

AN INNOVATIVE STUDY ON REVERSE DERIVATION IN NEARRINGS

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Abstract

An additive function f from a ring R to itself is known as a derivation if $f(ab)=f(a)b+af(b)$ for all a,b in R . An additive function f from a ring R to itself is known as a reverse derivation if $f(ab)=f(b)a+bf(a)$ for all a,b in R . An additive mapping $d:R$ to R is called a right generalized derivation if there exists a derivation $f:R$ to R satisfying $d(ab)=d(a)b+af(b)$ for all a,b in R . An additive mapping $d:R$ to R is called a left generalized derivation if there exists a derivation $f:R$ to R such that $d(ab)=f(a)b+ad(b)$ for all a,b in R . An additive map $d:R$ to R is called a generalized derivation if it is both right and left generalized derivations. This research article mainly explores on reverse derivation in nearrings. The prime objective of this talk is to present an elegant proof for "Let M be a prime nearring with a nonzero reverse derivation d . If $d(M)$ is a subset of Z , then $(M, +)$ is an abelian. Moreover, if M is a two torsion free, then M is commutative ring, If d acts as a homomorphism on M and anti-homomorphism on M then $d=0$ ". In order to present an innovative proof for this result five prerequisites in terms of lemmas are proposed. In this discourse we present some fundamental characteristic properties of reverse derivation in nearrings and some evolutions in the conjecture of reverse derivation in nearrings. The innovatory proofs proposed here ensures a way for young researchers to extend these ideas to Jordan generalized derivations in gamma nearrings.

Key words: Prime nearring, Commutative ring, Homomorphism, Anti-Homomorphism, Gamma nearrings

1. Introduction

The generalization of a ring is called a near ring. The nearring phenomenon now becomes a worldly-wise theory and has a large number of applications in various fields of Computer Science and Engineering. The deep-rooted theory of rings is termed as nearring theory. In 1987, H.E.Bell and G.Mason [1] invented the concept of derivation of nearrings. I.N.Herstein[2] discovered the idea of reverse derivation in prime rings. L.Madhuchelvi.et.al.,[3], in 2020, studied some concepts concerning with semi prime nearrings with generalized reverse derivations. C.Jayasubbareddy.,et.al, in 2015, investigated generalised reverse derivations on prime near rings. In 2015, A.M.Ibrahim discovered some results dealing with reverse derivations on prime gamma near rings.

An additive mapping $d: M \rightarrow M$ is said to be a reverse derivation if $d(pq) = d(q)p + qd(p)$, for all $p, q \in M$.

Here, M is a right nearring with multiplicative center Z .

Lemma (i): If M is a nearring and d is a reverse derivation of M , then $a(d(q)p + qd(p)) = a d(q)p + a q d(p)$, for all $a, p, q \in N$.

Lemma (ii): Let M be a prime nearring and d is a non-zero reverse derivation of M and $a \in M$. If $ad(M) = 0$ ($d(M)a = 0$), then $a = 0$.

Proof: Assume that $ad(M) = 0$.

For arbitrary $p, q \in M$. We have

$$0 = ad(pq) = ad(q)p + aqd(p)$$

By the supposition, $apd(q) = 0, \forall p, q \in N$.

Since M is prime nearring and d is non-zero, one can get a as zero

In the similar manner argument is true if $d(M)a = 0$.

Lemma (iii): Let M be a two torsion free prime nearring and d be a reverse derivation of M . If $d^2 = 0$, then $d = 0$.

Proof: For arbitrary $p, q \in N$, we have

$$\begin{aligned} 0 &= d^2(pq) = d(d(pq)) = d(d(q)p + qd(p)) \\ &= d(d(q)p) + d(qd(p)) \end{aligned}$$

$$=d(p)d(q)+pd^2(q)+d^2(p)q+d(p)d(q)$$

That implies $pd^2(q)+d^2(p)q+2d(p)d(q)=0$

That implies $2d(p)d(q)=0$ (since $d^2=0$)

That implies $d(p)d(q)=0$

Since M is a two torsion free nearring, then

$d(p)d(M)=0$, for all $p,q \in M$.

