A NOTE ON BIPOLAR VALUED FUZZY KM IDEAL ON K ALGEBRAS

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Abstract:
In this paper, Fuzzy KM-ideal on K algebras and bipolar valued fuzzy set are discussed and bipolar valued fuzzy KM-ideal on K-Algebras introduced and related topics are discussed

Keywords:

K-algebras, KM-ideals, Fuzzy KM-ideals, Cartesian product, Fuzzy relations, Strongest fuzzy relations,Bipolar KM Ideals on K algebras.

1.Introduction

Dar and Akram were introduced [3] a K-Algebra $(G,.,\odot.e)$,, is an Algebra built on a group (G,.,e) with identity e and adjoined with an induced binary operation $\odot$ on G. It is non-commutative and non-associative with a right identity e. It is proved in [1,3] that a K-algebra on an abelian group is equivalent to a p-semi simple BCI-algebra. For the convenience of study, authors renamed a K-algebra built on a group as K(G) algebra[2]. The K(G) has been characterized by using right and left mappings in [2]. After the introduction of fuzzy sets by Zadeh[4], the fuzzy set theory developed by Zadeh himself and others in many directions and found applications in various areas of sciences. Akram et al introduced the notions of sub algebras and fuzzy (maximal) ideals of K-algebras in [5] and further studied by Jun et al. in [61] Lee [7,8,9] introduced the operation in bipolar-valued fuzzy sets. Bipolar-valued fuzzy sets are an extension of fuzzy sets whose membership degree range is enlarged from the interval [0, 1] to [−1, 1]. In a bipolar-valued fuzzy set, the membership degree 0 means that elements are irrelevant to the corresponding property, the membership degree (0, 1] indicates that elements somewhat satisfy the property, and the membership degree [−1, 0) indicates that elements somewhat satisfy the implicit counter-property.

In this Paper, we discussed about the Bipolar-valued fuzzy KM-ideals on K-algebras are introduced and studied their properties with Cartesian product of KM-ideals and strongest fuzzy relation in detail.

2. Preliminaries

In this section we review some elementary aspects that are necessary for this paper:

Definition 2.1.

Let (G, ·, e) be a group with the identity e such that $x^2 \neq e$ for some $x(\neq e) \in G$.

A K-algebra built on G (briefly, K-algebra) is a structure K = (G, ., ⊙, e) where "⊙" is a binary operation on G which is induced from the operation "·", that satisfies the following:

(k1) $(\forall a, x, y \in G)\ ((a \odot x) \odot (a \odot y) = (a \odot (y^{-1} \odot x^{-1})) \odot a)$,

(k2) $(\forall a, x \in G)\ (a \odot (a \odot x) = (a \odot x^{-1}) \odot a)$,

(k3) $(\forall a \in G)\ (a \odot a = e)$,

(k4) $(\forall a \in G)\ (a \odot e = a)$,

(k5) $(\forall a \in G)\ (e \odot a = a^{-1})$.

If G is abelian, then conditions (k1) and (k2) are replaced by:

(k1') $(\forall a, x, y \in G)\ ((a \odot x) \odot (a \odot y) = y \odot x)$,

(k2') $(\forall a, x \in G)\ (a \odot (a \odot x) = x)$,

respectively. A nonempty subset H of a K-algebra K is called a subalgebra of K if it satisfies:

• $(\forall a, b \in H)\ (a \odot b \in H)$.

Note that every subalgebra of a K-algebra K contains the identity e of the group (G, ·). A mapping

f : K1 → K2 of K-algebras is called a homomorphism if $f(x \odot y) = f(x) \odot f(y)$ for all x, y ∈ K1. Note that if f is a homomorphism, then f(e) = e. A nonempty subset I of a K-algebra K is called an ideal of K if it satisfies:

(i) $e \in I$,

(ii) $(\forall x, y \in G)\ (x \odot y \in I,\ y \odot (y \odot x) \in I \Rightarrow x \in I)$.

Let μ be a fuzzy set on G, i.e., a map $\mu : G \to [0, 1]$. A fuzzy set μ in a K-algebra K is called a fuzzy subalgebra of K if it satisfies:

• $(\forall x, y \in G)\ (\mu(x \odot y) \geq \min\{\mu(x), \mu(y)\})$.

