EPQ MODEL – AN ENDOMMAGER ETYMOLOGY

W.Ritha and R.Saarumathi
Department of Mathematics Holy Cross College (Autonomous) Affiliated to Bharathidasan University Tiruchirappalli – 620002 Tamilnadu, India

ABSTRACT:
Economic Order Quantity (EOQ), an inventory model is ideal to produce cent percent ordered items. But practically, this assumption becomes invalid as process deterioration, production processes and other factors pop up. The customer's expectation and marketing demands can be satisfied, if only the production rate becomes a function of demand rate. As the demand and deterioration products are time dependent function, eventually allowed shortages will be partially backlogged. Rate of backlog is contingent on the waiting time up to the arrival next lot. To prevail over these sparsities, this paper signifies the application of EPQ model to contemplate the optimal production run time, production quantity and shortage period aiming to minimise the total average cost.
KEYWORDS:
Economic Production Quantity (EPQ), deteriorating items, trapezoidal fuzzy number, Yager's ranking index.

1. INTRODUCTION:

It is very important to understand economic production quantity from the inventory management side of things. To hold maximum inventory is costly to carry. Holding minimum can incur stock outs, lost sales and shutdown of production plant. Inventory managers and practitioners can overcome such short comings through EPQ model that is used to determine the optimal order quantity that an organization should place with a supplier to minimise inventory costs, while balancing inventory holding and average fixed costs.

An inventory model emphasizes on the concept of deterioration and shortages. Although the rate of deterioration cannot be neglected, it's imperative to scrutinize the product's rate of deterioration in the determination of lot size of products.

EOQ model was presented by Singh et.al (2016) for deteriorating products which experience stock dependent demand with trade credit period and preservation technology.
Khurana and Chaudhary (2016) bestowed an optimal pricing and ordering policy for deteriorating items with price and stock dependent demand and partial backlogging.

Due to customer'sexpectation in advanced customisation, a significant increase in market survey is materialised. Hence to renumerate the customer's demand, it is requisite to scrutinize the production rate as a function of occurring demand. This paper centralises (fixates) a production inventory model for deteriorating products over a finite planning perspective in which the production rate is demand rate dependent. The existing shortages are acknowledged and partially backlogged at a perpetual outlay.

2. NOTATIONS:

I - Inventory level during the given time t

s_c - Setup cost

h - Holding inventory cost
k - coefficient of deterioration, $k \succ 0$
α - production coefficient
a - initial rate of demand
b - coefficient of demand
M - Maximum level of inventory
T - Time span of inventory cycle
d - deterioration cost per tariff
p - production cost per unit time
S - shortage cost
l_s - lost sale cost
θ - rate of backlogging
v - measure for which inventory level becomes zero
t_1 - maximum time period of production

3. ASSUMPTIONS:

1. The product demand is linear in nature and time dependent $ie., D = a + bt$
2. Production rate $P = \alpha(a + bt)$ when production rate is considered as a function of demand rate.
3. The product deterioration rate is inclined as k_t when it is a function of $'t'$
4. Partial backlog when shortages are permitted.
5. Inexhaustible capacity of the warehouse.
6. Absolute rejection of the deteriorated items.

4. MATHEMATICAL MODEL:

4.1 PROPOSED INVENTORY MODEL IN CRISP SENSE:

At $t = 0$, production rate commences and continues upto $t = t_1$ to satisfy demand and deterioration. At this instant $[t_1, v]$ the inventory level expands to out – turn the demand and deterioration for shortages. Applying the concept of differential equation, this method extends the change in inventory level with respect to time. Hence, the differential equation directing the transition as per the system is given by,

Let set up cost be s_c

Holding cost is expressed as,

$$h\left\{\left[(\alpha-1)\left(\frac{at_1^{\,2}}{2}+\frac{bt_1^{\,3}}{6}-\frac{kat_1^{\,4}}{12}-\frac{kbt_1^{\,5}}{40}\right)\right]+\left(Mve^{kt_1^2/2}+a\left(t_1v-\frac{v^2}{2}\right)+\frac{b}{2}\left(t_1^{\,2}v-\frac{v^3}{3}\right)+\frac{ak}{6}\left(t_1^{\,3}v-\frac{v^4}{4}\right)+\frac{bk}{8}\left(t_1^{\,4}v-\frac{v^5}{5}\right)-\frac{Mkv^3}{6}e^{kt_1^2/2}-\frac{ka}{2}\left(t_1\frac{v^3}{3}-\frac{v^4}{4}\right)-\frac{kb}{4}\left(\frac{t_1^{\,2}v^3}{3}-\frac{v^5}{5}\right)-Mt_1e^{kt_1^2/2}-\frac{at_1^{\,2}}{2}-\frac{bt_1^{\,3}}{3}-\frac{akt_1^{\,4}}{12}-\frac{bkt_1^{\,5}}{15}+\frac{Mkt_1^{\,3}}{6}e^{kt_1^2/2}\right)\right\}$$