From Lemma 2, we get $d=0$.

Lemma (iv): Let d is a reverse derivation of M then $(d(q)p+qd(p))a$ is $d(q)pa+qd(p)a$, $\forall p,q \in M$.

Proof: For $a,p,q \in M$, then

$$\begin{aligned} d(a(pq)) &= d(pq)a + pqd(a) \\ &= [d(q)p + qd(p)]a + pqd(a) \end{aligned}$$

On the other hand,

$$\begin{aligned} d((ap)q) &= d(q)ap + qd(ap) \\ &= d(q)ap + q[d(p)a + pd(a)] \\ &= d(q)ap + qd(p)a + qp d(a) \end{aligned}$$

With the above two values of $d(apq)$, one can get that

$$(d(q)p+qd(p))a = d(q)pa+qd(p)a, \forall p,q \in M.$$

Lemma (v): Let M be a prime nearring, d be a non zero reverse derivation of M and $a \in M$. Then

(i) If $ad(M)=0$ then $a=0$.

(ii) If $d(M)a=0$ then $a=0$.

Proof: i) $\forall p,q \in M$, then

$$0 = ad(pq) = ad(q)p + aqd(p) \text{ and so } aMd(M) = 0.$$

Since M is prime nearring and d is nonzero, one can see a as zero.

ii) $\forall p,q \in M$, we get

$$0 = d(pq)a = d(q)pa + qd(p)a \text{ and } Md(M)a = 0$$

Since M is prime nearring and d is non zero, one can see a equals zero

2. Main Result (i)

Let M be a prime nearring with a non zero reverse derivation d . If $d(M)$ is contained in Z , then $(M,+)$ is an abelian. Moreover, if M is two torsion free, then M is commutative ring.

Proof: It is obvious from the above lemmas that

$d(a) \in Z \setminus \{0\}$, $d(a)+d(a) \in Z \setminus \{0\}$, then we have

$$d(a)p+d(a)q=d(a)q+d(a)p$$

$$d(a)(p,q)=0 \quad \forall p,q \in M.$$

Since $d(a) \in Z \setminus \{0\}$ and M is prime nearringone can get $(p,q)=0 \quad \forall p,q \in M$. Thus $(M,+)$ is an abelian

Now if $p,q,r \in N$.

$r d(pq)=d(pq)r$, by the lemma, one can get

$$r(d(q)p+qd(p))=(d(q)p+qd(p))r$$

$$r d(q)p+rqd(p)=d(q)pr+qd(p)r$$

By $d(M) \subset Z$ and $(M,+)$ is an abelian, one can get

$$d(q)pr-rd(q)p=rqd(p)-qd(p)r$$

and so $d(q)pr-rd(q)p=d(p)[q,r]$, $\forall p,q,r \in M$.

$$[p,r]d(q)=[q,r]d(p) \quad (1)$$

$$[p,r]d(q)=[d(q),r]d(p)$$

$$[p,r]d(q)=0$$

Replaced (q) for q in (1) and using $d(M) \subset Z$, one can see

$$[r,p]d(d(q))=0, \quad \forall p,q,r \in M.$$

Since $d(q) \in Z$ and so $d(d(q)) \in Z$, we have

$$d(d(q))=0, \quad \forall q \in M \text{ or } M \text{ is commutative ring.}$$

Let us assume $d(d(q))=0, \quad \forall q \in M$, then

$$0=d(d(pq))=d(d(q)p+qd(p))$$

$$=d^2(q)p+d(q)d(p)+qd^2(p)=0, \quad \forall p,q,r \in M.$$

Substitute q by qr here and applying it one can obtain

$$0=d^2(qr)p+d(qr)d(p)+qrd^2(p)$$

$$=d(d(qr))p+(d(r)q+rd(q))d(p)+qrd^2(p)$$

$$=d(d(r)q+rd(q))p+d(r)qd(p)+rd(q)d(p)+qrd^2(p)$$

$$=d^2(r)qp+rd^2(q)p+d(r)qd(p)+rd(q)d(p)+qrd^2(p)$$

$$=d(r)qp+d(p)+rd^2(q)p+d(r)qd(p)+rd(q)d(p)+qrd^2(p)$$

$$=[d^2(q)p+d(q)d(p)+qd^2(p)]r+2d(r)qd(p)$$

$$=2d(p)qd(r)$$

Since M is a two torsion free nearring, one can have
 $d(M)Md(M)=0$.

