Boost-Sustain Missile Motor Performance with Fixed Predetermined Coast Time Interval

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Abstract: Singular perturbation is used for synthesis and analysis of a near optimal midcourse guidance law for realistic air-to-air engagement. The proposed midcourse guidance law uses a five-degree-of-freedom (5-DOF) mathematical model of tactical air-to-air missile. This paper considers the midcourse guidance law derivation from optimal control viewpoint formulation of boost-sustain motor for air-to-air missile based on singular perturbation techniques. The slightly advance in down range and final velocity and specific energy of the missile was obtained through introducing the coast time interval between the booster and sustainer motors. This means that the sustainer motor ignited only after the introduced fixed coast interval. Therefore, the effect of variable coast time interval on the performance of the missile is studied in this paper.

Keywords: Midcourse guidance, Boost-sustain motor, Air-to-air missile, Coast time interval.

1. INTRODUCTION

The midcourse guidance law based on a minimum flight time optimal control formulation is presented in this paper. Specifically, the optimal control problem is considered at each guidance cycle of the guidance computer, with the updated missile and target states. The resulting guidance command is used until the next guidance cycle when new state information is available [1]. Conventional air-to-air missiles use proportional navigation guidance or some of its modifications [2, 3]. The singular perturbation techniques are used to simplify the optimal control problem through the elimination of the need for solving two-point boundary-value problem [4, 5]. Conventional medium range air-to-air missile concepts use boost-sustain motors for propulsion. Midcourse guidance's for intercepting conventional aviation targets or ballistic missiles are relatively mature. For example, concerning air-to-air missile, singular perturbation theory was applied to the derivation of near-optimal midcourse guidance, but they did not consider the angle alignment constraint and focusing on the optimal guidance problem with angular constraint [6, 7]. N. Indig, et al. [8] proposed an analytic guidance law with a linear dynamic model, which would cause control saturation at the time of interception. Liu et al. [9] designed a biased proportional navigation guidance method with attitude constraint and line-of-sight angle rate control, but it mainly aimed at stationary ground target, not to mention the capability of energy management. Singular perturbation based technique is used for synthesis and analysis of a near optimal midcourse guidance law for realistic air-to-air engagement [6]. Using the singular perturbation method, the guidance problem may be broken into sub problem based on the speed of the variable solution. They are, the slowest time-scale problem is solved first, followed by a sequential solution of the fast sub problems. So, the process yields a relatively robust near optimal nonlinear feedback law for the midcourse guidance problem [10]. The performance of a boost-sustain motor for air-to-air missile is studied through introducing a fixed predetermined coast time interval (i.e., fixed time interval between the burning of the booster and sustainer missile motors) is considered. This means the sustainer motor will be ignited only after introduce fixed predetermined coast time interval. The effect of the introduced coast interval was studying in this paper through the advanced in the down range, final velocity and specific energy against non-maneuvering target with constant velocity.

2. PROBLEM FORMULATION

To simplify the discussion, we consider the missile model is based on a set of state variables that can be partitioned through time-scale separation for the particular mission we are concerned with. The simplified point mass dynamics of the missile can be expressed as [10]:

$$\dot{x} = v \cos\gamma \cos\phi \qquad x(t_o) = x_o \qquad ....(1)$$

$$\dot{y} = v \cos\gamma \sin\phi \qquad y(t_o) = y_o \qquad ....(2)$$

$$\dot{h} = v \sin\gamma \qquad h(t_o) = h_o \qquad ....(3)$$

$$\dot{E} = (T - D)v/(mg) \qquad E(t_o) = E_o \qquad ....(4)$$

$$\dot{\phi} = L \sin\sigma / (mv \cos\gamma) \qquad \phi(t_o) = \phi_o \qquad ....(5)$$

$$\dot{\gamma}^{\cdot} = (L \cos\sigma - mg \cos\gamma) / (mv) \qquad \gamma(t_o) = \gamma_o \qquad ....(6)$$

The input control to be optimized is lift vector in the two-dimensional subspace normal to the missile velocity vector, defined by magnitude L and orientation σ. The thrust T and mass m time-histories are predefined functions of time shown in Fig.1.