Every fuzzy subalgebra μ of a K-algebra K satisfies the following inequality:

(∀x ∈ G) (μ(e) ≥ μ(x)).

Definition 2.2. A fuzzy set μ in a K-algebra is called a fuzzy KM ideal of K if it satisfies:

(i) (∀x ∈ G) (μ(e) ≥ μ(x)),

(ii) (∀x, y ∈ G) (μ(y) ≥ min{μ(y ⊙ x), μ(x ⊙ (x ⊙ y))}).

Definition 2.3.A bipolar valued fuzzy set $(B_iFS)\mu$ in P is defined as an object of the form μ = {<p, μ^+(p), μ^-(p) >/ p∈P}, where μ^+: P→ [0, 1] and μ^-: P→ [−1, 0]. The positive membership degree μ^+(p) denotes the satisfaction degree of an element p to the property corresponding to a bipolar valued fuzzy set μ and the negative membership degree μ^-(p) denotes the satisfaction degree of an element p to some implicit counter-property corresponding to a bipolar valued fuzzy set μ.

Definition 2.4.A bipolar valued fuzzy set $(B_iFS)\mu$ in K-Algebra P is said to be a bipolar valued fuzzy KM-ideal (B_iFI_{km}) of P if for every p, qϵ P

1. $\mu_B^+(0) \geq \mu_B^+(p)$ and $\mu_B^+(0) \leq \mu_B^+(p)$
2. $\mu_B^+(q) \geq \min\{\mu_B^+(q \odot p), \mu_B^+(p \odot (p \odot q)\}$ and
   $\mu_B^-(q) \leq \max\{\mu_B^-(q \odot p), \mu_B^-(p \odot (p \odot q)\}$

Theorem 2.5. Every B_iFI_{km} of B-Algebra P with q ≤ p, for every p, q∈P is

(i) Order reversing then $\mu_B^+(q) \geq \mu_B^+(p)$

(ii) Order preserving then $\mu_B^-(q) \leq \mu_B^-(p)$

Proof.

Let p, q ϵ P such that q ≤ p then q ⊙ p = 0.

Thus, $\mu_B^+(q) = \mu_B^+(0 \odot q) \geq \min\{\mu_B^+(q \odot p), \mu_B^+(p \odot (p \odot q)\}$

$= \min\{\mu_B^+(0), \mu_B^+(p)\}$

$= \mu_B^+(p)$

Hence $\mu_B^+(q) \geq \mu_B^+(p)$.

And $\mu_B^-(q) = \mu_B^-(0 \odot q) \leq \max\{\mu_B^-(q \odot p), \mu_B^-(p \odot (p \odot q)\}$

$= \max\{\mu_B^-(0), \mu_B^-(p)\}$

$= \mu_B^-(p)$

Hence $\mu_B^-(q) \leq \mu_B^-(p)$.

Theorem 2.6. The intersection of any two $B_iFI_{km}s$ of P is also a B_iFI_{km}.

Proof.

Let μ = {< p, μ^+(p), μ^-(p) >/ p∈P} and δ = {< p, δ$^+$(p), δ$^-$(p) >/ p∈P}

Let W = μ ∩δ and W={< p, W$^+$(p), W$^-$(p) >/ p∈P }

Then $W_B^+(0) = \min\{\mu_B^+(0), \delta_B^+(0)\}$

$\geq \min\{\mu_B^+(p), \delta_B^+(p)\}$

$= W_B^+(p)$

And $W_B^-(0)$ = max {$\mu_B^-(0)$,$\delta_B^-(0)$ }
≤max {$\mu_B^-(p)$, $\delta_B^-(p)$ }
= $W_B^-(p)$
Also $W_B^+(q)$ = min {$\mu_B^+(q)$,$\delta_B^+(q)$ }
≥min {min{ μ_B^+ (q ⊙ p), μ_B^+ (p ⊙ (p ⊙ q)}{δ_B^+ (q ⊙ p), δ_B^+ (p ⊙ (p ⊙ q)}
= min {min{ μ_B^+ (q ⊙ p),δ_B^+ (q ⊙ p)}, min{ μ_B^+ (p ⊙ (p ⊙ q),δ_B^+ (p ⊙ (p ⊙ q)}}
= min{{μ_B^+(q ⊙ p), μ_B^+(p ⊙ (p ⊙ q))}

