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# OPTIMIZATION OF FUZZY INVENTORY MODEL WITHOUT SHORTAGES

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***Abstract:The goal of this undertaking is to discover the enhancement of EOQ model under uncertainty with no reasonable deficiencies for Pentagonal Fuzzy Numbers . Here the parameters like purchasing cost, storing cost and yearly interest are thought to be Pentagonal Fuzzy Numbers . The PFN can be defuzzify by utilizing the Graded Mean Integration Representation technique to get the advancement in the least difficult manner. The fuzzy optimal solution of the stock model is arrived by utilizing the expansion of Lagrangean strategy . To make this strategy progressively reasonable, numerical instances of both PFN is shown by utilizing the fuzzy values.***

***Keywords:Defuzzification, Economic order quantity, Lagrangean method, fuzzy numbers.***

## Introduction

The utmost extensively recycled inventory model is the Economic Order Quantity (EOQ) model in which the sub sequential exploitation are classified as supply and demand. EOQ is one of the most established equation in inventory management. It was first created by Ford W. Harris in 1913. EOQ is the span of request which limits the absolute yearly expenses of conveying stock and cost of requesting. Consequently EOQ show is valuable, all things considered, circumstance. Pursued by Harris et al, Wilson indicated enthusiasm for creating EOQ demonstrate in scholastics and businesses. Hadely et al dissected many stock models.

The fuzzy set hypothesis presented by LoftiZadeh in 1965 as an augmentation of traditional documentation set and it is adequate. Fuzzy set hypothesis is an expansion of customary set speculation where components have level of participation. Human reasoning and thinking regularly included fuzzy data.Among the different kinds of fuzzy sets, distinctive sorts of fuzzy sets are characterized so as to clear the ambiguity of current issues. Membership function of these sets, which have the structre A:R→[0,1] and it has a quantitative importance and seen as fuzzy numbers.

## Pentagonal Fuzzy Number

A Pentagonal fuzzy number of a fuzzy set $\mathfrak{P}(\widetilde{U})$ is defined as
$\widetilde{U} = (u_1, u_2, u_3, u_4, u_5)$ where $u_1, u_2, u_3, u_4, u_5$ are real numbers and its membership function is given by

$$\chi_{\mathfrak{P}}(\mathfrak{x}) = \begin{cases} 0, & for\ \mathfrak{x} < u_1 \\ \dfrac{(\mathfrak{x} - u_1)}{(u_2 - u_1)}, & for\ u_1 \le \mathfrak{x} \le u_2 \\ \dfrac{(\mathfrak{x} - u_2)}{(\mathfrak{x} - u_3)}, & for\ u_2 \le \mathfrak{x} \le u_3 \\ 1, & for\ \mathfrak{x} = u_3 \\ \dfrac{(u_4 - \mathfrak{x})}{(u_4 - u_3)}, & for\ u_3 \le \mathfrak{x} \le u_4 \\ \dfrac{u_5 - \mathfrak{x}}{u_5 - u_4}, & for\ u_4 \le \mathfrak{x} \le u_5 \\ 0, & for\ \mathfrak{x} > u_5 \end{cases}$$

**Graded mean integration representation formula for PFN:**

The graded mean integration representation of $\mathfrak{P}(\widetilde{U})$ with grade $e_u$, where

$$\mathfrak{P}(\widetilde{U}) = \frac{u_1 + 2u_2 + 2u_3 + 2u_4 + u_5}{8}$$

**The fuzzy arithmetic operations under function principle:**

.Suppose$\widetilde{U} = (u_1, u_2, u_3, u_4, u_5)$ and $\tilde{V} = (v_1, v_2, v_3, v_4, v_5)$ are two pentagonal fuzzy numbers. Then,

