deng and Liu [11] suggested a centroid-index technique. Additionally, the centroid notion was used in the ranking indices developed by Liang et al. Chinoy and Chinoy.

In order to rank generalised trapezoidal fuzzy numbers, [14] proposed an algorithm. To rank trapezoidal fuzzy numbers, Abbasbandy and Hajjari developed a novel method based on the left and right spreads at various -levels. Fuzzy risk analysis based on ranking generalised fuzzy numbers with various heights and spreads was introduced by Chen and Chen [16].

The ranking approach presented by Chen and Chen is proven to be flawed in this study using a number of counter instances.

There is a problem with [16]. New methods for ranking generalised trapezoidal fuzzy numbers are the focus of this study. Because the suggested technique provides the right ordering of generalised and normal trapezoidal fuzzy numbers, it is a significant benefit.

The following is how the paper is laid out: Second, the definitions, arithmetic operations, and a sorting algorithm are covered in this section. Chen & Chen's technique [16] is examined in section III. Generalized trapezoidal fuzzy numbers are discussed in further detail in Section IV. On the other hand, the suggested ranking function is proven to satisfy all the reasonable features of fuzzy quantities in Section V. Section VI focuses on the conclusion.

II. PRELIMINARIES

In this section some basic definitions, arithmetic operations and ranking function are reviewed.

Basic Definitions

In this section some basic definitions are reviewed.

Definition 1. [4] The characteristic function μA of a crisp set $A \subseteq X$ assigns a value either 0 or 1 to each member in X. This function can be generalized to a function $\mu A\tilde{}$ such that the value assigned to the element of the universal set X fall within a specified range i.e. $\mu A\tilde{} : X \to [0, 1]$. The assigned value indicate the membership grade of the element in the set A.

The function $\mu A\tilde{}$ is called the membership function and the

set $\tilde{A} = \{(x, \mu_{\tilde{A}}(x));$
$x \in X\}$ defined by $\mu_{\tilde{A}}(x)$ for each $x \in X$ is called a fuzzy set.

Definition 2. [4] A fuzzy set $\tilde{A}$, defined on the universal set of real numbers R, is said to be a fuzzy number if its membership function has the following characteristics:

1. $\mu_{\tilde{A}} : R \longrightarrow [0, 1]$ is continuous.
2. $\mu_{\tilde{A}}(x) = 0$ for all $x \in (-\infty, a] \bigcup [d, \infty)$.
3. $\mu_{\tilde{A}}(x)$ strictly increasing on $[a, b]$ and strictly decreasing on $[c, d]$.
4. $\mu_{\tilde{A}}(x) = 1$ for all $x \in [b, c]$, where $a < b < c < d$.

Definition 3. [4] A fuzzy number $\tilde{A} = (a, b, c, d)$ is said to be a trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{(x-a)}{(b-a)}, & a < x < b \\ 1 & b \leq x \leq c \\ \frac{(x-d)}{(c-d)} & c < x < d \end{cases}$$

Definition 4. [16] A fuzzy set $\tilde{A}$, defined on the universal set of real numbers R, is said to be generalized fuzzy number if its membership function has the following characteristics:
1. $\mu_{\tilde{A}} : R \longrightarrow [0, w]$ is continuous.
2. $\mu_{\tilde{A}}(x) = 0$ for all $x \in (-\infty, a] \bigcup [d, \infty)$.
3. $\mu_{\tilde{A}}(x)$ strictly increasing on $[a, b]$ and strictly decreasing on $[c, d]$.
4. $\mu_{\tilde{A}}(x) = w$, for all $x \in [b, c]$, where $0 < w \leq 1$.

Definition 5. [17] A fuzzy number $\tilde{A} = (a, b, c, d; w)_{LR}$ is said to be a L-R type generalized fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} wL(\frac{b-x}{b-a}), & \text{for } a < x < b \\ w & \text{for } b \leq x \leq c \\ wR(\frac{x-c}{d-c}) & \text{for } c < x < d. \end{cases}$$

where L and R are reference functions.