Deterioration cost is given by $d\left\{\alpha\left(at_1+\frac{bt_1^{\,2}}{2}\right)-\left(av+\frac{bv^2}{2}\right)\right\}$

Production cost is expressed as $\alpha\left(at_1+\frac{bt_1^{\,2}}{2}\right)p$

Shortage cost is defined as $S\theta\left(aT+\frac{bT^2}{2}-av-\frac{bv^2}{2}\right)$

Lost sale cost is identified as $l_s(1-\theta)\left(aT+\frac{bT^2}{2}-av-\frac{bv^2}{2}\right)$

Total cost per unit is given by,

$$TC=\frac{1}{T}\left\{\left[\alpha\left(at_1+\frac{bt_1^{\,2}}{2}\right)p\right]+\left[d\left\{\alpha\left(at_1+\frac{bt_1^{\,2}}{2}\right)-\left(av+\frac{bv^2}{2}\right)\right\}\right]+\left\{h\left[(\alpha-1)\left(\frac{at_1^{\,2}}{2}+\frac{bt_1^{\,3}}{6}-\frac{kat_1^{\,4}}{12}-\frac{kbt_1^{\,5}}{40}\right)\right]+\left(Mve^{kt_1^2/2}+a\left(t_1v-\frac{v^2}{2}\right)+\frac{b}{2}\left(t_1^{\,2}v-\frac{v^3}{3}\right)+\frac{ak}{6}\left(t_1^{\,3}v-\frac{v^4}{4}\right)+\frac{bk}{8}\left(t_1^{\,4}v-\frac{v^5}{5}\right)-\frac{Mkv^3}{6}e^{kt_1^2/2}-\frac{ka}{2}\left(t_1\frac{v^3}{3}-\frac{v^4}{4}\right)-\frac{kb}{4}\left(\frac{t_1^{\,2}v^3}{3}-\frac{v^5}{5}\right)-Mt_1e^{kt_1^2/2}-\frac{at_1^{\,2}}{2}-\frac{bt_1^{\,3}}{3}-\frac{akt_1^{\,4}}{12}-\frac{bkt_1^{\,5}}{15}+\frac{Mkt_1}{6}e^{kt_1^2/2}\right]\right\}+s_c+S\theta\left(aT+\frac{bT^2}{2}-av-\frac{bv^2}{2}\right)+l_s(1-\theta)\left(aT+\frac{bT^2}{2}-av-\frac{bv^2}{2}\right)\right\}$$

4.2 PROPOSED INVENTORY MODEL IN FUZZY SENSE USING YAGER'S RANKING INDEX:

Here s_c, S, h, l_s, p, d are considered as trapezoidal fuzzy numbers where s_c is the set up cost per production unit, S is the shortage cost per unit, h is the holding cost, l_s is the lost sale cost, p is the production cost, d is the deterioration cost and are defined by their α cuts as follows:

