HAMILTONIANI COMPLETE FUZZY CYCLES ON COMPLETE FUZZY GRAPHS

S. Venu Madhava Sarma[1], K. Prasad[2], K. Pushphaltha[3], T.Anitha[4],Harisainadh Komatineni[5]
[1,2,3,4]*Department of Mathematics, Koneru Lakshmaiah Education Foundation, Vaddeswaram, Guntur(Dt), A.P., India*
[5]*Department of Computer Science, Koneru Lakshmaiah Education Foundation, Vaddeswaram, Guntur(Dt), A.P., India.*

ABSTRACT: A Hamiltonian Path is the path passing through every vertex of the graph. In this paper the notion of translation shows a dual part in both theoretical and practical requests of complete Fuzzy Charts. In this research work proved with Hamiltonian complete fuzzy sequence on K_{2n+1} complete fuzzy chart. Compared go existed methods this work gives more improvement and compete with present applications.

Key Words: Graph, Fuzzy Graph

1. INTRODUCTION

Fuzzy graph analysis gives the more applications in different areas, these applications are helpful for many technologies such as IT, neural networks, artificial intelligence, machine learning and cluster analysis etc. The modern science, some connectivity relations are related to fuzzy cut nodes and fuzzy bridges are founded by Bhatta Charya. The theory like decomposition integration and numerical analysis depending on Hamiltonian concepts. In this theoretical practical trees cycles parts and bridges or introduced for random analysis. In this research work fuzzy graphs Hamiltonian techniques are introduced for advanced applications. Earlier Rosenfeld had done some phenomenal work on fuzzy equivalents of numerous elementary chart- the notion of disintegration of charts into Hamiltonian cycles, paths into regular charts and hypothetical concepts like pledges, tracks, series, trees, and connectedness. Klas Markstrom presented. Among the variety of exemplary changes in science and technology, the concept of uncertainty played a significant role, which led to the development of fuzzy sets, which in turn helped in the transition from graph theory to fuzzy graph theory. This paper familiarizes an improved concept in fuzzy graphs, called contraction. Two types of contraction namely edge contraction and neighborhood contraction are introduced. Edge contraction is an act of merging the two end vertices of an edge, neighborhood contraction is the act of merging the two adjacent vertices

Notation 1.1: Hamiltonian computations are covers the vertices of $G\ in\ G*(V,E)$ and evolution paths p in graph

Notation 1.2: G_f = (σ,μ) be a fuzzy graph and let uv be the edge of G_f, then the edge contracted fuzzy graph w.r.t the edge uv is denoted by $G_f\backslash uv$ and is a chart with vertex set $V' = [V\backslash\{u,v\} \cup \{w\}]$ where $\sigma(G_f\backslash uv) = \sigma(G_f)\ \forall$ vertices $x \in V$, and

Notation 1.3 $G_f\backslash uv$ and is a graph with vertex set
$V' = [V\backslash\{u,v\} \cup \{w\}]$ where $\sigma(G_f\backslash uv) = \sigma(G_f)\ \forall$ vertices $x \in V$, and

$\beta\ (x,y)\ \le\ \alpha\ (x)\ \wedge\ \alpha\ (y)\ for\ all\ x,y\ \in\ S\ where\ \wedge$
$stands\ for\ minimum. The\ underlying\ crisp\ graph\ of\ the\ fuzzy\ graph\ G{:}\ (\alpha,\beta)\ is\ denoted\ as\ G*$
$:(V,E). Where\ E\ \subseteq=\ V\ \times\ V.$

Notation 1.4

$: P\ is\ the\ path\ length\ 'n'\ is\ a\ sequence\ of\ distinct\ nodes\ u0, u1, \dots \dots un\ such\ that$

$$\beta\ (ui\ -1, ui) >\ 0\ i1,2 \ldots\ldots n\ is\ called\ a\ fuzzy\ path\ and\ the\ degree\ of\ a\ membership\ of\ a\ minimum\ arc\ is\ defined\ as\ its\ length.$$

Notation 1.5: $If\ P\ is\ a\ cycle\ and\ u0\ =\ un\ and\ n \geq\ 3, then\ P\ is\ called\ a\ fuzzy\ cycle\ (f - cycle)\ if\ it\ contains\ more\ than\ one\ weakest\ arc.$

Notation 1.6: A fluffy way P in a fluffy chart G covers all the vertices of G totally once then the way is called Hamiltonian Fuzzy Path.