Definition 6. [17] A L-R type generalized fuzzy number $\tilde{A} = (a, b, c, d; w)_{LR}$ is said to be a generalized trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} w\frac{(x-a)}{(b-a)}, & a < x < b \\ w & b \leq x \leq c \\ w\frac{(x-d)}{(c-d)} & c < x < d \end{cases}$$

B. Arithmetic operations

In this subsection, arithmetic operations between two generalized trapezoidal fuzzy numbers, defined on universal set of real numbers *R*, are reviewed [16].

Let $\tilde{A}_1 = (a_1, b_1, c_1, d_1; w_1)$ and $\tilde{A}_2 = (a_2, b_2, c_2, d_2; w_2)$ be two generalized trapezoidal fuzzy numbers then

(i) $\tilde{A}_1 \oplus \tilde{A}_2 = (a_1+a_2, b_1+b_2, c_1+c_2, d_1+d_2; min\,(w_1, w_2))$
(ii) $\tilde{A}_1 \ominus \tilde{A}_2 = (a_1-d_2, b_1-c_2, c_1-b_2, d_1-a_2; min\,(w_1, w_2))$
(iii) $\lambda\tilde{A}_1 = \begin{cases} (\lambda a_1, \lambda b_1, \lambda c_1, \lambda d_1; w_1) & \lambda > 0 \\ (\lambda d_1, \lambda c_1, \lambda b_1, \lambda a_1; w_1) & \lambda < 0. \end{cases}$

C. Ranking function

An efficient approach for comparing the fuzzy numbers is by the use of a ranking function [2], _ : $F(R) \rightarrow R$,

where $F(R)$ is a set of fuzzy numbers defined on set of real numbers, which maps each fuzzy number into the real line, where a natural order exists i.e.,

(i) $\tilde{A} \succ \tilde{B}$ iff $\Re(\tilde{A}) > \Re(\tilde{B})$
(ii) $\tilde{A} \prec \tilde{B}$ iff $\Re(\tilde{A}) < \Re(\tilde{B})$
(iii) $\tilde{A} \sim \tilde{B}$ iff $\Re(\tilde{A}) = \Re(\tilde{B})$

Remark 1. [18] For all fuzzy numbers $\tilde{A}, \tilde{B}, \tilde{C}$ and $\tilde{D}$ we have

(i) $\tilde{A} \succ \tilde{B} \Rightarrow \tilde{A} \oplus \tilde{C} \succ \tilde{B} \oplus \tilde{C}$
(ii) $\tilde{A} \succ \tilde{B} \Rightarrow \tilde{A} \ominus \tilde{C} \succ \tilde{B} \ominus \tilde{C}$
(iii) $\tilde{A} \sim \tilde{B} \Rightarrow \tilde{A} \oplus \tilde{C} \sim \tilde{B} \oplus \tilde{C}$
(iv) $\tilde{A} \succ \tilde{B}, \tilde{C} \succ \tilde{D} \Rightarrow \tilde{A} \oplus \tilde{C} \succ \tilde{B} \oplus \tilde{D}$

III. SHORTCOMINGS OF CHEN AND CHEN APPROACH

In this section, the shortcomings of Chen and Chen approach [16], on the basis of reasonable properties of fuzzy quantities [18] and on the basis of height of fuzzy numbers, are pointed out

On the basis of reasonable properties of fuzzy quantities Let ˜ A and ˜B be any two fuzzy numbers then

$\tilde{A} \succ \tilde{B} \Rightarrow \tilde{A} \ominus \tilde{B} \succ \tilde{B} \ominus \tilde{B}$ (Using remark 1)
i.e., $\Re(\tilde{A}) > \Re(\tilde{B}) \Rightarrow \Re(\tilde{A} \ominus \tilde{B}) > \Re(\tilde{B} \ominus \tilde{B})$

In this subsection, several examples are choosen to prove that the ranking function proposed by Chen and Chen does not satisfy the reasonable property, $\tilde{A} \succ \tilde{B} \Rightarrow (\tilde{A} \ominus \tilde{B}) \succ (\tilde{B} \ominus \tilde{B})$, for the ordering of fuzzy quantities i.e., according to Chen Chen approach $\tilde{A} \succ \tilde{B} \not\Rightarrow \tilde{A} \ominus \tilde{B} \succ \tilde{B} \ominus \tilde{B}$, which is contradiction according to Wang and Kerre [18].

