INVERSE DOMINATION TO THE TRANSFORMATION OF SOME GRAPH

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Abstract - A set D of vertices in a graph $G(V,E)$ is a dominating set of G , if every vertex $v \in V - D$ is adjacent to at least one vertex in D. If $V - D$ contains a dominating set D' of G then D' is called an inverse dominating set with respect to D. In this paper we have established some theorems and properties of the inverse domination parameters to the transformation of path (by removing any one vertex) and cubic symmetric graph.

Keywords: Graph, Domination, Inverse Domination, cubic symmetric graph, Transformation .

1. INTRODUCTION

Graph Theory is one of the important tool in Applied Mathematics. In a current scenario, domination and inverse domination of sets place a vital role in day - to – day life. Allan [1], Cockayne [2] have derived various domination parameters of graphs. Kulli and Sigarkanthi [3] first introduced the concept of Inverse domination. The transformation graph G^{xyz} of G was introduced by Wu and Meng in 2001. For each graph we can obtain eight transformation graph by using symbols $(+,-)$ for all values of x, y, z.

Definition 1.1 : A graph G is an ordered pair $(V(G), E(G))$ consisting of a non empty set $V(G)$ of vertices and a set $E(G)$ disjoint from $V(G)$ of edges together with an incidence function ψ_G associates with each edge of G.

Definition 1.2 : A cubic symmetric graph is regular graph of order three. Cubic graphs must have an even number of vertices. It consists of 8 vertices with each of degree 3.

Definition 1.3 : $A\ set\ D$ of vertices in a graph G is a dominating set if every vertex not in D is adjacent to atleast one vertex in D. The number of vertices in a minimum dominating set of G is called a domination number of G, denoted by $\gamma(G)$.

Definition 1.4 : Let D be a minimum dominating set of G. If $V - D$ contains a dominating set say $D'\ of\ G$ then D' is called an inverse dominating set with respect to D. The inverse domination number $\gamma'(G)\ of\ G$ is the order of a smallest inverse dominating set of G.

Definition 1.5 : Let $G = (V(G), E(G))$ be a graph and x, y, z be three variables taking the values + or -. Vertex set of the transformation graph G^{xyz} of G is $V(G) \cup E(G)$ and each pair of $(\alpha, \beta) \in V(G^{xyz})$ is adjacent if and only if the following holds.

i). $\alpha, \beta \in V(G), \alpha\ and\ \beta$ are adjacent in G if $x = +$; $\alpha\ and\ \beta$ are not adjacent in $G\ if\ x = -$.

ii). $\alpha, \beta \in E(G),\ \alpha$ and β are adjacent in G if $y = +$; α and β are not adjacent in $G\ if\ y = -$.

iii). $\alpha \in V(G), \beta \in E(G), \alpha$ and β are adjacent in G if $z = +$; α and β are not adjacent in G if $z = -$.

2. Inverse Domination Parameters to the transformation of path (by removing any one vertex) :

Theorem 2.1 :

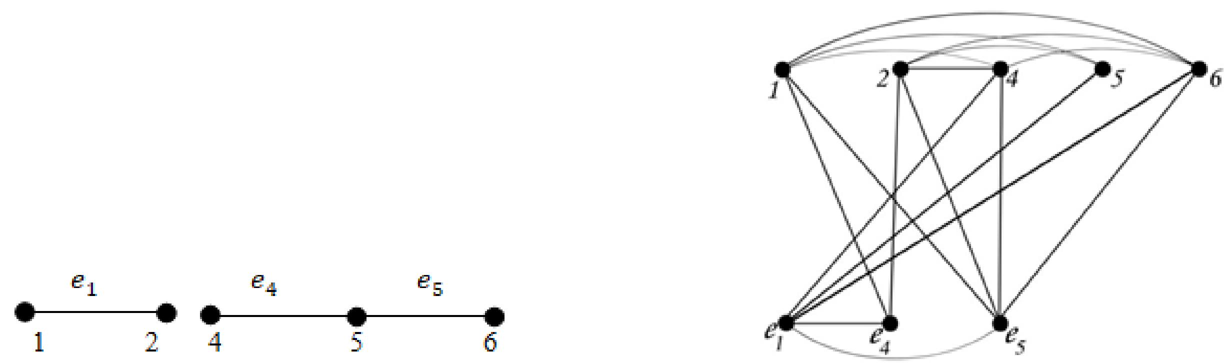
Let $G = P_n - \{v\}, n \geq 5$ *and* G^{---} is the transformation of G then $\gamma'(G^{---}) = 2$.