Notation 1.7:
$A\ fuzzy\ cycle\ C\ \boldsymbol{i}n\ a\ fuzzy\ graph\ G\ covers\ all\ the\ vertices\ of\ G\ absolutely\ once\ except\ the\ end\ vertices\ then\ the\ cycle\ is\ called\ Hamiltonian\ fuzzy\ cycle.$

Notation 1.8: $\boldsymbol{F}uzzy\ Graph\ G: (\alpha, \beta)\ is\ said\ to\ be\ Complete\ if\ \beta\ (x, y)\ = \alpha(x) \wedge \alpha(y)\ for\ all\ x\ \&\ y.$

Notation 1.9: $The\ complement\ of\ a\ fuzzy\ graph\ is\ denoted\ by\ Gc : (\alpha c, \beta c)\ , where\ \alpha c\ I = \alpha\ and\ \beta c\ (x, y)\ = \wedge [\ \alpha\ (x), \alpha(y)]\ -\ \beta(x, y)\ .$

2. RESULTS & APPLICATIONS

Notation 2.1: Kn is a finished fuzzy_chart with 'n' vertices.

Example 2.2: $If\ G_f\ is\ a\ uniform\ vertex\ fuzzy\ graph, then\ every\ vertex\ v\ will\ be\ a\ neighborhood\ contracted\ \gamma - fixed\ vertex\ of\ G_{f_{(v)}}\ for\ any\ vertex\ v.$

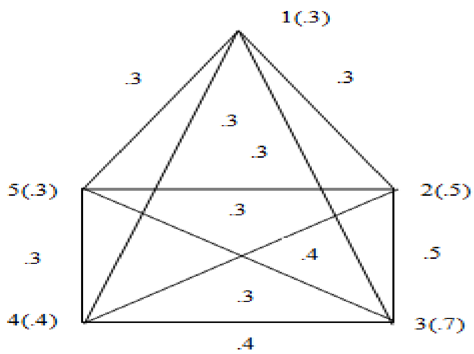
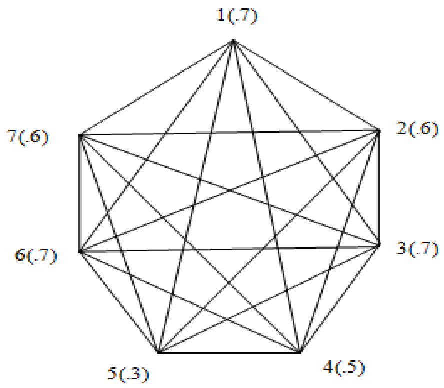


Fig. 1.K5 Fuzzy graph

Example2.3: In figure.2, A graph K7 is decomposed into $3c_7$. i.e. three Hamiltonian complete fuzzy cycles. Label the vertices clockwise



next vertex in the first copy of the Hamiltonian complete fuzzy

Fig.2. K7 Fuzzy Graph

Theorem 2.4: For every n ≥ 1, Complete fuzzy graph can be partitioned into n Hamiltonian complete fuzzy cycles C_{2n+1}.
Proof: Let G = K_{2n+1} be a complete fuzzy graph.

$$Label\ the\ vertices, 0,1,2,3,\ldots.,2n-1.$$
$$In\ this\ complete\ fuzzy\ graph\ form\ the\ Hamiltonian\ complete\ fuzzy\ cycles\ as\ follows.$$
$$(C1)\ \ is\ a\ complete\ fuzzy\ cycle\ that\ is$$
$$(C1), 0,2n-1,1,2n-2,2,2n-3,\ldots n-1,n,$$
$$(C2), 1,0,2,2n-1,3,2n-2,\ldots,n,n+1,$$
$$(C3)\ ,2,1,3,0,4,2n-1,\ldots..n+1,n+2,$$