Example 1. Let $\tilde{A} = (0.1, 0.3, 0.3, 0.5; 1)$ and $\tilde{B} = (0.2, 0.3, 0.3, 0.4; 1)$ be two generalized trapezoidal fuzzy numbers then according to Chen and Chen approach $\tilde{B} \succ \tilde{A}$ but $\tilde{B} \ominus \tilde{A} \prec \tilde{A} \ominus \tilde{A}$ i.e., $\tilde{B} \succ \tilde{A} \nRightarrow \tilde{B} \ominus \tilde{A} \succ \tilde{A} \ominus \tilde{A}$.

Example 2. Let $\tilde{A} = (0.1, 0.3, 0.3, 0.5; 0.8)$ and $\tilde{B} = (0.1, 0.3, 0.3, 0.5; 1)$ be two generalized trapezoidal fuzzy numbers then according to Chen and Chen approach $\tilde{B} \succ \tilde{A}$ but $\tilde{B} \ominus \tilde{A} \prec \tilde{A} \ominus \tilde{A}$ i.e., $\tilde{B} \succ \tilde{A} \nRightarrow \tilde{B} \ominus \tilde{A} \succ \tilde{A} \ominus \tilde{A}$.

Example 3. Let $\tilde{A} = (-0.8, -0.6, -0.4, -0.2; 0.35)$ and $\tilde{B} = (-0.4, -0.3, -0.2, -0.1; 0.7)$ be two generalized trapezoidal fuzzy numbers then according to Chen and Chen approach $\tilde{A} \succ \tilde{B}$ but $\tilde{A} \ominus \tilde{B} \prec \tilde{B} \ominus \tilde{B}$ i.e., $\tilde{A} \succ \tilde{B} \nRightarrow \tilde{A} \ominus \tilde{B} \succ \tilde{B} \ominus \tilde{B}$.

Example 4. Let $\tilde{A} = (0.2, 0.4, 0.6, 0.8; 0.35)$ and $\tilde{B} = (0.1, 0.2, 0.3, 0.4; 0.7)$ be two generalized trapezoidal fuzzy numbers then according to Chen and Chen approach $\tilde{B} \succ \tilde{A}$ but $\tilde{B} \ominus \tilde{A} \prec \tilde{A} \ominus \tilde{A}$ i.e., $\tilde{B} \succ \tilde{A} \nRightarrow \tilde{B} \ominus \tilde{A} \succ \tilde{A} \ominus \tilde{A}$.

On the basis of height of fuzzy numbers

Chen and Chen method [16] asserts that the ordering of fuzzy numbers relies on the height of fuzzy numbers in certain circumstances, although this is not always the case, as shown in this part.

Let $\tilde{A} = (a_1, a_2, a_3, a_4; w_1)$ and $\tilde{B} = (a_1, a_2, a_3, a_4; w_2)$ be two generalized trapezoidal fuzzy numbers then according to Chen and Chen [16] there may be two cases

Case (i) If $(a_1 + a_2 + a_3 + a_4) \neq 0$ then

$$\begin{cases} \tilde{A} \prec \tilde{B}, & \text{if } w_1 < w_2 \\ \tilde{A} \succ \tilde{B}, & \text{if } w_1 > w_2 \\ \tilde{A} \sim \tilde{B}, & \text{if } w_1 = w_2. \end{cases}$$

Case (ii) If $(a_1 + a_2 + a_3 + a_4) = 0$ then $\tilde{A} \sim \tilde{B}$ for all values of w_1 and w_2.

Fuzzy numbers are ranked according to height in the first instance and not according to height at all in the second case, which is a contradiction, according to Chen and Chen [16].