In this, one can get $d=0$. It contradicts d is non zero

So, we must have N is commutative.

3. Mainresult (ii)

If M is a prime nearring and d is a reverse derivation of M and d behaves as a homomorphism on M , then $d=0$.

Proof: If d works as a homomorphism on M then

$$d(pq)=d(p)d(q)=d(q)p+qd(p) \quad \forall p, q \in M. \quad (2)$$

Replacing pq by p in (2), one can see

$$d(q)p+qd(p)=d(p)d(q)$$

$$d(q)p+qd(p)=(d(q)p+qd(p))d(q)$$

$$d(q)p+qd(p)=d(q)pd(q)+qd(p)d(q)$$

$$d(q)p+qd(p)=d(q)pd(q)+qd(pq)$$

$$0=d(q)pd(q) \quad (\text{since } d(q)p=d(M)a=0, \text{ by Lemma 2}).$$

Since M is a prime nearring, one can get $d=0$.

4. Main Result (iii)

Let M be prime nearring and d be a reverse derivation of M associated with d . If d works as a anti-homomorphism on M , then $d=0$.

Proof: It is obvious that

$$d(pq)=d(q)d(p)$$

$$d(q)d(p)=d(q)p+qd(p), \quad \forall p, q \in M. \quad (3)$$

Substitute p by pq in (3), then

$$\begin{aligned} d(q)pq+qd(pq) &= d(q)d(pq) \\ &= d(q)(d(q)p+qd(p)) \\ &= d(q)d(q)p+d(q)qd(p) \\ &= d(q)pd(q)+qd(pq) \end{aligned}$$

$$\text{And so, } pq \, d(q)=d(q)pd(q), \quad \forall p, q \in M. \quad (4)$$

If we take tp instead of p in (4), we have

$$tpq \, d(q)=d(q)tpd(q)$$

$$t(d(q)pd(q))=d(q)tpd(q)$$

$$td(q)pd(q)=d(q)tpd(q)$$

$$[t,d(q)]pd(q)=0$$

And so $[t,d(q)]pd(q)=0, \forall p,q,t \in M$.

As M is a prime nearring, one can arrive at $d(q) \in Z$ or $d(q)=0, \forall q \in M$.

Let's define $C=\{p \in M/d(p)=0\}$, $D=\{p \in M/d(p) \in Z\}$.

It is obvious that each of C and D is additive sub group of M such that $M=C \cup D$.

However a group cannot be the set theoretic union of two proper sub groups.

Hence $M=C$ or $M=D$.

In the latter case, $d(M) \subset Z$, which influences that d serves as anti-homomorphism on M and so $d=0$, if $M=A$ then $d=0$.

5.Conclusions and Future Research

In order to present innovative proofs for three main results, five prerequisites in terms of lemmas are proposed. Moreover novel proofs for these five lemmas have been proposed. In this discourse we crave to present some fundamental characteristic properties of reverse derivation in nearrings and some evolutions in the conjecture of reverse derivation in nearrings. The innovatory proofs proposed here ensures a way for young researchers .In the context of future research we can establish one result:

” Let N be a two torsion free Γ -nearring. If N has two elements p and q so that for any $\alpha \in \Gamma, r, \gamma \in \Gamma, [p,q]_{\alpha} r = 0$ or $[p,q]_{\alpha} r \gamma \gamma r = 0$ implies $r=0$ for all $y \in N, \gamma \in \Gamma$ then every Jordan generalized (σ, τ) derivation on N is a generalized (σ, τ) derivation”.

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