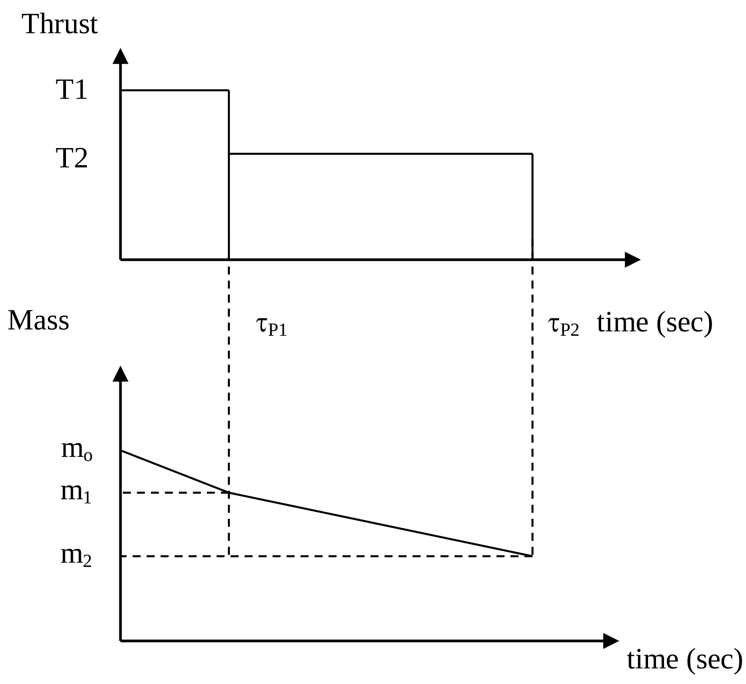


Fig. 1 Thrust and Mass Profiles

The drag is defined by the usual quadratic dependence on angle of attack.

$$D = qS\ CD \tag{7}$$

Where

$$CD = CD_O + CD\left|\propto\right| + CD_{\alpha 2}\ \alpha^2 \tag{8}$$

$$q = {}^{1}/_{2}\ \rho(h).V^2 \tag{9}$$

Where α is the total angle of attack and the air density ρ is a function of attitude. The total angle of attack is related to lift magnitude L as follows:

$$L = qS\, CL_{\propto}\alpha \tag{10}$$

For this study, the intercept point as predicted during midcourse is based on a constant velocity target and constant speed missile. The terminal condition can thus be identifies: -

$$(x, y, h)(t_f) = (x_f, y_f, h_f). \tag{11}$$

The performance index is the minimum time flight as:

$$J = \int_{t_o}^{t_f} dt \tag{12}$$

where the final terminal time t_f is free parameter. The optimal control problem is to solve the control variable that minimize J in Eq.(12), subject to the differential constraints of Eqs.(1-6) and the terminal condition of Eq.(11). The control variable is the command missile lift L. [5] discussed about a system, a low power area reduced and speed improved serial type daisy chain memory register also known as shift Register is proposed by using modified clock generator circuit and SSASPL (Static differential Sense Amplifier based Shared Pulsed Latch). This latch based shift register consumes low area and low power than other latches. There is a modified complementary pass logic based 4 bit clock pulse generator with low power and low area is proposed that generates small clock pulses with small pulse width. These pulses are given to the conventional shift register that results high speed. The system is designed by the Cadence virtuoso 180 nm technology. The Maximum supply voltage for the system, clock source and input source are 1.8V. The complementary pass logic based proposed system reduces the area about 7% for the total system and about 23% for the 4 bit clock pulse generator circuit. The Power is reduced by 26% than the conventional system. The speed is improved about 7% than the existing system.

For purposes of real-time control, we desire an approximate feedback solution, thus we are led to the use of singular perturbation methods. In this problem we regard position components x and y and specific energy E as the slowest variable, the attitude has slow variable and flight path angle γ and heading angle ϕ are the fast variable [6]. Thus a perturbation parameter is artificially introduced into the dynamics.