Example 5. Let $\tilde{A} = (1, 1, 1, 1; w_1)$ and $\tilde{B} = (1, 1, 1, 1; w_2)$ be two generalized trapezoidal fuzzy numbers then according to Chen and Chen approach $\tilde{A} \prec \tilde{B}$ if $w_1 < w_2$, $\tilde{A} \succ \tilde{B}$ if $w_1 > w_2$ and $\tilde{A} \sim \tilde{B}$ if $w_1 = w_2$.

Example 6. Let $\tilde{A} = (-.4, -.2, -.1, .7; w_1)$ and $\tilde{B} = (-.4, -.2, -.1, .7; w_2)$, be two generalized trapezoidal fuzzy numbers then $\tilde{A} \sim \tilde{B}$ for all values of w_1 and w_2.

IV. PROPOSED APPROACH

In this section, a new approach is proposed for the ranking of generalized trapezoidal fuzzy numbers

Let $\tilde{A} = (a_1, b_1, c_1, d_1; w_1)$ and $\tilde{B} = (a_2, b_2, c_2, d_2; w_2)$ be two generalized trapezoidal fuzzy numbers then

(i) $\tilde{A} \succ \tilde{B}$ if RM$(\tilde{A})$ >RM$(\tilde{B})$
(ii) $\tilde{A} \prec \tilde{B}$ if RM$(\tilde{A})$ <RM$(\tilde{B})$
(iii) $\tilde{A} \sim \tilde{B}$ if RM$(\tilde{A})$ =RM$(\tilde{B})$ (1)

A. Method to find values of RM$(\tilde{A})$ and RM$(\tilde{B})$

Let $\tilde{A} = (a_1, b_1, c_1, d_1; w_1)$ and $\tilde{B} = (a_2, b_2, c_2, d_2; w_2)$ be two generalized trapezoidal fuzzy numbers then use the following steps to find the values of RM$(\tilde{A})$ and RM$(\tilde{B})$

Step 1 Find $w = min(w_1, w_2)$

Step 2 Find $\Re(\tilde{A}) = \frac{1}{2}\int_0^w \{L^{-1}(x) + R^{-1}(x)\}dx$, where $L^{-1}(x) = a_1 + \frac{(b_1-a_1)}{w}x$, $R^{-1}(x) = c_1 + \frac{(b_1-c_1)}{w}x$ $\Rightarrow \Re(\tilde{A}) = \frac{w(a_1+b_1+c_1+d_1)}{4}$ and

$\Re(\tilde{B}) = \frac{1}{2}\int_0^w \{L^{-1}(x) + R^{-1}(x)\}dx$, where $L^{-1}(x) = a_2 + \frac{(b_2-a_2)}{w}x$,
$R^{-1}(x) = c_2 + \frac{(b_2-c_2)}{w}x \Rightarrow \Re(\tilde{B}) = \frac{w(a_2+b_2+c_2+d_2)}{4}$.

Step 3 If $\Re(\tilde{A}) \neq \Re(\tilde{B})$ then RM$(\tilde{A}) = \Re(\tilde{A})$ and RM $(\tilde{B}) = \Re(\tilde{B})$
otherwise RM$(\tilde{A})$ =mode$(\tilde{A}) = \frac{1}{2}\int_0^w b_1 dx + \frac{1}{2}\int_0^w c_1 dx = \frac{w(b_1+c_1)}{2}$ and RM$(\tilde{B})$ =mode$(\tilde{B}) = \frac{1}{2}\int_0^w b_2 dx + \frac{1}{2}\int_0^w c_2 dx = \frac{w(b_2+c_2)}{2}$

Remark 2

Two fuzzy numbers may be joined by using the -cut technique [4] to get arithmetic operations between them, and the highest value of, which is common to both fuzzy numbers, can be found by determining w1 and w2's minimum heights.

V. RESULTS AND DISCUSSION

Fuzzy numbers described in section 3 are correctly arranged in this part. A ranking function suggested here meets the fuzzy quantity features provided by Wang and Kerre [18] as shown in Table 1.