$s_c(\alpha_{s_c})=[L^{-1}{}_{s_c}(\alpha_{s_c});R^{-1}{}_{s_c}(\alpha_{s_c})]$

$S(\alpha_S)=[L^{-1}{}_S(\alpha_S);R^{-1}{}_S(\alpha_S)]$

$h(\alpha_h)=[L^{-1}{}_h(\alpha_h);R^{-1}{}_h(\alpha_h)]$

$l_s(\alpha_{l_s})=[L^{-1}{}_{l_s}(\alpha_{l_s});R^{-1}{}_{l_s}(\alpha_{l_s})]$

$$p(\alpha_p)=[L^{-1}{}_p(\alpha_p);R^{-1}{}_p(\alpha_p)]$$

$$d(\alpha_d)=[L^{-1}{}_d(\alpha_d);R^{-1}{}_d(\alpha_d)]$$

The fuzzy total cost is given by,

$$\tilde{TC}=\frac{1}{T}\left\{\left[\left[\alpha\left(at_1+\frac{bt_1^{\,2}}{2}\right)\tilde{p}\right]+\left[\tilde{d}\left\{\alpha\left(at_1+\frac{bt_1^{\,2}}{2}\right)-\left(av+\frac{bv^2}{2}\right)\right\}\right]+\left\{\tilde{h}\left[(\alpha-1)\left(\frac{at_1^{\,2}}{2}+\frac{bt_1^{\,3}}{6}-\frac{kat_1^{\,4}}{12}-\frac{kbt_1^{\,5}}{40}\right)\right]+\right.\right.$$
$$\left(Mve^{kt_1^2/2}+a\left(t_1v-\frac{v^2}{2}\right)+\frac{b}{2}\left(t_1^{\,2}v-\frac{v^3}{3}\right)+\frac{ak}{6}\left(t_1^{\,3}v-\frac{v^4}{4}\right)+\frac{bk}{8}\left(t_1^{\,4}v-\frac{v^5}{5}\right)-\frac{Mkv^3}{6}e^{kt_1^2/2}-\right.$$
$$\left.\left.\frac{ka}{2}\left(t_1\frac{v^3}{3}-\frac{v^4}{4}\right)-\frac{kb}{4}\left(\frac{t_1^{\,2}v^3}{3}-\frac{v^5}{5}\right)-Mt_1e^{kt_1^2/2}-\frac{at_1^{\,2}}{2}-\frac{bt_1^{\,3}}{3}-\frac{akt_1^{\,4}}{12}-\frac{bkt_1^{\,5}}{15}+\frac{Mkt_1}{6}e^{kt_1^2/2}\right]\right\}+\tilde{s}_c+$$
$$\left.\tilde{S}\theta\left(aT+\frac{bT^2}{2}-av-\frac{bv^2}{2}\right)+\tilde{l}_s(1-\theta)\left(aT+\frac{bT^2}{2}-av-\frac{bv^2}{2}\right)\right\}$$

Using Yager's ranking index for the above equation we have,

$$K_1(\alpha_p)=\frac{1}{2}[\int_0^1 L_{\alpha_p}{}^{-1}(\alpha_p)d\alpha_p+R_{\alpha_p}{}^{-1}(\alpha_p)d\alpha_p]$$

$$K_2(\alpha_d)=\frac{1}{2}[\int_0^1 L_{\alpha_d}{}^{-1}(\alpha_d)d\alpha_d+R_{\alpha_d}{}^{-1}(\alpha_d)d\alpha_d]$$

$$K_3(\alpha_h)=\frac{1}{2}[\int_0^1 L_{\alpha_h}{}^{-1}(\alpha_h)d\alpha_h+R_{\alpha_h}{}^{-1}(\alpha_h)d\alpha_h]$$

$$K_4(\alpha_{s_c})=\frac{1}{2}[\int_0^1 L_{\alpha_{s_c}}{}^{-1}(\alpha_{s_c})d\alpha_{s_c}+R_{\alpha_{s_c}}{}^{-1}(\alpha_{s_c})d\alpha_{s_c}]$$

$$K_5(\alpha_S)=\frac{1}{2}[\int_0^1 L_{\alpha_S}{}^{-1}(\alpha_S)d\alpha_S+R_{\alpha_S}{}^{-1}(\alpha_S)d\alpha_S]$$

$$K_6(\alpha_{l_s})=\frac{1}{2}[\int_0^1 L_{\alpha_{l_s}}{}^{-1}(\alpha_{l_s})d\alpha_{l_s}+R_{\alpha_{l_s}}{}^{-1}(\alpha_{l_s})d\alpha_{l_s}]$$