And also$\mu_B^-(q)$ = max {$\mu_B^-(q)$,$\delta_B^-(q)$ }
≤max {max{ μ_B^- (q ⊙ p), μ_B^- (p ⊙ (p ⊙ q)}{δ_B^- (q ⊙ p), δ_B^- (p ⊙ (p ⊙ q)}
= max {max{ μ_B^- (q ⊙ p),δ_B^- (q ⊙ p)}, max{ μ_B^- (p ⊙ (p ⊙ q),δ_B^- (p ⊙ (p ⊙ q)}}
= max{{ μ_B^+(q ⊙ p), μ_B^+(p ⊙ (p ⊙ q))}
Hence W = $\mu \cap \delta$is also a B_iFI_{km}.

3.Cartesian Product

Definition 3.1. Let μ and δ be the B_iFSs in P and Q respectively. The cartesian product μ x δ: Px Q → [0,1] is defined by (μx δ)={(p,q), $(\mu \text{ x } \delta)_B^+(p,q)$, $(\mu \text{ x } \delta)_B^-(p,q)$ / ∀ p ∈ P and ∀q∈Q } where $(\mu \text{ x } \delta)_B^+(p, q)$ = min{ $\mu_B^+(p)$, $\delta_B^+(q)$ } and $(\mu \text{ x } \delta)_B^-(p, q)$ = max{ $\mu_B^-(p)$, $\delta_B^-(q)$ },∀p ∈ P , q ∈ Q.

Definition 3.2. Let μ be the B_iFS in a set P, the strongest bipolar valued fuzzy relation on P, that is the strongest bipolar valued fuzzy relation on μ is J = {⟨ (p, q), $J_B^+(p, q)$, $J_B^-(p, q)$ ⟩/ p, q ∈P}given by $J_B^+(p, q)$ = min{$\mu_B^+(p)$, $\mu_B^+(q)$} and $J_B^-(p, q)$ = min{ $\mu_B^-(p)$, $\mu_B^-(q)$},∀p, q ∈ P.

Theorem 3.3.IF μ and δare the $B_iFI_{km}s$ of P and Q respectively, then μ x δ is a B_iFI_{km} of P x Q.

Proof.
For any (p, q) ∈P x Q, we have
$(\mu \text{ x } \delta)_B^+(0, 0)$ = min { $\mu_B^+(0)$, $\delta_B^+(0)$ }
≥min {$\mu_B^+(p)$, $\delta_B^+(q)$ }
= $(\mu \text{ x } \delta)_B^+(p, q)$
And
$(\mu \text{ x } \delta)_B^-(0, 0)$ = max{ $\mu_B^-(0)$, $\delta_B^-(0)$ }
≤max{$\mu_B^-(p)$, $\delta_B^-(q)$ }
= $(\mu \text{ x } \delta)_B^-(p, q)$

Also, let (p_1, p_2), (q_1, q_2) ∈ P x Q.

$(\mu \text{ x } \delta)_B^+((q_1, q_2))$= $(\mu \text{ x } \delta)_B^+(q_1, q_2)$
= min { $\mu_B^+(q_1)$,$\delta_B^+(q_2)$ }
≥min {min{ μ_B^+ (q_1⊙ p_1), μ_B^+ (p_1⊙ (p_1⊙ q_1)}{δ_B^+ (q_2⊙ p_2), δ_B^+ (p_2⊙ (p_2⊙ q_2)}

$\geq$min {min{ μ_B^+ (q$_1\odot$ p$_1$), δ_B^+ (q$_2\odot$ p$_2$),)},min{ μ_B^+ (p$_1\odot$ (p$_1\odot$ q$_1$)), δ_B^+ (p$_2\odot$ (p$_2\odot$ q$_2$)}}= min{(μ x δ)$_B^+$ (q$_1\odot$ p$_1$),(q$_2\odot$ p$_2$)(μ x δ)$_B^+$((p$_1\odot$ (p$_1\odot$ q$_1$), (p$_2\odot$ (p$_2\odot$ q$_2$))}}

And

(μ x δ)$_B^-$((q$_1$, q$_2$))

= (μ x δ)$_B^-$(q$_1$, q$_2$)