1. The addition of $\widetilde{U}$ and $\tilde{V}$ is
$$\widetilde{U} \oplus \tilde{V} = (u_1 + v_1, u_2 + v_2, u_3 + v_3, u_4 + v_4, u_5 + v_5),$$
where$u_1, u_2, u_3, u_4, u_5, v_1, v_2, v_3, v_4$ and $v_5$ are any real numbers.
2. The multiplication of $\widetilde{U}$ and $\tilde{V}$ is
$$\widetilde{U} \otimes \tilde{V} = (w_1, w_2, w_3, w_4, w_5),$$
where$u_1, u_2, u_3, u_4, u_5, v_1, v_2, v_3, v_4$ and $v_5$ are all nonzero positive real numbers, then
$$\widetilde{U} \otimes \tilde{V} = (u_1v_1, u_2v_2, u_3v_3, u_4v_4, u_5v_5)$$
3. $-\tilde{V} = (-v_1, -v_2, -v_3, -v_4, -v_5)$, then the subtraction of $\widetilde{U}$ and $\tilde{V}$ is
$$\widetilde{U} \ominus \tilde{V} = (u_1 - v_5, u_2 - v_4, u_3 - v_3, u_4 - v_2, u_5 - v_1),$$
where$u_1, u_2, u_3, u_4, u_5, v_1, v_2, v_3, v_4$ and $v_5$ are any real numbers.
4. $\frac{1}{\tilde{V}} = \tilde{V}^{-1} = \left(\frac{1}{v_5}, \frac{1}{v_4}, \frac{1}{v_3}, \frac{1}{v_2}, \frac{1}{v_1}\right)$, where $v_1, v_2, v_3, v_4$ and $v_5$ are all positive real numbers.
If $u_1, u_2, u_3, u_4, u_5, v_1, v_2, v_3, v_4$ and $v_5$ are all nonzero positive real numbers, then the division of $\widetilde{U}$ and $\tilde{V}$ is $\widetilde{U} \oslash \tilde{V} = \left(\frac{u_1}{v_5}, \frac{u_2}{v_4}, \frac{u_3}{v_3}, \frac{u_4}{v_2}, \frac{u_5}{v_1}\right)$
5. Let $\alpha \epsilon R$. Then
   i. $\alpha \ge 0,\ \alpha \otimes \widetilde{U} = (\alpha u_1, \alpha u_2, \alpha u_3, \alpha u_4, \alpha u_5),$
   ii. $\alpha < 0,\ \alpha \otimes \widetilde{U} = (\alpha u_5, \alpha u_4, \alpha u_3, \alpha u_2, \alpha u_1).$

**Extension of Lagrangean method:**

Suppose that the problem is given by

Minimize $y = f(x)$, Subject to $g_i(x) \ge 0, i = 1,2, \ldots . m$.

The non negativity constraints$x \ge 0$, if any, are covered in the m constraints.

Step 1: Solve the unconstrained problem.

Minimize $y = f(x)$

If the subsequent optimum fulfills every one of the constraints, stop since all imperatives are redundant. Otherwise, set $k = 1$ and move on to step 2.

Step 2: Initiate any $k$ constraints and then convert them into equality and optimize $f(x)$ subject to the $k$ dynamic requirements by the Lagrangean method. If the resulting solution is feasible with respective to the remaining constraints, stop because it is a nearby ideal.

Something else, initiate another arrangement of $k$ requirements and rehash the progression. On the off chance that all arrangements of dynamic requirements taken $k$ at once are considered without experiencing on attainable arrangement, go to step 3.

Step 3: If $k = m$, stop; no feasible solution exists. Otherwise, set $k = k + 1$ and go to step 2.

**Representations Required:**

| | |
|---|---|
| $Y_D$ | Yearly demand of crisp value |
| $\widetilde{Y_D}$ | Yearly demand of fuzzy value |
| $B_C$ | Buying cost of crisp value |
| $\widetilde{B_C}$ | Buying cost of fuzzy value |
| $S_C$ | Storage cost of crisp value |
| $\widetilde{S_C}$ | Storage cost of fuzzy value |
| $L_T$ | Longevity of time period |
| $WA$ | Whole amount of crisp value |
| $\widetilde{WA}$ | Whole amount of fuzzy value |
| $E_{OQ}$ | Economic order quantity of crisp value |
| $\widetilde{E_{OQ}}$ | Economic order quantity of fuzzy value |

**Assumptions:**

- Shortages are not allowed
- Demand will be constant
- Lead time always zero
- Cost of ordering and holding cost are always known

**The fuzzy inventory model for crisp Economic Order Quantity:**

The total fuzzy inventory cost can be expressed by using function principle for crisp quantity $\widetilde{WA} = Y_D \otimes B_C \oslash E_{OQ} \oplus E_{OQ} \otimes S_C \otimes L_T \oslash 2$