Therefore, on substituting these ranking values the fuzzy total cost becomes,

$$\tilde{TC} = \frac{1}{T}\left\{\left[\alpha\left(at_1 + \frac{bt_1^{\,2}}{2}\right)K_1(\alpha_p)\right] + \left[K_2(\alpha_d)\left\{\alpha\left(at_1 + \frac{bt_1^{\,2}}{2}\right) - \left(av + \frac{bv^2}{2}\right)\right\}\right] + \right.$$

$$\left\{K_3(\alpha_h)\left[(\alpha-1)\left(\frac{at_1^{\,2}}{2} + \frac{bt_1^{\,3}}{6} - \frac{kat_1^{\,4}}{12} - \frac{kbt_1^{\,5}}{40}\right)\right] + \right.$$

$$\left(Mve^{kt_1^{\,2}/2} + a\left(t_1 v - \frac{v^2}{2}\right) + \frac{b}{2}\left(t_1^{\,2}v - \frac{v^3}{3}\right) + \frac{ak}{6}\left(t_1^{\,3}v - \frac{v^4}{4}\right) + \frac{bk}{8}\left(t_1^{\,4}v - \frac{v^5}{5}\right) - \frac{Mkv^3}{6}e^{kt_1^{\,2}/2} - \right.$$

$$\left.\left.\frac{ka}{2}\left(t_1\frac{v^3}{3} - \frac{v^4}{4}\right) - \frac{kb}{4}\left(\frac{t_1^{\,2}v^3}{3} - \frac{v^5}{5}\right) - Mt_1e^{kt_1^{\,2}/2} - \frac{at_1^{\,2}}{2} - \frac{bt_1^{\,3}}{3} - \frac{akt_1^{\,4}}{12} - \frac{bkt_1^{\,5}}{15} + \frac{Mkt_1}{6}e^{kt_1^{\,2}/2}\right]\right\} + K_4(\alpha_{s_c}) +$$

$$\left.K_5(\alpha_S)\theta\left(aT + \frac{bT^2}{2} - av - \frac{bv^2}{2}\right) + K_6(\alpha_{l_s})(1-\theta)\left(aT + \frac{bT^2}{2} - av - \frac{bv^2}{2}\right)\right\}$$

5. NUMERICAL EXAMPLE:

CRISP SENSE:

$a = 100$ units, $b = 2$ unit, $k = 0.01$, $l_s =$ Rs.4 /unit, $h =$ Rs.0.2 /unit, $d =$ Rs.12 /unit,

$S =$ Rs.3 /unit, $p =$ Rs.10 /unit, $T = 30$, $\alpha = 1.2$, $A =$ Rs.250 /production run

The total cost is Rs.1152.97

FUZZY SENSE:

$a = 100$ units, $b = 2$ unit, $k = 0.01$, $T = 30$, $\alpha = 1.2$, $\tilde{p}(\alpha_p) = (7,9,11,13)$

$\tilde{d}(\alpha_d) = (9,11,13,15)$, $\tilde{h}(\alpha_h) = (0.05,0.09,0.06,0.85)$, $\tilde{s_c}(\alpha_{s_c}) = (175,225,275,325)$,

$\tilde{S}(\alpha_S) = (1.5,2.5,3.5,4.5)$, $\tilde{l_s}(\alpha_{l_s}) = (1,3,5,7)$

we have,

$L^{-1}{}_p(\alpha_p) = 7 + 2\alpha$; $R^{-1}{}_p(\alpha_p) = 13 - 2\alpha$

$L^{-1}{}_d(\alpha_d) = 9 + 2\alpha$; $R^{-1}{}_p(\alpha_p) = 15 - 2\alpha$

$L^{-1}{}_h(\alpha_h) = 0.05 + 0.04\alpha$; $R^{-1}{}_p(\alpha_p) = 0.85 - 0.79\alpha$

$L^{-1}{}_{s_c}(\alpha_{s_c}) = 170 + 50\alpha$; $R^{-1}{}_{s_c}(\alpha_{s_c}) = 325 - 50\alpha$

$L^{-1}{}_S(\alpha_S) = 1.5 + \alpha$; $R^{-1}{}_S(\alpha_S) = 4.5 - \alpha$

$L^{-1}{}_{l_s}(\alpha_{l_s}) = 1 + 2\alpha$; $R^{-1}{}_{l_s}(\alpha_{l_s}) = 7 - 2\alpha$

$$K_1(\alpha_p) = \frac{1}{2}[\int_0^1 7 + 2\alpha\, d\alpha + \int_0^1 13 - 2\alpha\, d\alpha] = 10$$

$$K_2(\alpha_d) = \frac{1}{2}[\int_0^1 9 + 2\alpha\, d\alpha + \int_0^1 15 - 2\alpha\, d\alpha] = 12$$