Proof :

Let $G = P_n - \{v\}$, $V(G) = \{v_i /1 \leq i \leq n-1\}$, $E(G) = \{e_j/v_i v_{i+1}/1 \leq j \leq n-3\}$ are the vertices and edges of G. Then $V(G^{---}) = \{v_i, e_j/ 1 \leq i \leq n-1, j = 1,2, \ldots .(n-3)\}$ is the vertex set of the transformation of G. $|V(G^{---})| = 2n-4$, $d(v_1) = d(v_n) = 2n-7$, $d(e_1) = d(e_n) = 2n-8$ for all $v_{n,} e_n \in V(G^{---})$.

Illustration 2.2 :

$G = P_6 - \{v\}$ and its transformation G^{---} is given in Figure – 1.



$G = P_6 - \{3\}$ G^{---}

Figure - 1

$N(v_1) = V(G^{---}) - \{v_2, e_1 /v_{1,} v_{2,} e_1 \in V(G^{---})\}$ and $\{v_2, e_1\} \in N(v_n)$ for all $e_1, v_2, v_n \in V(G^{---})$. Since each $v_i, 2 \leq i \leq n-2$ is adjacent with v_{i-1} and v_{i+1} and incident with e_{i-1} and ei in G, $v_{i-1,}\ v_{i+1}, e_{i-1}$ *and* $e_{i,} \notin N(v_i)$ *in* (G^{---}). Similarly, each $e_i \in E(G)$ is adjacent with $e_{i-1,}\ e_{i+1}$ is incident with v_i *and* v_{i+1} *in* G, e_{i-1}, $e_{i+1,} v_i$ *and* $v_{i+1} \notin N(e_i)$ *in* G^{---} . Therefore either $\{v_i\}$ or $\{e_i\}$ is not a dominating set of G^{---}. But, two vertices dominate the entire graph. Figure - (1) shows that such vertices form a minimum dominating set of G^{---}. Thus $\gamma(G^{---}) = 2$. Also, there exists one more dominating set D' which has the same number of vertices and hence forms an inverse dominating set of G^{---}. Thus, $\gamma'(G^{---}) = 2$.

Theorem 2.3 :

The inverse domination to the transformation G^{---} of path $P_n - \{v\}$ *is* $\left\lceil \frac{n}{3} \right\rceil$

Proof :

Let $G = P_n - \{v\}$ be the undirected transformation of path graph. Let $\{v_i | 1 \leq i \leq n-1\}$ be the vertices of G. By theorem 2.1, $\gamma'(G^{---}) = 2$. To determine the required result, we consider the following cases,

Case (i) : When $n = 3k+1, k > 0$

Consider the vertex v_{3k+1}. It is adjacent to the vertices $v_{3k,} v_{3k+2}$ and its incident edges are e_{3k}, e_{3k+1} in G. But $v_{3k}, v_{3k+2,} e_{3k}, e_{3k+1} \notin N(v_{3k+1})$ in G^{---}. Thus, we obtain the inverse domination to the transformation path and hence $|D'| = \left\lceil \frac{n}{3} \right\rceil$

Case (ii) When $n = 3k+2, k > 0$

Consider the vertex $v_{3k+2.}$ It is adjacent to the vertices $v_{3k+1,}v_{3k+3}$ and its incident edges are e_{3k+1}, e_{3k+2} $in\ G$. But, $v_{3k+1,}v_{3k+3}, e_{3k+1}, e_{3k+2} \notin$ N$(v_{3\text{k}+2})$. Thus, we obtain the inverse domination to the transformation of path and hence $|D'| = \left\lceil \frac{n}{3} \right\rceil$.

Corollary 2.4 :

(i) Transformation G^{---} of $P_4 - \{v\}$ becomes a star graph $k_{1,3}$ and its inverse domination is not possible.

(ii) The transformation of G^{+++} of $P_n - \{v\}$ is disconnected.

3. Inverse Domination parameters to the transformation of cubic symmetric graph:

Theorem 3.1 :

If G is any cubic symmetric graph then $\gamma'\ (G^{---}) = 2$.

Proof :

Let G^{---} be the transformation of the cubic symmetric graph. The vertex and edge set are respectively denoted by $V(G) = \{v_i\ /i = 1,2, \ldots .8\}$ and $E(G) = \{e_j\ /j = 1,2, \ldots .12\}$.