In this method yearly demand, longevity of time period and costs are represented by $\widetilde{Y_D} = (y_{d_1}, y_{d_2}, y_{d_3}, y_{d_4}, y_{d_5})$,$\widetilde{B_C} = (b_{c_1}, b_{c_2}, b_{c_3}, b_{c_4}, b_{c_5})$,$\widetilde{S_C} = (s_{c_1}, s_{c_2}, s_{c_3}, s_{c_4}, s_{c_5})$, are non-negative pentagonal fuzzy numbers. Then we solve the optimal production quantity of formula (1) as the following steps. The fuzzy total inventory cost,

$$\widetilde{WA} = \left(\frac{y_{d_1} b_{c_1}}{E_{OQ}} + \frac{E_{OQ} s_{c_1} L_T}{2}, \frac{y_{d_2} b_{c_2}}{E_{OQ}} + \frac{E_{OQ} s_{c_2} L_T}{2}, \frac{y_{d_3} b_{c_3}}{E_{OQ}} + \frac{E_{OQ} s_{c_3} L_T}{2}, \frac{y_{d_4} b_{c_4}}{E_{OQ}} + \frac{E_{OQ} s_{c_4} L_T}{2}, \frac{y_{d_5} b_{c_5}}{E_{OQ}} + \frac{E_{OQ} s_{c_5} L_T}{2}\right) \quad \ldots\ldots\ldots (1)$$

Next, we defuzzify the fuzzy total inventory cost by GMIR formula for pentagonal fuzzy number. The result is

$$\mathfrak{P}(\widetilde{WA}) = \frac{1}{8}\left(\frac{y_{d_1} b_{c_1}}{E_{OQ}} + \frac{E_{OQ} s_{c_1} L_T}{2} + \frac{2y_{d_2} b_{c_2}}{E_{OQ}} + \frac{2E_{OQ} s_{c_2} L_T}{2} + \frac{2y_{d_3} b_{c_3}}{E_{OQ}} + \frac{2E_{OQ} s_{c_3} L_T}{2} + \frac{2y_{d_4} b_{c_4}}{E_{OQ}} + \frac{2E_{OQ} s_{c_4} L_T}{2} + \frac{y_{d_5} b_{c_5}}{E_{OQ}} + \frac{E_{OQ} s_{c_5} L_T}{2}\right) \quad \ldots\ldots\ldots (2)$$

Differentiate (2) partially with respect to $E_{OQ}$ and equating to zero, in order to find the minimum optimum cost for this unconstrained problem.We get,

$$E_{OQ}^{*}=\sqrt{\frac{2\left(y_{d_1}b_{c_1}+2y_{d_2}b_{c_2}+2y_{d_3}b_{c_3}+2y_{d_4}b_{c_4}+y_{d_5}b_{c_5}\right)}{\left(s_{c_1}L_T+2s_{c_2}L_T+2s_{c_3}L_T+2s_{c_4}L_T+s_{c_5}L_T\right)}}$$

**The fuzzy inventory model for fuzzy crisp economic order quantity**

Suppose fuzzy quantity $\widetilde{E_{OQ}}$ be a pentagonal fuzzy number,

$\widetilde{E_{OQ}}=\left(e_{oq_1},e_{oq_2},e_{oq_3},e_{oq_4},e_{oq_5}\right)$ with

$0<e_{oq_1}\le e_{oq_2}\le e_{oq_3}\le e_{oq_4}\le e_{oq_5}$ ……………. (*)

Then the fuzzy total inventory cost for the fuzzy quantity is

$$\widetilde{WA}=\left(\frac{y_{d_1}b_{c_1}}{e_{oq_5}}+\frac{e_{oq_1}s_{c_1}L_T}{2},\frac{y_{d_2}b_{c_2}}{e_{oq_4}}+\frac{e_{oq_2}s_{c_2}L_T}{2},\frac{y_{d_3}b_{c_3}}{e_{oq_3}}+\frac{e_{oq_3}s_{c_3}L_T}{2},\frac{y_{d_4}b_{c_4}}{e_{oq_2}}+\frac{e_{oq_4}s_{c_4}L_T}{2},\frac{y_{d_5}b_{c_5}}{e_{oq_1}}+\frac{e_{oq_5}s_{c_5}L_T}{2}\right)\ \dots\dots\dots\dots(3)$$

By using GMIR formula we get,

$$\mathfrak{P}\left(\widetilde{WA}\right)=\frac{1}{8}\left(\frac{y_{d_1}b_{c_1}}{e_{oq_5}}+\frac{e_{oq_1}s_{c_1}L_T}{2}+\frac{2y_{d_2}b_{c_2}}{e_{oq_4}}+\frac{2e_{oq_2}s_{c_2}L_T}{2}+\frac{2y_{d_3}b_{c_3}}{e_{oq_3}}+\frac{2e_{oq_3}s_{c_3}L_T}{2}+\frac{2y_{d_4}b_{c_4}}{e_{oq_2}}+\frac{2e_{oq_4}s_{c_4}L_T}{2}+\frac{y_{d_5}b_{c_5}}{e_{oq_1}}+\frac{e_{oq_5}s_{c_5}L_T}{2}\right)\ \dots\dots\dots\dots(4)$$