$$\dot{x} = v_1 \cos\gamma \cos\phi \tag{13}$$

$$\dot{y} = v_1 \cos\gamma \sin\phi \tag{14}$$

$$\varepsilon\dot{h} = \sin\gamma \tag{15}$$

$$\dot{E} = k_E(T_1 - D_1) \tag{16}$$

$$\varepsilon^2\dot{\phi} = L_1 \sin\sigma \,/\, \cos\gamma \tag{17}$$

$$\varepsilon^2\dot{\gamma}^{\cdot} = (L_1 \cos\sigma - k_\gamma \cos\gamma \tag{18}$$

where

$$v_1 = \frac{v}{r} \tag{19}$$

$$k_E = \frac{2g}{v(1 + 2gh/v^2)} \tag{20}$$

$$L_1 = \frac{L}{L_{\max}} \quad (21)$$

$$k_\gamma = \frac{mg}{L_{\max}} \quad (22)$$

$$T_1 = \frac{T}{mg} \quad (23)$$

$$D_1 = \frac{D}{mg} \quad (24)$$

$$h_1 = \frac{h}{h_{\max}} \quad (25)$$

and an approximate solution is found by an asymptotic expansion of the state equations and necessary conditions about ε = 0, and enforcing all the boundary conditions. Since the expansion is non-uniform at t = 0 and at $t = t_f$, boundary layer solution are required to satisfy the end conditions. This is accomplished by replacing t by the stretched time variables. [2] discussed that the activity related status data will be communicated consistently and shared among drivers through VANETs keeping in mind the end goal to enhance driving security and solace. Along these lines, Vehicular specially appointed systems (VANETs) require safeguarding and secure information correspondences. Without the security and protection ensures, the aggressors could track their intrigued vehicles by gathering and breaking down their movement messages. A mysterious message confirmation is a basic prerequisite of VANETs. To conquer this issue, a protection safeguarding confirmation convention with expert traceability utilizing elliptic bend based chameleon hashing is proposed. Contrasted and existing plans Privacy saving confirmation utilizing Hash Message verification code, this approach has the accompanying better elements: common and unknown validation for vehicle-to-vehicle and vehicle-to-roadside interchanges, vehicle unlinkability, specialist following capacity and high computational effectiveness

$$\tau_i = t / \varepsilon^i \qquad i = 1, 2 \quad (26)$$

In the i^{th} boundary layer, the outer and first boundary-Layer solutions are summarized below.

Outer Solution

$$\phi_o = \tan^{-1}\left[(y_f - y_o)/(x_f - x_o)\right] \quad (27)$$

$$\lambda_x^o = -\cos\phi_o / V_1(t_f) \quad (28)$$

$$\lambda_y^o = -\sin\phi_o / V_1(t_f) \quad (29)$$

$$\lambda_E^o = \left(\frac{v_1}{v_1(t_f)} - 1\right)\frac{1}{k_E(T_1 - D_1)}$$

$$\frac{\partial D_1}{\partial h}(\mathrm{h}_1) = \frac{\mathrm{g}(\mathrm{T}_1 - D_1)v_1(t_f)}{v_1^{\,2}(v_1 - v_1(t_f))} \quad (30)$$

First Boundary Layer

$$\tan\gamma_1 = \frac{\lambda^1{}_h}{\lambda_x^o \cos\phi_o + \lambda_y^o \sin\phi_o} \tag{31}$$

$$\frac{\sec\gamma_1 - 1}{v_1(t_f)} = \frac{v_1 - v_2}{v_1 v_2} + \frac{(D_1 - D_2)g}{v_1^3 \partial D_1 / \partial h} \tag{32}$$

$$\Delta\psi = \cos^{-1}(v_c.v_d).$$

Second Boundary Layer

$$L_2 = -\sqrt{\frac{qs\,C_{L\infty}^2}{C_{D\infty}^2}}\sqrt{\frac{V_1(T_1 - D_1)}{2[V_1 - V_1(t_f)]}}\quad \Delta\psi. \tag{33}$$

3. SIMULATION RESULTS

The guidance law obtained through the application of singular perturbation techniques on the optimal control problem. The simulation uses a five-degree-of-freedom (5-DOF) mathematical model of tactical air-to-air missile. This simulation studied the performance of a baseline boost-sustain motor which is summarized in table 1.