. . .

$$\ldots.$$
$$(Cn)\ I,n-1,n-2,n,n-3,n+1,n-4,\ldots..2n-2,2n-1,.(so, each\ time\ we\ add\ 1\ to\ every\ label\ and\ find\ the\ remainder\ modulo\ 2n).$$
$$For\ example, label\ vertices\ of\ K9\ with, 0,1,2,3,4,5,6,7\ (n=4)\ and\ decompose\ it\ into$$
$$0\ 7\ 1\ 6\ 2\ 5\ 3\ 4.$$
$$1\ 0\ 2\ 7\ 3\ 6\ 4\ 5.$$
$$2\ 1\ 3\ 0\ 4\ 7\ 5\ 6.$$
$$3\ 2\ 4\ 1\ 5\ 0\ 6\ 7.(you\ can\ visualize\ this\ by\ putting\ vertices\ 0\ through\ 2n-1\ clockwise\ along\ a\ circle\ with\ outside\ the\ circle.)$$

Corollary 2.5: For any n ≥1, K_{2n} can be partitioned into n Hamiltonian complete fuzzy paths P_{2n-1}
Proof: Take the composition of IK_{2n+1} of theorem 1 and delete the vertex K_{2n+1} will become K_{2n}, While each Hamiltonian complete fuzzy path P_{2n-1} of K_{2n}

3. Cycles in regular fuzzy graph

Notation 3.1: $Every\ complete\ fuzzy\ graph\ G\ is\ called\ r-regular\ if\ every\ vertex\ of\ G\ has\ degree\ r$
Notation 3.2: $A\ 3-\ regular\ complete\ fuzzy\ graph\ is\ called\ cubic\ fuzzy\ graph$

Notation 3.3: $A\ complete\ fuzzy\ graph\ g\ is\ called\ r-regular\ if\ every\ vertex\ of\ G\ has\ degree\ r$

Notation 3.4: $1-factor\ of\ a\ complete\ fuzzy\ graph\ is\ a\ spanning\ 1-regular\ fuzzy\ sub\ graph\ of\ G.$

Theorem 3.5: $For\ any\ n\ \geq 1, K2n\ can\ be\ partitioned\ into\ 2n-1\ 1-factors.$
$Proof: label\ vertices, 0,1,2,\ldots,2n-2. Start\ with\ a\ 1-factor, 0;\ 1,2n-1;\ 2,2n-2;\ \ldots n-1,n$
$use\ the\ turning\ trick\ to\ obtain\ the\ following\ decomposition: ,0;\ 1,2n-1;\ 2,2n-2;\ \ldots n-1,n\ ,1;\ 2,0;\ 3,2n-2;\ \ldots n,n+1$

. . . .
…
….

$2n-2;\ 0,2n-3;\ 1,2n-4;\ \ldots n-2,n-1.$

Theorem 3.6: Let G (α, β) be a complete fuzzy graph where G*: (V, E) is an odd cycle. Then G is regular iff β is a constant function.
$Proof: If\ \beta\ is\ a\ constant\ function, say\ \beta\ (u,v)\ = c, for\ all\ u,v\ \in E, then\ d(v) = 2c, for\ every\ v \in V. So, G\ is\ regular.$
$Conversely, suppose\ that\ G\ is\ a\ k-regular\ complete\ fuzzy\ graph.$
$Let\ e1, e2,\ldots, e2n+1\ be\ the\ edges\ of\ G*\ in\ that\ order.$
$Let\ \beta(e1)\ =\ k1. Since\ G\ is\ k-regular,$
$\beta(e2)\ =\ k-k1$