Step 1: Solve the unconstrained problem

Differentiate (4) partially with respect to $e_{oq_1},e_{oq_2},e_{oq_3},e_{oq_4},e_{oq_5}$ and equating to zero to find the minimum $\mathfrak{P}\left(\widetilde{WA}\right)$ we obtain, $e_{oq_1}=\sqrt{\frac{2y_{d_5}b_{c_5}}{s_{c_1}L_T}}$ Similarly we get,

$e_{oq_2}=\sqrt{\frac{2y_{d_4}b_{c_4}}{s_{c_2}L_T}},e_{oq_3}=\sqrt{\frac{2y_{d_3}b_{c_3}}{s_{c_3}L_T}},e_{oq_4}=\sqrt{\frac{2y_{d_2}b_{c_2}}{s_{c_4}L_T}},e_{oq_5}=\sqrt{\frac{2y_{d_1}b_{c_1}}{s_{c_5}L_T}}$

But we have $e_{oq_1}>e_{oq_2}>e_{oq_3}>e_{oq_4}\ge e_{oq_5}$, it does't satisfy the constraint $0<e_{oq_1}\le e_{oq_2}\le e_{oq_3}\le e_{oq_4}\le e_{oq_5}$, therefore set $k=1$ and go to step 2.

Step 2: Convert the inequality constraint $e_{oq_2}-e_{oq_1}\ge 0$ into an equality constraint $e_{oq_2}-e_{oq_1}=0$ and minimize $\mathfrak{P}\left(\widetilde{WA}\right)$ with respect to $e_{oq_2}-e_{oq_1}=0$ by the Lagrangean method. We have Lagrangean function as,

$$L\left(e_{oq_1},e_{oq_2},e_{oq_3},e_{oq_4},e_{oq_5},\Upsilon_1\right)=\mathfrak{P}\left(\widetilde{WA}\right)-\Upsilon_1\left(e_{oq_2}-e_{oq_1}\right)$$

$$L\left(e_{oq_1},e_{oq_2},e_{oq_3},e_{oq_4},e_{oq_5},\Upsilon_1\right)=\frac{1}{8}\left(\frac{y_{d_1}b_{c_1}}{e_{oq_5}}+\frac{e_{oq_1}s_{c_1}L_T}{2}+\frac{2y_{d_2}b_{c_2}}{e_{oq_4}}+\frac{2e_{oq_2}s_{c_2}L_T}{2}+\frac{2y_{d_3}b_{c_3}}{e_{oq_3}}+\frac{2e_{oq_3}s_{c_3}L_T}{2}+\frac{2y_{d_4}b_{c_4}}{e_{oq_2}}+\frac{2e_{oq_4}s_{c_4}L_T}{2}+\frac{y_{d_5}b_{c_5}}{e_{oq_1}}+\frac{e_{oq_5}s_{c_5}L_T}{2}\right)-\Upsilon_1\left(e_{oq_2}-e_{oq_1}\right)\ \dots\dots\dots(5)$$

Differentiate equation (5) partially with respect to $e_{oq_1},e_{oq_2},e_{oq_3},e_{oq_4},e_{oq_5},\Upsilon_1$ and equating them to zero, we get $e_{oq_1}=e_{oq_2}=\sqrt{\frac{2\left(y_{d_5}b_{c_5}+2y_{d_4}b_{c_4}\right)}{\left(s_{c_1}L_T+2s_{c_2}L_T\right)}}$

$$e_{oq_3} = \sqrt{\frac{2y_{d_3}b_{c_3}}{s_{c_3}L_T}}, e_{oq_4} = \sqrt{\frac{2y_{d_2}b_{c_2}}{s_{c_4}L_T}}, e_{oq_5} = \sqrt{\frac{y_{d_1}b_{c_1}}{s_{c_5}L_T}}$$

In the above equations show that $e_{oq_1} = e_{oq_2} > e_{oq_3} > e_{oq_4} > e_{oq_5}$, it does not satisfy the given condition (*). Now, fix $\hbar = 2$ and move to next step.