Table 1: Typical Midcourse Parameter Values

r = 70 Km				
h_{max} = 10 Km				
S = 0.024 m^2				
t_{p1} = 1.5 sec				
t_{p2} = 7 sec				
t_c = 3, 6, 9 sec				
T = 33000 N	and	$\dot{m}$ = -14.53 Kg	for	$t_{p1} > t \geq 0$
T = 0	and	$\dot{m}$ = 0	for	$t_{p1} + t_c > t \geq t_{p1}$
T = 7500 N	and	$\dot{m}$ = -3.250 Kg	for	$t_{p1} + t_c + t_{p2} > t \geq t_{p1} + t_c$
T = 0	and	$\dot{m}$ = 0	for	$t \geq t_{p1} + t_c + t_{p2}$
Total impulse 102 kN.sec				
Specific impulse 213 N.sec/Kg				

The near-optimal guidance law based on singular perturbation methodology was formulated to optimized midcourse performance. The performance of the missile was evaluated against non-maneuvering target with constant speed. The fly out example considered in this paper is coalitude head on launch at 3Km against target of height 6 Km with constant speed at 300 m/sec, and at launch range of 70 Km. The performance of a boost-sustain motor for air-to-air missile is studied through introducing a Fixed Predetermined Coast Time Interval (PCTI), i.e., fixed time interval between the burning of the booster and sustainer missile motors. This means the sustainer motor will be ignited only after introduced fixed predetermined coast time interval.

The missile thrust is illustrated in Fig. 2. Fig. 3 and Fig.4 shows the missile and target trajectories, missile speed and specific energy profile for different predetermined coast time interval (3 sec, 6 sec, 9 sec, and 12 sec) for snap up and snap down case respectively. The missile parameters percentage changes are given in table 1 and 2 respectively.

Table 2a: Percentage of the Parameter change for Snap up Case

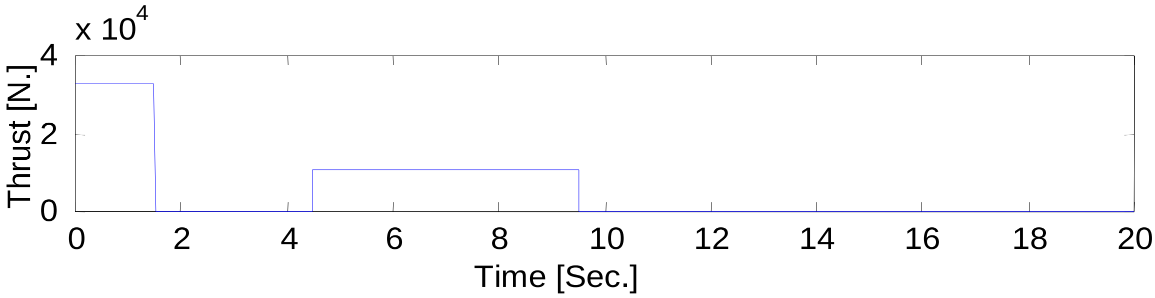
MBMBT	MSMBT	Coast time interval	Parameters
Fixed at 1.5 sec	**Fixed at 5 sec**	**Increased from 3sec to 6 sec**	The down range: is increased by 1.866% The flight time: is increased by 0.42% The final velocity: is increased by 1.323% The final specific energy: is increased by 0.0254%
Fixed at 1.5 sec	**Fixed at 5 sec**	**Increased from 6 sec to 9 sec**	The down range: is increased by 3.493% The flight time: is increased by 0.84% The final velocity: is increased by 2.152% The final specific energy: is increased by 0.0575%
Fixed at 1.5 sec	**Fixed at 5 sec**	**Increased from 9 sec to 12sec**	The down range: is increased by 5.078% The flight time: is increased by 1.26% The final velocity: is increased by 4.02% The final specific energy: is increased by 0.108%