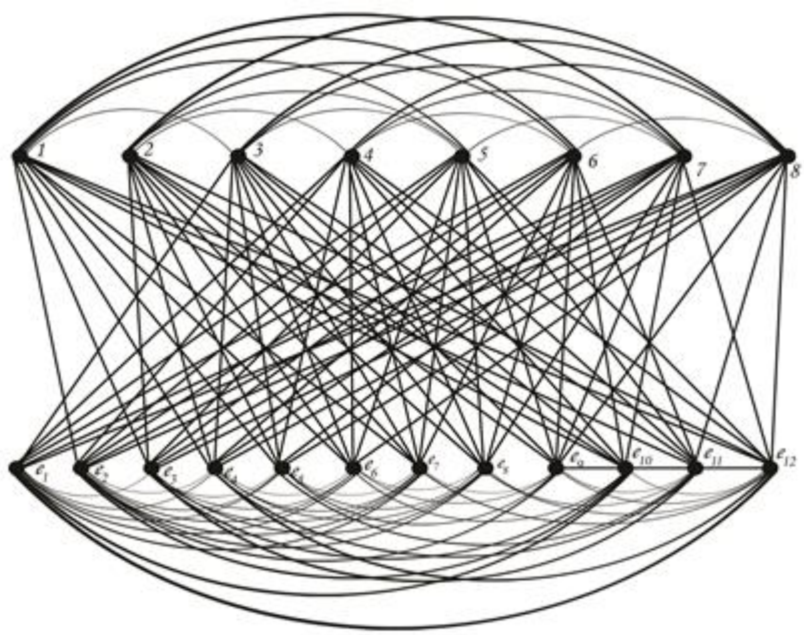


Figure – 2 : G^{---}

In Figure – 2, $deg\ (v_i) = deg(e_j) = 13$, It is enough to have only two vertices to dominate all the remaining vertices of G^{---}. Suppose V is the set of all vertices and D is the dominating set, then there exists another set D' which still dominates the vertices of G^{---}. Thus, D' is the inverse dominating set with minimum cardinality 2 and hence $\gamma'(G^{---}) = 2$.

Theorem 3.2 :

(i) If G is a cubic symmetric graph, then the minimal cardinality of the inverse domination of the transformation G^{+++} is 4. That is $\gamma'(G^{+++}) = 4$.

(ii) In G^{+++}, $deg(v_i) = deg(e_j) = 6$, $\forall\ i = 1,2, \ldots \ldots ,8$, and $j = 1,2, \ldots \ldots \ldots . . ,12$.

Proof :

Graph of G^{+++} is given in Figure – 3. Clearly, $\gamma'(G^{+++}) = 4$ and $eg(v_i) = deg(e_j) = 6$, $\forall\, i = 1,2, \dots \dots, 8$, and $j = 1,2, \dots \dots \dots .., 12$ in G^{+++}.

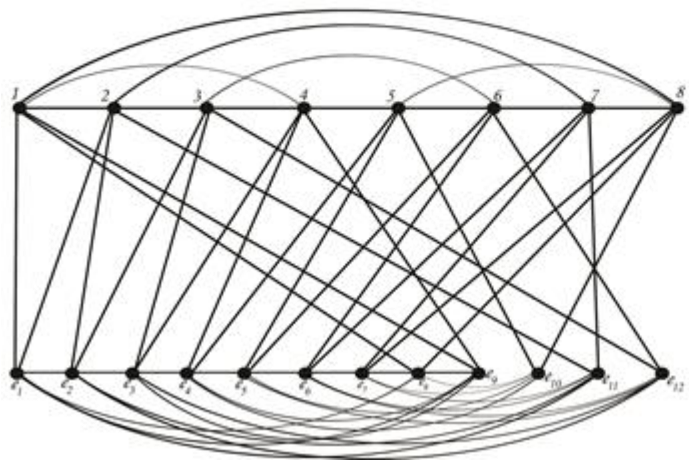


Figure – 3 : G^{+++}

Theorem 3.3 :

(i) $\gamma'(G^{++-}) = 2$

(ii) $\gamma'(G^{+-+}) = 4$

(iii) $\gamma'(G^{-++}) = 4$

(iv) $\gamma'(G^{-+-}) = 2$

(v) $\gamma'(G^{--+}) = 4$

(vi) $\gamma'(G^{+--}) = 2$

Corollary 3.4 :

If G is a cubic symmetric graph, then the inverse domination number for the transformation is

(i) $\gamma'(G^{+++}) = \gamma'(G^{+-+}) = \gamma'(G^{-++}) = \gamma'(G^{--+}) = 4$

(ii) $\gamma'(G^{++-}) = \gamma'(G^{-+-}) = \gamma'(G^{+--}) = \gamma'(G^{---}) = 2$.

4. CONCLUSION

Wu and Meng introduced the concept of the transformation graph G^{xyz} of G in 2001. Up to now, many researchers research and develop this theory. In this paper, we developed some useful theorems and discussed few properties of inverse domination parameters to the transformation of path (by removing any one vertex) and cubic symmetric graph.

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