Step 3: Now, convert the inequality constraints $e_{oq_2} - e_{oq_1} \geq 0$ and $e_{oq_3} - e_{oq_2} \geq 0$ into equality constraints, $e_{oq_2} - e_{oq_1} = 0$ and $e_{oq_3} - e_{oq_2} = 0$. Minimize $\mathfrak{P}(\widetilde{WA})$ with respect to $e_{oq_1}, e_{oq_2} \& e_{oq_3}$ by Lagrangean method.Consider the Lagrangean function as,

$L(e_{oq_1}, e_{oq_2}, e_{oq_3}, e_{oq_4}, e_{oq_5}, \Upsilon_1, \Upsilon_2) = \mathfrak{P}(\widetilde{WA}) - \Upsilon_1(e_{oq_2} - e_{oq_1}) - \Upsilon_2(e_{oq_3} - e_{oq_2})$….(6)

Differentiate (6) partially w.r.t $e_{oq_1}, e_{oq_2}, e_{oq_3}, e_{oq_4}, e_{oq_5}, \Upsilon_1, \Upsilon_2$ and equating them to zero, we get $e_{oq_1} = e_{oq_2} = e_{oq_3} = \sqrt{\frac{2(y_{d_5}b_{c_5}+2y_{d_4}b_{c_4}+2y_{d_3}b_{c_3})}{s_{c_1}L_T+2s_{c_2}L_T+2s_{c_3}L_T}}$ $e_{oq_4} = \sqrt{\frac{2y_{d_2}b_{c_2}}{s_{c_4}L_T}}, e_{oq_5} = \sqrt{\frac{2y_{d_1}b_{c_1}}{s_{c_5}L_T}}$

Therefore, it does not satisfy the condition (*). Therefore, the optimum solution is not obtained. Now, fix $\hbar = 3$ and move to next step.

Step 4: Convert the inequality constraints into equality constraints.

i.e., $e_{oq_2} - e_{oq_1} \geq 0$ , $e_{oq_3} - e_{oq_2} \geq 0$ and $e_{oq_4} - e_{oq_3} \geq 0$ into equality constraints $e_{oq_2} - e_{oq_1} = 0$ , $e_{oq_3} - e_{oq_2} = 0$ and $e_{oq_4} - e_{oq_3} = 0$ to find the minimization by using Lagrangean method.

Therefore the Lagrangean function is $L(e_{oq_1}, e_{oq_2}, e_{oq_3}, e_{oq_4}, e_{oq_5}, \Upsilon_1, \Upsilon_2, \Upsilon_3) = \mathfrak{P}(\widetilde{WA}) - \Upsilon_1(e_{oq_2} - e_{oq_1}) - \Upsilon_2(e_{oq_3} - e_{oq_2}) - \Upsilon_3(e_{oq_4} - e_{oq_3})$

$$L(e_{oq_1}, e_{oq_2}, e_{oq_3}, e_{oq_4}, e_{oq_5}, \Upsilon_1, \Upsilon_2, \Upsilon_3) = \frac{1}{8}\left(\frac{y_{d_1}b_{c_1}}{e_{oq_5}} + \frac{e_{oq_1}s_{c_1}L_T}{2} + \frac{2y_{d_2}b_{c_2}}{e_{oq_4}} + \frac{2e_{oq_2}s_{c_2}L_T}{2} + \frac{2y_{d_3}b_{c_3}}{e_{oq_3}} + \frac{2e_{oq_3}s_{c_3}L_T}{2} + \frac{2y_{d_4}b_{c_4}}{e_{oq_2}} + \frac{2e_{oq_4}s_{c_4}L_T}{2} + \frac{y_{d_5}b_{c_5}}{e_{oq_1}} + \frac{e_{oq_5}s_{c_5}L_T}{2}\right) - \Upsilon_1(e_{oq_2} - e_{oq_1}) - \Upsilon_2(e_{oq_3} - e_{oq_2}) - \Upsilon_3(e_{oq_4} - e_{oq_3}) \quad \ldots\ldots\ldots(7)$$

Differentiate equation (7) partially w.r.t $e_{oq_1}, e_{oq_2}, e_{oq_3}, e_{oq_4}, e_{oq_5}, \Upsilon_1, \Upsilon_2, \Upsilon_3$ and equating them to zero, we get $e_{oq_1} = e_{oq_2} = e_{oq_3} = e_{oq_4} = \sqrt{\frac{2(y_{d_5}b_{c_5}+2y_{d_4}b_{c_4}+2y_{d_3}b_{c_3}+2y_{d_2}b_{c_2})}{s_{c_1}L_T+2s_{c_2}L_T+2s_{c_3}L_T+2s_{c_4}L_T}}$

$$e_{oq_5} = \sqrt{\frac{2y_{d_1}b_{c_1}}{s_{c_5}L_T}}$$

The above equations show that $e_{oq_1} = e_{oq_2} = e_{oq_3} = e_{oq_4} > e_{oq_5}$, it does not satisfy the given condition (*). Now, fix $\hbar = 4$ and move to next step.