Table 2b: Percentage of the Parameter Change for Snap down Case

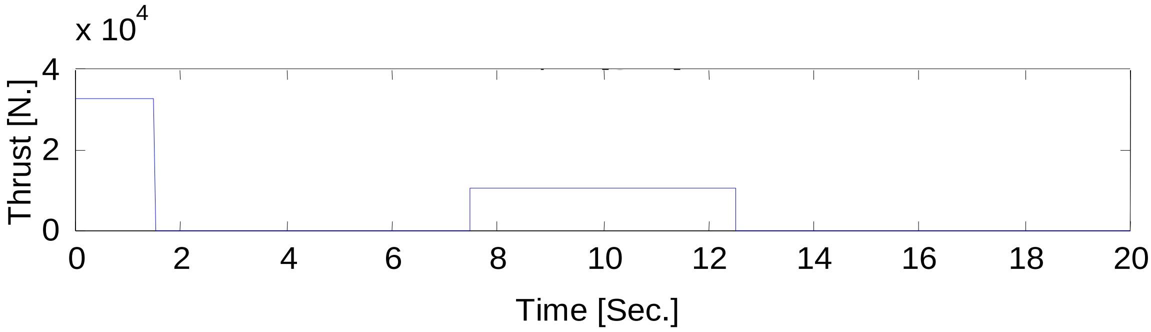
MBMBT	MSMBT	Coast time Interval	Parameters
Fixed at 1.5 sec	**Fixed at 5 sec**	**Increased from 3 sec to 6 sec**	The down range: is reduced by 1.032% The flight time: is reduced by 2.097% The final velocity: is increased by 3.548% The final specific energy: is increased by 0.143%
Fixed at 1.5 sec	**Fixed at 5 sec**	**Increased from 6 sec to 9 sec**	The down range: is reduced by 1.406% The flight time: is reduced by 3.496% The final velocity: is increased by 8.18% The final specific energy: is increased by 0.336%
Fixed at 1.5 sec	**Fixed at 5 sec**	**Increased from 9 sec to 12 sec**	The down range: is reduced by 2.062% The flight time: is reduced by 6.293% The final velocity: is increased by 11.82% The final specific energy: is increased by 0.769%

4. CONCLUSION

The application of singular perturbation technique for midcourse guidance law for air-to-air missile is to obtain an optimal control formulation in order to completely eliminate the need for solving two-point boundary-value problem. The resulting guidance law is near optimal and sufficiently simple for implementation. It has seen that introducing the fixed coast time interval between the booster and sustainer motors will give an advanced in missile performance in the range, velocity and specific energy. It has seen increasing the coast time interval will gives a more advanced in missile performance. Therefore, for this reason the simplified optimal guidance formulation in the development of a real-time on-line pulse motor algorithm for medium range missiles is applied.



(a)



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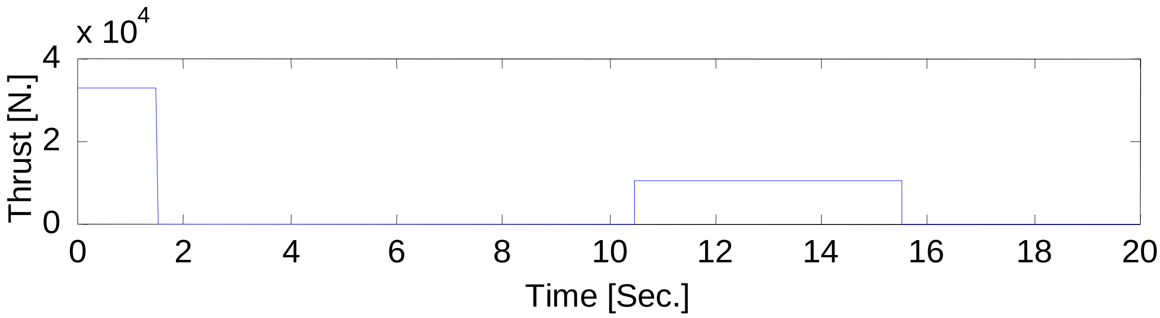