Example 7. Let $\tilde{A} = (0.1, 0.3, 0.3, 0.5; 1)$ and $\tilde{B} = (0.2, 0.3, 0.3, 0.4; 1)$ be two generalized trapezoidal fuzzy numbers
Step 1 $min(1, 1) = 1$
Step 2 $\Re(\tilde{A}) = 0.3$ and $\Re(\tilde{B}) = 0.3$. Since $\Re(\tilde{A}) = \Re(\tilde{B}) \Rightarrow$ $\mathrm{RM}(\tilde{A}) = \mathrm{mode}(\tilde{A}) = 0.3$ and $\mathrm{RM}(\tilde{B}) = \mathrm{mode}(\tilde{B}) = 0.3$. Now $\mathrm{RM}(\tilde{A}) = \mathrm{RM}(\tilde{B}) \Rightarrow \tilde{A} \sim \tilde{B}$.

Example 8. Let $\tilde{A} = (0.1, 0.3, 0.3, 0.5; 0.8)$ and $\tilde{B} = (0.1, 0.3, 0.3, 0.5; 1)$ be two generalized trapezoidal fuzzy numbers
Step 1 $min(0.8, 1) = 0.8$
Step 2 $\Re(\tilde{A}) = 0.24$ and $\Re(\tilde{B}) = 0.24$. Since $\Re(\tilde{A}) = \Re(\tilde{B}) \Rightarrow \mathrm{RM}(\tilde{A}) = \mathrm{mode}(\tilde{A}) = 0.24$ and $\mathrm{RM}(\tilde{B}) = \mathrm{mode}(\tilde{B}) = 0.24$. Now $\mathrm{RM}(\tilde{A}) = \mathrm{RM}(\tilde{B}) \Rightarrow \tilde{A} \sim \tilde{B}$.

Example 9. Let $\tilde{A} = (-0.8, -0.6, -0.4, -0.2; 0.35)$ and $\tilde{B} = (-0.4, -0.3, -0.2, -0.1; 0.7)$ be two generalized fuzzy numbers
Step 1 $min(0.35, 0.7) = 0.35$
Step 2 $\Re(\tilde{A}) = -0.175$ and $\Re(\tilde{B}) = -0.0875$. Since $\Re(\tilde{A}) \neq \Re(\tilde{B}) \Rightarrow \mathrm{RM}(\tilde{A}) = \Re(\tilde{A})$ and $\mathrm{RM}(\tilde{B}) = \Re(\tilde{B})$. Now $\mathrm{RM}(\tilde{A}) < \mathrm{RM}(\tilde{B}) \Rightarrow \tilde{A} \prec \tilde{B}$.

Example 10. Let $\tilde{A} = (0.2, 0.4, 0.6, 0.8; 0.35)$ and $\tilde{B} = (0.1, 0.2, 0.3, 0.4; 0.7)$ be two generalized trapezoidal fuzzy numbers then
Step 1 $min(0.35, 0.7) = 0.35$
Step 2 $\Re(\tilde{A}) = 0.175$ and $\Re(\tilde{B}) = 0.0875$. Since $\Re(\tilde{A}) \neq \Re(\tilde{B}) \Rightarrow \mathrm{RM}(\tilde{A}) = \Re(\tilde{A})$ and $\mathrm{RM}(\tilde{B}) = \Re(\tilde{B})$. Now $\mathrm{RM}(\tilde{A}) > \mathrm{RM}(\tilde{B}) \Rightarrow \tilde{A} \succ \tilde{B}$.

Example 11. Let $\tilde{A} = (1, 1, 1, 1; w_1)$ and $\tilde{B} = (1, 1, 1, 1; w_2)$ be two generalized trapezoidal fuzzy numbers.
Step 1 $min(w_1, w_2) = w$ (say)
Step 2 $\Re(\tilde{A}) = w$ and $\Re(\tilde{B}) = w$. Since $\Re(\tilde{A}) = \Re(\tilde{B}) \Rightarrow$ RM$(\tilde{A})$ = mode$(\tilde{A}) = w$ and RM$(\tilde{B})$ = mode$(\tilde{B}) = w$. Now RM$(\tilde{A}$ = RM$(\tilde{B}) \Rightarrow \tilde{A} \sim \tilde{B}$.