Step 5: Now, convert the inequality constraints $e_{oq_2} - e_{oq_1} \geq 0$, $e_{oq_3} - e_{oq_2} \geq 0$, $e_{oq_4} - e_{oq_3} \geq 0$ and $e_{oq_5} - e_{oq_4} \geq 0$ into equality constraints $e_{oq_2} - e_{oq_1} = 0$ , $e_{oq_3} - e_{oq_2} = 0$, $e_{oq_4} - e_{oq_3} = 0$ and $e_{oq_5} - e_{oq_4} = 0$ to find the minimization by using Lagrangean method. Consider the Lagrangean function as,

$$L(e_{oq_1}, e_{oq_2}, e_{oq_3}, e_{oq_4}, e_{oq_5}, \Upsilon_1, \Upsilon_2, \Upsilon_3, \Upsilon_4) = \mathfrak{P}(\widetilde{WA}) - \Upsilon_1(e_{oq_2} - e_{oq_1}) - \Upsilon_2(e_{oq_3} - e_{oq_2}) - \Upsilon_3(e_{oq_4} - e_{oq_3}) - \Upsilon_4(e_{oq_5} - e_{oq_4})$$

$$L(e_{oq_1}, e_{oq_2}, e_{oq_3}, e_{oq_4}, e_{oq_5}, \Upsilon_1, \Upsilon_2, \Upsilon_3, \Upsilon_4) = \frac{1}{8}\left(\frac{y_{d_1} b_{c_1}}{e_{oq_5}} + \frac{e_{oq_1} s_{c_1} L_T}{2} + \frac{2y_{d_2} b_{c_2}}{e_{oq_4}} + \frac{2e_{oq_2} s_{c_2} L_T}{2} + \frac{2y_{d_3} b_{c_3}}{e_{oq_3}} + \frac{2e_{oq_3} s_{c_3} L_T}{2} + \frac{2y_{d_4} b_{c_4}}{e_{oq_2}} + \frac{2e_{oq_4} s_{c_4} L_T}{2} + \frac{y_{d_5} b_{c_5}}{e_{oq_1}} + \frac{e_{oq_5} s_{c_5} L_T}{2}\right) - \Upsilon_1(e_{oq_2} - e_{oq_1}) - \Upsilon_2(e_{oq_3} - e_{oq_2}) - \Upsilon_3(e_{oq_4} - e_{oq_3}) - \Upsilon_4(e_{oq_5} - e_{oq_4}) \ldots\ldots\ldots (8)$$

Differentiate equation (8) partially w.r.t $e_{oq_1}, e_{oq_2}, e_{oq_3}, e_{oq_4}, e_{oq_5}, \Upsilon_1, \Upsilon_2, \Upsilon_3, \Upsilon_4$ and equating them to zero, we get

$$e_{oq_1} = e_{oq_2} = e_{oq_3} = e_{oq_4} = e_{oq_5} = \sqrt{\frac{2(y_{d_5} b_{c_5} + 2y_{d_4} b_{c_4} + 2y_{d_3} b_{c_3} + 2y_{d_2} b_{c_2} + y_{d_1} b_{c_1})}{s_{c_1} L_T + 2s_{c_2} L_T + 2s_{c_3} L_T + 2s_{c_4} L_T + s_{c_5} L_T}}$$

The above solution $E_{OQ} = e_{oq_1}, e_{oq_2}, e_{oq_3}, e_{oq_4}, e_{oq_5}$ satisfies all the inequality constraints. Therefore it is an optimum solution of the inventory model with fuzzy according to Extension of the Lagrangean method. Then the optimal fuzzy quantity is

$\widetilde{E_{OQ}} = e_{oq}, e_{oq}, e_{oq}, e_{oq}, e_{oq}$, where $\widetilde{E_{OQ}} = \sqrt{\frac{2(y_{d_5} b_{c_5} + 2y_{d_4} b_{c_4} + 2y_{d_3} b_{c_3} + 2y_{d_2} b_{c_2} + y_{d_1} b_{c_1})}{s_{c_1} L_T + 2s_{c_2} L_T + 2s_{c_3} L_T + 2s_{c_4} L_T + s_{c_5} L_T}}$

**Numerical Example**

Prolgae Spirulina Supplies Pvt Ltd is the Food producer company that produces in units. Retailers requires 2000 units of Spirulina per year. It has been assessed the expense of putting in a request is Rs.50 and the cost of storing the stock is Rs.5. The longevity of time period for the production is 2 months. Find the optimal solution and the total stock expense that was calculated by the seller.