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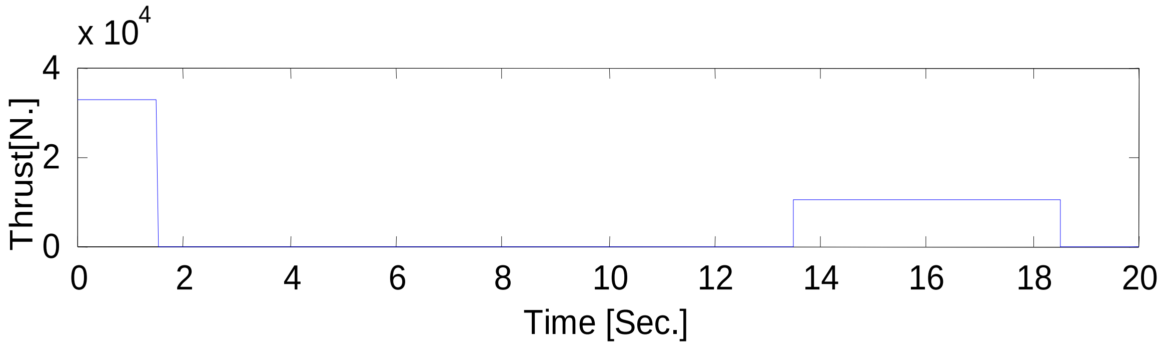
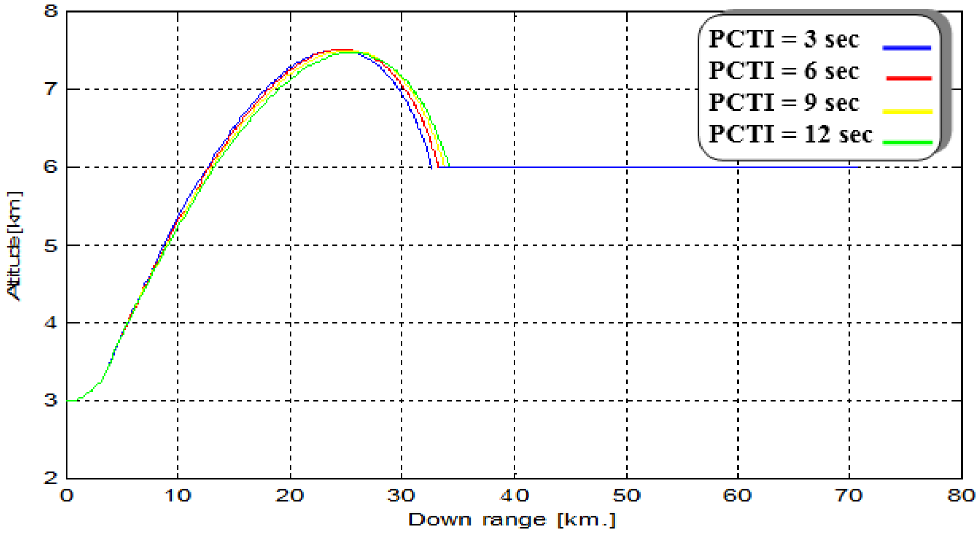


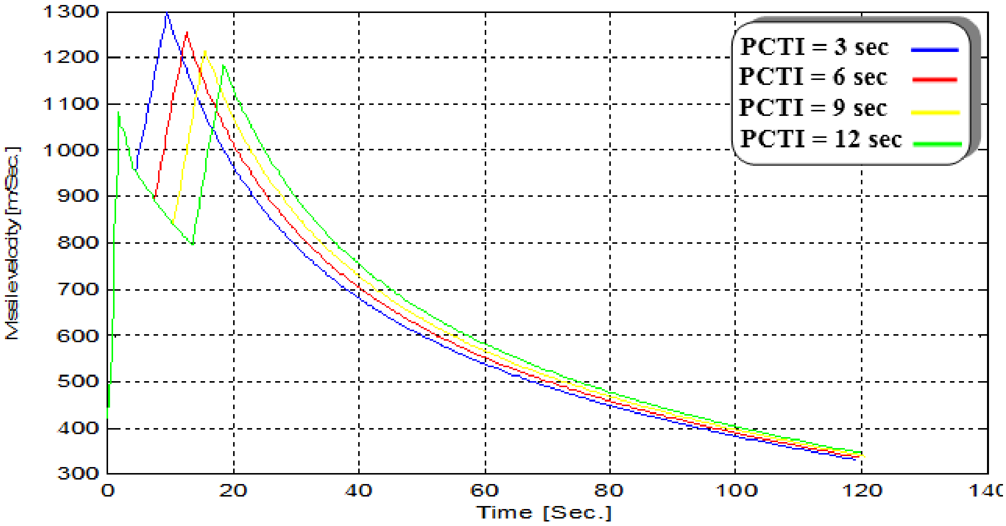
(d)