= max { μ_B^-(q$_1$),δ_B^-(q$_2$) }

$\leq$max {max{ μ_B^- (q$_1\odot$ p$_1$), μ_B^- (p$_1\odot$ (p$_1\odot$ q$_1$)}{δ_B^- (q$_2\odot$ p$_2$), δ_B^- (p$_2\odot$ (p$_2\odot$ q$_2$)}

$\leq$max {max{ μ_B^- (q$_1\odot$ p$_1$),δ_B^- (q$_2\odot$ p$_2$),)},max{ μ_B^- (p$_1\odot$ (p$_1\odot$ q$_1$)),δ_B^- (p$_2\odot$ (p$_2\odot$ q$_2$)}}

= max{(μ x δ)$_B^-$ (q$_1\odot$ p$_1$),(q$_2\odot$ p$_2$)(μ x δ)$_B^-$((p$_1\odot$ (p$_1\odot$ q$_1$), (p$_2\odot$ (p$_2\odot$ q$_2$))}}

Therefore μ x δ is a B_iFI_{km} of P x Q.

Theorem 3.4. For any given B_iFS of a K-algebra P, let J be the strongest bipolar valued fuzzy relation on P. If μ is a B_iFI_{km}ofPxP, then J$_B^+$(P, P) $\leq$ J$_B^+$(0, 0),$\forall$p $\in$ P and J$_B^-$(P, P) $\geq$ J$_B^-$(0, 0),$\forall$p $\in$ P.

Proof.

Here J is the strongest bipolar valued fuzzy relation on P x P, then

J$_B^+$(P, P) = min { μ_B^+(p) , μ_B^+(p) }

$\leq$ min{ μ_B^+(0) , μ_B^+(0) }

= J$_B^+$(0, 0),$\forall$p $\in$ P.

$\Rightarrow$J$_B^+$(P, P) $\leq$ J$_B^+$(0, 0),$\forall$p $\in$ P

In the same way,

J$_B^-$(P, P) = max {μ_B^-(p), μ_B^-(p)}

$\geq$ max{ μ_B^-(0), μ_B^-(0)}

= J$_B^-$(0, 0),$\forall$p $\in$ P.

$\Rightarrow$J$_B^-$(P, P) $\geq$ J$_B^-$(0, 0),$\forall$p $\in$ P.

Theorem 3.5. Let μ be the B_iFS in a K-Algebra P and J be the strongest bipolar valued fuzzy relation on P. If μ is a B_iFI_{km} of P iff J is a B_iFI_{km} of P x P.

Proof.

Suppose μ is a B_iFI_{km} of P.

Then J$_B^+$(0, 0) = min { μ_B^+(0) , μ_B^+(0) }

$\geq$ min { μ_B^+(p) , μ_B^+(q) }

= J$_B^+$(p, q) ,$\forall$p, q$\in$ P.

And J$_B^-$(0, 0) = max { μ_B^-(0) , μ_B^-(0) }

$\leq$ max{ μ_B^-(p) , μ_B^-(q) }

= J$_B^-$(p, q) ,$\forall$ p, q $\in$ P.

Also for any (p$_1$, p$_2$), (q$_1$, q$_2$) $\in$ P x P

J$_B^+$(q$_1$, q$_2$) = min { μ_B^+(q$_1$),μ_B^+(q$_2$) }

$\geq$min {{min{ μ_B^+ (q$_1\odot$ p$_1$), μ_B^+ (p$_1\odot$ (p$_1\odot$ q$_1$)}{δ_B^+ (q$_2\odot$ p$_2$), δ_B^+ (p$_2\odot$ (p$_2\odot$ q$_2$)}

$\geq$min {min{ μ_B^+ (q$_1\odot$ p$_1$),δ_B^+ (q$_2\odot$ p$_2$),)},min{ μ_B^+ (p$_1\odot$ (p$_1\odot$ q$_1$)),δ_B^+ (p$_2\odot$ (p$_2\odot$ q$_2$)}}

= min{(J$_B^+$(q$_1\odot$ p$_1$),(q$_2\odot$ p$_2$))}{J$_B^+$((p$_1\odot$ (p$_1\odot$ q$_1$),(p$_2\odot$ (p$_2\odot$ q$_2$))}