Example 12. Let $\tilde{A} = (-.4, -.2, -.1, .7; w_1)$ and $\tilde{B} = (-.4, -.2, -.1, .7; w_2)$, be two generalized fuzzy numbers then
Step 1 $min(w_1, w_2) = w$ (say)
Step 2 $\Re(\tilde{A}) = 0$ and $\Re(\tilde{B}) = 0$. Since $\Re(\tilde{A}) = \Re(\tilde{B}) \Rightarrow$ RM$(\tilde{A})$ = mode$(\tilde{A}) = 0$ and RM$(\tilde{B})$ = mode$(\tilde{B}) = 0$. Now RM$(\tilde{A})$ = RM$(\tilde{B}) \Rightarrow \tilde{A} \sim \tilde{B}$.

A. Validation of the results

In the above examples it can be easily check that

(i) $\tilde{A} \sim \tilde{B} \Rightarrow \tilde{A} \ominus \tilde{B} \sim \tilde{B} \ominus \tilde{B}$.
i.e., RM$((\tilde{A} \ominus \tilde{B}) \ominus (\tilde{B} \ominus \tilde{B}))$ = RM$((\tilde{A} \ominus \tilde{B}) \ominus (\tilde{B} \ominus \tilde{B}))$

(ii) $\tilde{A} \succ \tilde{B} \Rightarrow \tilde{A} \ominus \tilde{B} \succ \tilde{B} \ominus \tilde{B}$.
i.e., RM$((\tilde{A} \ominus \tilde{B}) \ominus (\tilde{B} \ominus \tilde{B})) \succ$ RM$((\tilde{A} \ominus \tilde{B}) \ominus (\tilde{B} \ominus \tilde{B}))$

(iii) $\tilde{A} \prec \tilde{B} \Rightarrow \tilde{A} \ominus \tilde{B} \prec \tilde{B} \ominus \tilde{B}$.
i.e., RM$((\tilde{A} \ominus \tilde{B}) \ominus (\tilde{B} \ominus \tilde{B})) \prec$ RM$((\tilde{A} \ominus \tilde{B}) \ominus (\tilde{B} \ominus \tilde{B}))$

B. Validation of the proposed ranking function

For the validation of the proposed ranking function, in Table 1, it is shown that proposed ranking function satisfies the all reasonable properties of fuzzy quantities proposed by Wang and Kerre [18].

TABLE I
FULFILMENT OF THE AXIOMS FOR THE ORDERING IN THE FIRST AND SECOND CLASS [18]

Index	A_1	A_2	A_3	A_4	A'_4	A_5	A_6	A'_6	A_7
Y_1	Y	Y	Y	Y	Y	Y	N	N	N
Y_2	Y	Y	Y	Y	Y	Y	Y	Y	N
Y_3	Y	Y	Y	N	N	Y	N	N	N
Y_4	Y	Y	Y	Y	Y	Y	N	N	N
C	Y	Y	Y	N	N	Y	N	N	N
FR	Y	Y	Y	Y	Y	Y	Y	Y	N
CL	Y	Y	Y	Y	Y	Y	Y	Y	N
LW^{λ}	Y	Y	Y	Y	Y	Y	Y	Y	N
CM_1^{λ}	Y	Y	Y	Y	Y	Y	Y	Y	N
CM_2^{λ}	Y	Y	Y	Y	Y	Y	Y	Y	N
K	Y	Y	Y	N	N	N	N	N	N
W	Y	Y	Y	Y	N	N	N	N	N
J^k	Y	Y	Y	Y	Y	N	N	N	N
CH^k	Y	Y	Y	Y	Y	N	N	N	N
KP^k	Y	Y	Y	Y	Y	N	N	N	N
Proposed Approach	Y	Y	Y	Y	Y	Y	Y	Y	N

VI. CONCLUSION

A novel ranking method for obtaining the right order of generalised trapezoidal fuzzy numbers is provided in this study, highlighting the inadequacies of Chen and Chen [16]. The suggested ranking function meets all of the acceptable features of fuzzy quantities established by Wang and Kerre [18] as shown.

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