Fig. 2: Thrust Profile

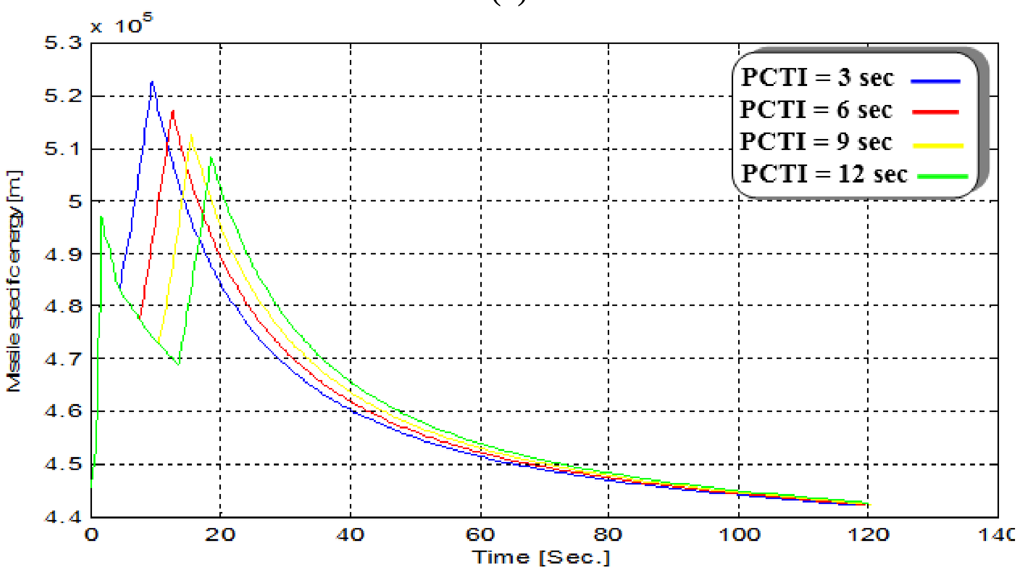
a. coast Time Interval is 3 sec. b. Coast Time Interval is 6 sec. c. Coast Time Interval is 9 sec. d. Coast Time Interval is 12 sec.


PCTI = 3 sec
PCTI = 6 sec
PCTI = 9 sec
PCTI = 12 sec
Altitude [km]
Down range [km.]

(a)


PCTI = 3 sec
PCTI = 6 sec
PCTI = 9 sec
PCTI = 12 sec
Missile velocity [m/Sec.]
Time [Sec.]

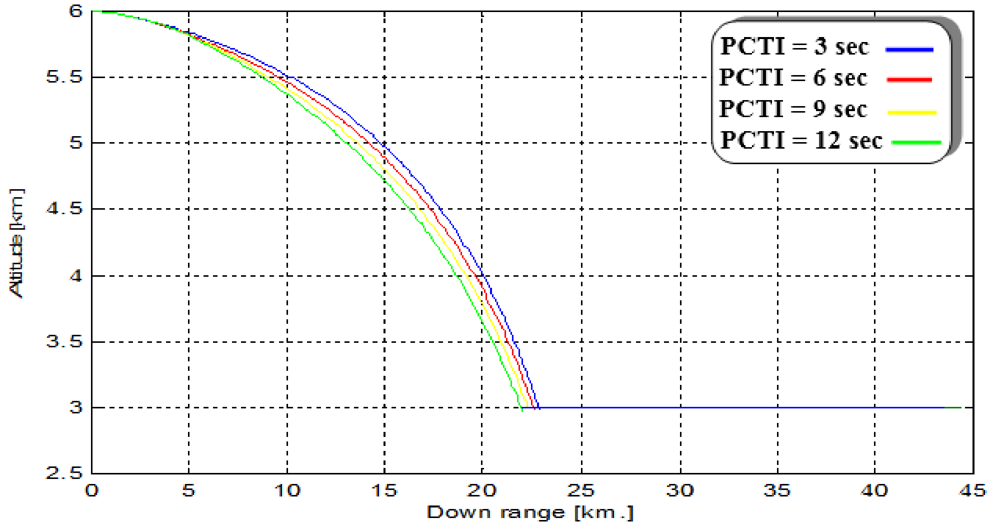
(b)


x 10^5
PCTI = 3 sec
PCTI = 6 sec
PCTI = 9 sec
PCTI = 12 sec
Missile specific energy [m]
Time [Sec.]