And also

$J_B^-(q_1, q_2) = \min\{ \mu_B^-(q_1),\mu_B^-(q_2) \}$

$\leq \max \{\{\max\{ \mu_B^-(q_1 \odot p_1), \mu_B^- (p_1 \odot (p_1 \odot q_1)\}\{\delta_B^- (q_2 \odot p_2), \delta_B^-(p_2 \odot (p_2 \odot q_2)\}$

$\leq \max \{\max\{ \mu_B^- (q_1 \odot p_1),\delta_B^- (q_2 \odot p_2),)\},\max\{ \mu_B^- (p_1 \odot (p_1 \odot q_1)),\delta_B^- (p_2 \odot (p_2 \odot q_2)\}\}$

$= \max\{(J_B^-(q_1 \odot p_1),(q_2 \odot p_2))\}\{J_B^-((p_1 \odot (p_1 \odot q_1),(p_2 \odot (p_2 \odot q_2))\}$

Therefore,J is a B_iFI_{km} of P x P.

Conversely, suppose that J is a B_iFI_{km} of P x P, then

$J_B^+(P, P) \leq J_B^+(0, 0)$ where (0,0) is the zero element of P x P.

Which means that

$\min \{ \mu_B^+(p) , \mu_B^+(q) \} \leq \min \{ \mu_B^+(0) , \mu_B^+(0) \}$

$\Rightarrow \mu_B^+(p) \leq \mu_B^+(0), \forall p \in P$ and also $\mu_B^-(p) \geq \mu_B^-(0), \forall p \in P$.

Now, let $(p_1, p_2), (q_1, q_2) \in P \times P$

Then,

$\min \{ \mu_B^+(q_1),\mu_B^+(q_2) \} = J_B^+(q_1, q_2)$

$\geq \min\{\{(J_B^+(q_1 \odot p_1),(q_2 \odot p_2))\}\{J_B^+((p_1 \odot (p_1 \odot q_1),(p_2 \odot (p_2 \odot q_2))\}$

$= \min \{\min\{ \mu_B^+ (q_1 \odot p_1),\delta_B^+ (q_2 \odot p_2),)\},\min\{ \mu_B^+ (p_1 \odot (p_1 \odot q_1)),\delta_B^+ (p_2 \odot (p_2 \odot q_2)\}\}$ In particular, if we take $p_2=q_2=0$,then

$\mu_B^+(q_1) \geq \min \{ \mu_B^+(q_1),\mu_B^+(q_2) \}$

Also,$\max\{ \mu_B^-(q_1),\mu_B^-(q_2) \} = J_B^-(q_1, q_2)$

$\leq \max\{\{(J_B^-(q_1 \odot p_1),(q_2 \odot p_2))\}\{J_B^-((p_1 \odot (p_1 \odot q_1),(p_2 \odot (p_2 \odot q_2))\}$

$= \max \{\max\{ \mu_B^- (q_1 \odot p_1),\delta_B^- (q_2 \odot p_2),)\},\max\{ \mu_B^- (p_1 \odot (p_1 \odot q_1)),\delta_B^- (p_2 \odot (p_2 \odot q_2)\}\}$ In particular, if we take $p_2=q_2=0$,then

$\mu_B^-(q_1) \leq \max\{ \mu_B^-(q_1),\mu_B^-(q_2) \}$

This proves μ is a B_iFI_{km} of P.

Theorem 3.6. Let μ and δ be the B_iFSs in a K-Algebra P such that $\mu \times \delta$ is a B_iFI_{km} of P x P then$\forall p \in P$,

(i) Either $\mu_B^+(0) \geq \mu_B^+(p)$ or $\delta_B^+(0) \geq \delta_B^+(p)$ and $\mu_B^-(0) \leq \mu_B^-(p)$ or $\delta_B^-(0) \leq \delta_B^-(p)$.

(ii) If $\mu_B^+(0) \geq \mu_B^+(p)$ then either $\delta_B^+(0) \geq \mu_B^+(p)$ or $\delta_B^+(0) \geq \delta_B^+(p)$ and $\mu_B^-(0) \leq \mu_B^-(p)$ then either $\delta_B^-(0) \leq \mu_B^-(p)$ or $\delta_B^-(0) \leq \delta_B^-(p)$.

(iii) If $\delta_B^+(0) \geq \delta_B^+(p)$ then either $\mu_B^+(0) \geq \mu_B^+(p)$ or $\mu_B^+(0) \geq \delta_B^+(p)$ and $\delta_B^-(0) \leq \delta_B^-(p)$ then either $\mu_B^-(0) \leq \mu_B^-(p)$ or $\mu_B^-(0) \leq \delta_B^-(p)$.