$$K_3(\alpha_h) = \frac{1}{2}[\int_0^1 0.05 + 0.04\alpha\, d\alpha + \int_0^1 0.85 - 0.79\alpha\, d\alpha] = 0.2625$$

$$K_4(\alpha_{s_c})=\frac{1}{2}[\int_0^1 170+50\alpha\, d\alpha+\int_0^1 325-50\alpha\, d\alpha]=250$$

$$K_5(\alpha_S)=\frac{1}{2}[\int_0^1 1.5+\alpha\, d\alpha+\int_0^1 4.5-\alpha\, d\alpha]=3$$

$$K_6(\alpha_{l_s})=\frac{1}{2}[\int_0^1 1+2\alpha\, d\alpha+\int_0^1 7-2\alpha\, d\alpha]=4$$

Fuzzy total cost is Rs.602.5024

CONCLUSION:

In a market economy, any price movement can be explained by a temporary difference between what providers are supplying and what consumers are demanding. Supply is the amount of articles people want to purchase. Deterioration of goods overtime and occurrence of shortages is a natural feature in one's life. This paper culminates a production inventory model EPQ using Yager's ranking method to spotlight that deterioration rate is time dependent and production rate is a function of demand rate. To agglomerate, the solution obtained through this unique method of Yager's ranking is found to be effectual for all conditions of shortages and deterioration.

REFERENCES:

[1] Goyal,S.K, 'An integrated inventory model for a single supplier – single customer problem', International Journal of Operational Research, Vol.15, No.1, pp.107-111, (1997)
[2] Goyal,S.K, 'A comment on economic order quantity under conditions of permissible delay in payments', Journal of Operational Research, Vol.36, No.4, pp.335-338, (1988)
[3] Hill,R.M, 'The optimal production and shipment policy for single vendor – single buyer integrated production – integrated inventory problrem', International Journal of Production Research, Vol.37, No.11, pp.2463-2475, (1999)
[4] Hsu,P.H., Wee,H.M and Teng,H.M, 'Optimal lot sizing for deteriorating items with expiration date', Journal of Information and Optimization sciences, Vol.27, No.2, pp.271-286, (2006)
[5] Sarkar,B. and Moon,I., 'An EPQ model with inflation in an imperfect production system', Applied Mathematics and Computation, Vol.217, No.13, pp.6159-6167, (2011)
[6] Singh,C. and Singh,S.R., 'An integrated supply chain model for perishable items with trade credit policy under imprecise environment', International Journal of Computer Applications, Vol.48, No.20, pp.41-45, (2011)
[7] Singh,N.,Vaish,B. and Singh,S.R., 'An economic production lot size (EPLS) model with rework and flexibility under allowable shortages', International Journal of Procurement Management, Vol.5, No.1, pp.104-122, (2012)
[8] Tayal,S., Singh,S.R. and Sharma,R., 'An inventory model for deteriorating items with seasonal products and an option of an alternative market, Uncertain Supply Chain Management, Vol.3, No.1, pp.69-86, (2014a)

[9] Tayal,S., Singh,S.R. and Sharma,R., 'Multi item inventory model for deteriorating items with expiration date and allowable shortages', Indian Journal of Science and Technology, Vol.7, No.4, pp.463-471, (2014b)

[10] Tayal,S., Singh,S.R., Chauhan,A. and Sharma,R., 'A deteriorating production inventory problem with space restriction', Journal of Information and Optimization Sciences, Vol.35, No.3, pp.203-229, (2014c)

[11] Tayal,S., Singh,S.R., and Sharma,R., 'An integrated production inventory model for perishable products with trade credit period and investment in preservation technology', International Journal of Mathematics in Operational Research, Vol.8, No.2, pp.137-163, (2016)

[12] Tayal,S., Singh,S.R., Sharma,R. and Singh,A.P., 'An EPQ model for non-instantaneous deteriorating items with time dependent holding cost and exponential demand rate', International Journal Operational Research, Vol.23, No.2, pp.145-161, (2015)

[13] Wee,H.M., 'Joint pricing and replenishment policy for deteriorating inventory with declining market', International Journal of Production Economics, Vol.40, No.2, pp.163-171, (1995)

[14] Zhang,Y., Balochain,S., Agarwal,P., Bhatnagar,V., and Housheya,O.J., 'Artificial intelligence and its applications,' Mathematical problems in Engineering, p.10, (2014)