**Crisp Value:** $E_{OQ} = 141\ units$, Whole amount of the stock $(WA) = \text{Rs.} 1414$

**Fuzzy Value :**

Yearly demand of fuzzy value

$\widetilde{Y_D} = (y_{d_1}, y_{d_2}, y_{d_3}, y_{d_4}, y_{d_5}) = (1700, 1900, 2050, 2100, 2200)$

Buying cost of fuzzy value

$\widetilde{B_C} = (b_{c_1}, b_{c_2}, b_{c_3}, b_{c_4}, b_{c_5}) = (35, 40, 51, 60, 63)$

Storage cost of fuzzy value $\widetilde{S_C} = (s_{c_1}, s_{c_2}, s_{c_3}, s_{c_4}, s_{c_5}) = (1, 3, 4, 8, 9)$

Longevity of time period $L_T = 2$ months

$$\widetilde{E_{OQ}} = (133, 133, 133, 133, 133)\ units$$

The Whole amount of the fuzzy is given by, $\mathfrak{P}(\widetilde{\mathrm{WA}}) = Rs. 1427$

**Sensitivity Analysis Table 1:**

| S.No | Change In Yearly Demand $(Y_D)$ | Economic Order Quantity $(E_{OQ})$ |
|---|---|---|
| 1 | 2000 | 131 |
| 2 | 2500 | 158 |
| 3 | 3000 | 173 |

| | | |
|---|---|---|
| 4 | 3500 | 187 |
| 5 | 4000 | 200 |
| 6 | 4500 | 212 |
| 7 | 5000 | 224 |