Proof.

Let $\mu \times \delta$ is a B_iFI_{km} of P x P.

Therefore,

$(\mu \times \delta)_B^+(0, 0) \geq (\mu \times \delta)_B^+(p, q) ,\forall (p, q) \in P \times P$

$(\mu \times \delta)_B^+((q_1, q_2)*(r_1, r_2)) \geq \min \{(\mu \times \delta)_B^+((p_1, p_2) * (q_1, q_2))(\mu \times \delta)_B^+((r_1, r_2) *(p_1, p_2))\}$

And $(\mu \times \delta)_B^-((q_1, q_2) * (r_1, r_2)) \leq \max \{(\mu \times \delta)_B^-((p_1, p_2) * (q_1, q_2))(\mu \times \delta)_B^-((r_1, r_2) *(p_1, p_2))\}$

For all $(p_1, p_2), (q_1, q_2), (r_1, r_2) \in P \times P$

(i) Suppose that $\mu_B^+(0) < \mu_B^+(p)$ and $\delta_B^+(0) < \delta_B^+(p)$ for some p, q$\in$ P.
$(\mu \text{ x } \delta)_B^+(p, q) = \min \{ \mu_B^+(p), \delta_B^+(q) \}$
$> \min \{\mu_B^+(0), \delta_B^+(0) \}$
$= (\mu \text{ x } \delta)_B^+(0, 0)$ which is a contradiction.
Therefore either $\mu_B^+(0) \geq \mu_B^+(p)$ or $\delta_B^+(0) \geq \delta_B^+(p)$, $\forall p \in$ P.
And, suppose $\mu_B^-(0) > \mu_B^-(p)$ or $\delta_B^-(0) > \delta_B^-(p)$.
$(\mu \text{ x } \delta)_B^-(p, q) = \max\{ \mu_B^-(p), \delta_B^-(q)\}$
$< \max\{\mu_B^-(0), \delta_B^-(0) \}$
$= (\mu \text{ x } \delta)_B^-(0, 0)$ which is a contradiction
Therefore either $\mu_B^-(0) \leq \mu_B^-(p)$ or $\delta_B^-(0) \leq \delta_B^-(p), \forall p \in$ P.

(ii) Assume that there exists p, q $\in$ P such that $\delta_B^+(0) < \mu_B^+(p)$ or $\delta_B^+(0) < \delta_B^+(p)$
Then $(\mu \text{ x } \delta)_B^+(0, 0) = \min \{ \mu_B^+(0), \delta_B^+(0) \}$
$= \delta_B^+(0)$
and hence $(\mu \text{ x } \delta)_B^+(p, q) = \min \{ \mu_B^+(p), \delta_B^+(q) \}$
$> \delta_B^+(0)$
$= (\mu \text{ x } \delta)_B^+(0, 0)$
which is a contradiction.
Hence if $\mu_B^+(0) \geq \mu_B^+(p)$ then either $\delta_B^+(0) \geq \mu_B^+(p)$ or $\delta_B^+(0) \geq \delta_B^+(p)$.
Also assume that $\delta_B^-(0) > \mu_B^-(p)$ or $\delta_B^-(0) > \delta_B^-(p)$
$(\mu \text{ x } \delta)_B^-(0, 0) = \max \{ \mu_B^-(0), \delta_B^-(0) \}$
$= \delta_B^-(0)$
and hence $(\mu \text{ x } \delta)_B^-(p, q) = \max \{ \mu_B^-(p), \delta_B^-(q) \}$
$> \delta_B^-(0)$
$= (\mu \text{ x } \delta)_B^-(0, 0)$ which is a contradiction.
Hence if $\mu_B^-(0) \leq \mu_B^-(p)$ then either $\delta_B^-(0) \leq \mu_B^-(p)$ or $\delta_B^-(0) \leq \delta_B^-(p)$.
Similarly we can prove that if $\delta_B^+(0) \geq \delta_B^+(p)$ then either $\mu_B^+(0) \geq \mu_B^+(p)$ or $\mu_B^+(0) \geq \delta_B^+(p)$ and $\delta_B^-(0) \leq \delta_B^-(p)$ then either $\mu_B^-(0) \leq \mu_B^-(p)$ or $\mu_B^-(0) \leq \delta_B^-(p)$ which gives (iii).