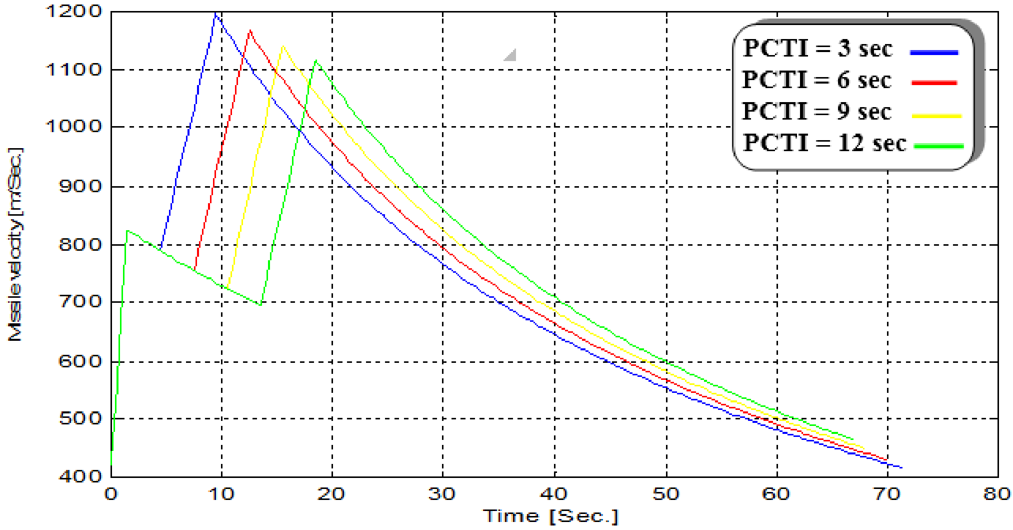
(c)

Fig. 3: Snap up case study

a. Missile and Target Trajectories. b. Missile Velocities. c. Missile Specific Energy.



(a)



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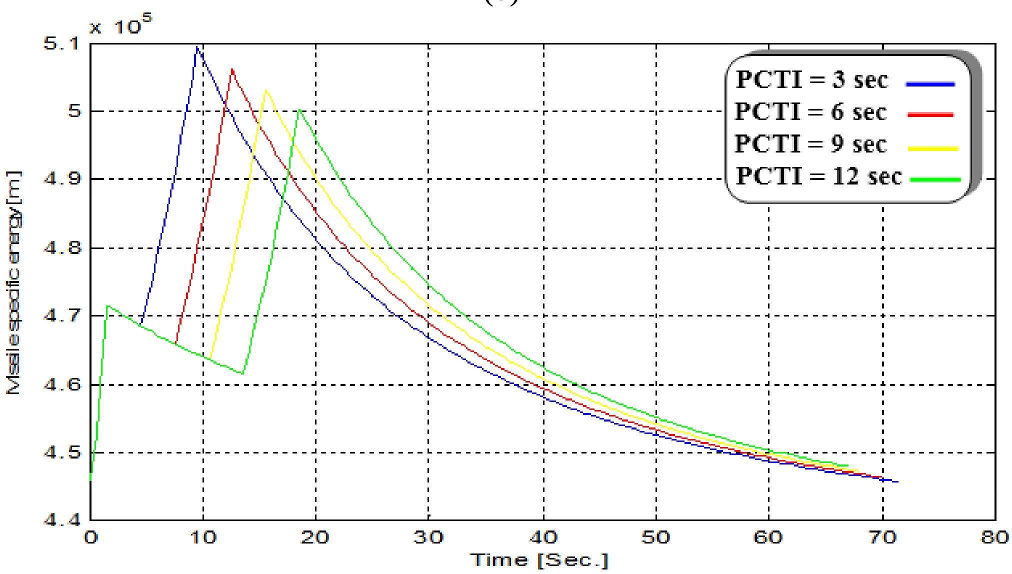

(c)

Fig. 4: Snap Down Case Study

a. Missile and target trajectories. b. Missile velocities. c. Missile specific energy.

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