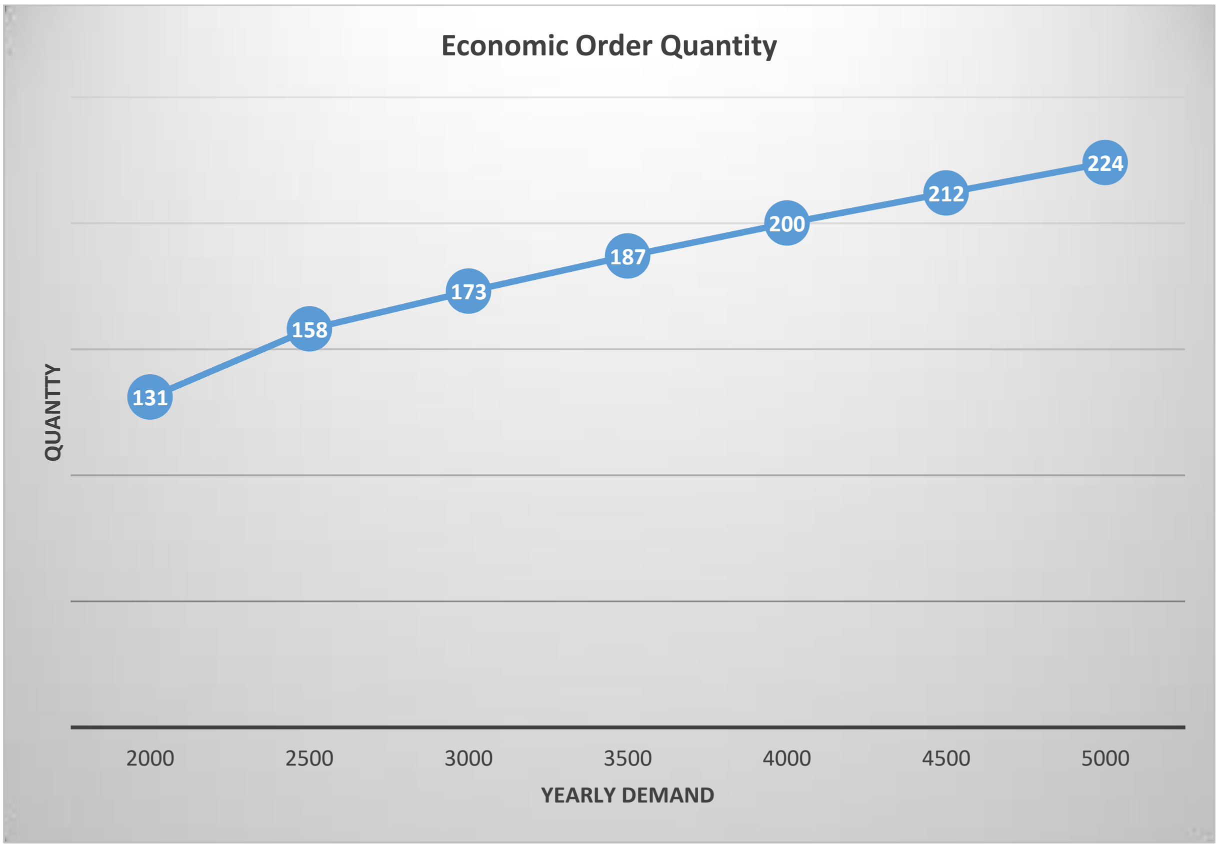
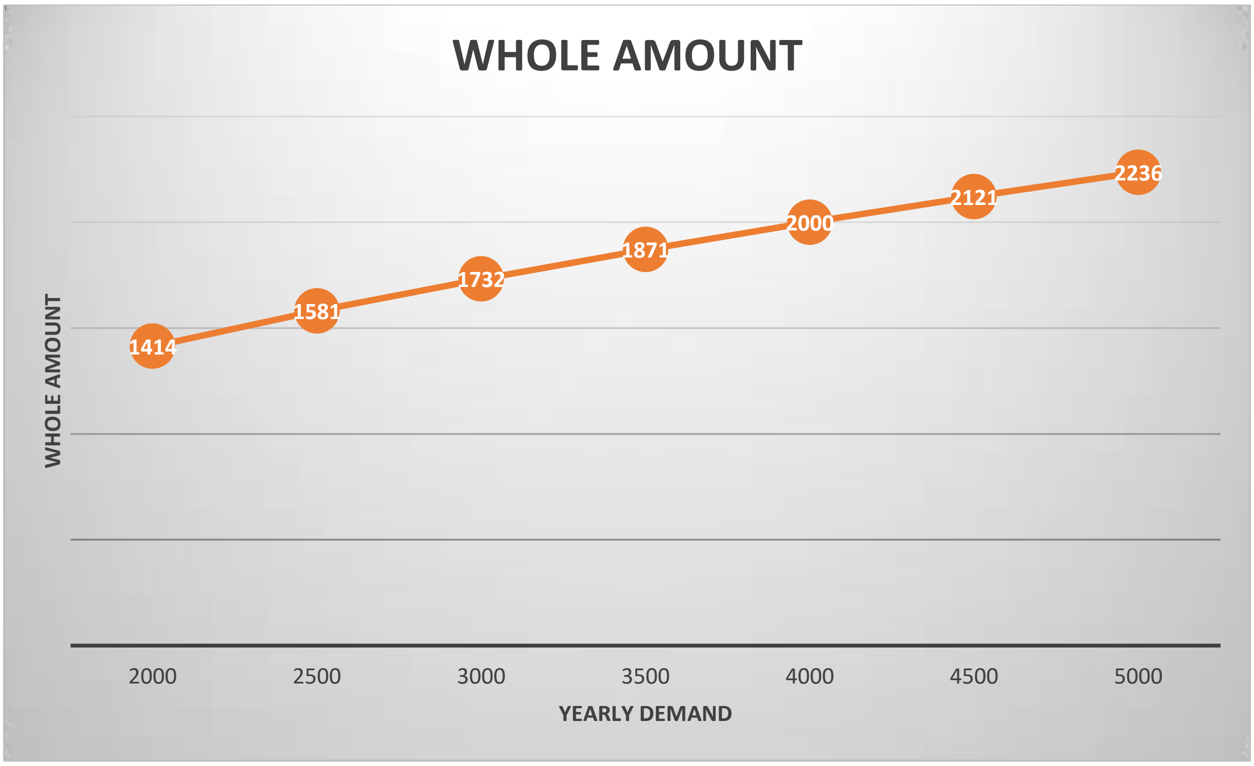



**Table 2:**

| S.No | Change In Yearly Demand $(Y_D)$ | Whole Amount $(WA)$ |
|---|---|---|
| 1 | 2000 | 1414 |
| 2 | 2500 | 1581 |
| 3 | 3000 | 1732 |
| 4 | 3500 | 1871 |
| 5 | 4000 | 2000 |
| 6 | 4500 | 2121 |
| 7 | 5000 | 2236 |




**Conclusion**

In this paper, we have gotten the optimal solution of the stock model with no passable deficiencies with fuzzy parameters. The model was explained for PFN utilizing the Lagrangean technique . Numerical models were represented for PFN and the outcomes were classified. Sensitivity analysis were made for EOQ and complete amount with the various qualities in the yearly interest. Table 1 and Table 2 demonstrates that there is an expansion in Economic Order Quantity and Total expense as interest increments. The optimal solution of our recommended model can be viable to meet the primary reason in which amplify the benefit and to limit the total cost. Accordingly fuzzy stock model is the standout amongst the best illuminating approach for the stock control.

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