Theorem 3.7. Let μ and δ be the B_iFSs in a K-Algebra P such that μ x δ is a B_iFI_{km} of P x P then $\forall$ p $\in$ P. Then μ or δ is a B_iFI_{km} of P.

Proof.
By theorem (3.6 (i)), without loss of generality we assume that
$\mu_B^+(p) \leq \mu_B^+(0)$ and $\mu_B^-(0) \leq \mu_B^-(p)$ for all p $\in$ P. From the theorem (3.6 (ii)) it follows that for all p $\in$ P, either $\delta_B^+(0) \geq \mu_B^+(p)$ or $\delta_B^+(0) \geq \delta_B^+(p)$ and $\mu_B^-(0) \leq \mu_B^-(p)$ then either $\delta_B^-(0) \leq \mu_B^-(p)$ or $\delta_B^-(0) \leq \delta_B^-(p)$.
If $\delta_B^+(0) \geq \mu_B^+(p)$ for all p $\in$ P
Then $(\mu \text{x } \delta)_B^+(0, p) = \min \{\delta_B^+(0), \mu_B^+(p) \}$
$= \mu_B^+(p)$
Let (p, q)$\in$ P x P. Since μ x δ is a B_iFI_{km} of P,

we get $(\mu \times \delta)_B^+(0, 0) \geq (\mu \times \delta)_B^+(p, q)$ and $(\mu \times \delta)_B^-(0, 0) \leq (\mu \times \delta)_B^-(p, q)$
Let $(p_1, p_2), (q_1, q_2) \in P \times P$
Using KM-ideal,
$(\mu \times \delta)_B^+(q_1, q_2) = \min\{\mu_B^+(q_1), \delta_B^+(q_2)\}$
$\geq \min\{\{\{\min\{\mu_B^+(q_1 \odot p_1), \mu_B^+(p_1 \odot (p_1 \odot q_1)\}\{\delta_B^+(q_2 \odot p_2), \delta_B^+(p_2 \odot (p_2 \odot q_2)\}\}$
$= \min\{(\mu \times \delta)_B^+(((q_1 \odot p_1),(p_1 \odot (p_1 \odot q_1))\}\{(\mu \times \delta)_B^+(((q_2 \odot p_2),(p_2 \odot (p_2 \odot q_2))\}\}$
In particular, if we take $p_1 = q_1 = 0$, then
$\delta_B^+(q_2) = (\mu \times \delta)_B^+(0, q_2)$
$\geq \min\{\min\{\mu_B^+(0), \delta_B^+(p_2)\}, \min\{\mu_B^+(0), \delta_B^+(q_2)\}\}$
$= \min\{\delta_B^+(p_2 \odot q_2)\}, \delta_B^+(p_2)\}\}$
And
$(\mu \times \delta)_B^-(q_1, q_2) = \min\{\mu_B^-(q_1), \delta_B^-(q_2)\}$
$\leq \max\{\{\{\max\{\mu_B^-(q_1 \odot p_1), \mu_B^-(p_1 \odot (p_1 \odot q_1)\}\{\delta_B^-(q_2 \odot p_2), \delta_B^-(p_2 \odot (p_2 \odot q_2)\}\}$
$= \max\{(\mu \times \delta)_B^-(((q_1 \odot p_1),(p_1 \odot (p_1 \odot q_1))\}\{(\mu \times \delta)_B^-(((q_2 \odot p_2),(p_2 \odot (p_2 \odot q_2))\}\}$
In particular, if we take $p_1 = q_1 = 0$, then
$\delta_B^-(q_2) = (\mu \times \delta)_B^-(0, q_2)$
$\leq \max\{\max\{\mu_B^-(0), \delta_B^-(p_2)\}, \max\{\mu_B^-(0), \delta_B^-(q_2)\}\}$
$= \max\{\delta_B^-(p_2 \odot q_2)\}, \delta_B^-(p_2)\}\}$
Similarly, second part can be proved.
This completes the proof.

5.Conclusion.

In this paper we have discussed Bipolar valued fuzzy KM ideal on K algebras and also discussed Cartesian product.

6.References.

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