$\beta(e3) = k - (k - k1) = k1$
$\beta(e4) = k - k1$
and so on.
$Therefore\ \beta(ei) = k1, if\ i\ is\ odd$
$= k - k1\ if\ i\ is\ even$
$Hence\ \beta(e1) = \beta(e2n + 1) = k1.$
$so\ if\ e1\ and\ e2n + 1\ incident\ at\ a\ vertex\ u, then\ d(u) = k. so\ d(e1) + d(e2n + 1) = k$
$ie\ k1 + k1 = k$
$2k1 = k$
$k1 = k/2.$
$Hence\ k - k1 = k/2. so\ \beta(ei) = k/2, for\ all\ i. Hence\ \beta\ is\ a\ constant\ function.$

Theorem 3.7: Let $G\ (\alpha, \beta)$ be a finished fluffy chart where $G *: (V, E)$ is an even cycle. At that point G is normal if β is a steady capacity or substitute edges have same enrollment esteems.

Validation: If either β is a steady capacity or substitute edges have same enrollment esteems, at that point G is a normal
$Conversely, Suppose\ G\ is\ a\ k - regular\ complete\ fuzzy\ graph.$
$Let\ e1, e2\ , \dots, e2n\ be\ the\ edges\ of\ even\ cycle\ G *\ in\ that\ order.$
$Proceeding\ as\ in\ theorem,$
$\beta(ei) = k1, if\ i\ is\ odd$
$= k - k1\ if\ i\ is\ even$
$f\ k1 = k - k1, then\ \beta\ is\ a\ constant\ function.$
$f\ k1 \neq k - k1, then\ alternate\ edges\ have\ same\ membership\ values.$

Theorem 3.8: Let G be a cubic complete fuzzy graph where G* is a cycle. Then G is a complete fuzzy cycle. It cannot be a complete fuzzy tree.
Validation: Assume that G be a cubic complete fuzzy graph where G* is a cycle.
That is G be a 3- regular complete fuzzy graph on a cycle G*, then by known theorem, either β is a constant function or alternate edges have same membership values.
So there does not exist a unique edges xy such that $\beta(xy) = \wedge \{\beta(uv) / \beta(uv) > 0\}$.
Therefore, G is a complete fuzzy cycle.
Hence by known lemma, G cannot be a complete fuzzy tree.

Example 3.9
Is a cubic complete fuzzy graph and G* is a cycle. Then G is a complete fuzzy cycle. Also, it cannot be a complete fuzzy tree.

5. Complement of complete fuzzy cycles

Construct the complement of the complete fuzzy cycles on 3, 4, 5 vertices as complete fuzzy cycles by choosing the membership values of vertices and edges suitably

Theorem 4.1
$Let\ G : (\alpha, \beta)\ be\ a\ complete\ fuzzy\ graph\ such\ that$
$*\ is\ a\ cycle\ with\ more\ than\ five\ vertices. Then\ (G *)\ c\ cannot\ be\ a\ cycle.$
$\boldsymbol{Proof}: Given\ G *\ is\ a\ cycle\ having\ n\ vertices\ where\ n \geq 6.$
$Then\ G *\ will\ have\ exactly\ n\ edges.$
$since\ all\ the\ vertices\ of\ G\ are\ also\ present\ in\ G\ c.$
$Therefore, number\ of\ vertices\ in\ G\ c\ is\ n. Let\ the\ vertices\ of\ G\ and\ G\ c\ be\ v1, v2, \dots, vn..$
$Then\ Gc\ must\ contain\ atleast\ the\ following\ edges.$
$(v1, v3), (v1, v4), (v1, v5), \dots., (v1, vn);\ (v2, v4), (v2, v5), \dots., (v2, vn);\ (v3, v5), (v3, v6), \dots, (v3, vn)$

$since\ n \geq 6\ the\ total\ number\ of\ edges\ in\ G*\ will\ be\ greater\ than\ n. Thus\ G\ c\ will\ not\ be\ a\ cycle.$

Corollary 4.2:

$Let\ G\ be\ complete\ fuzzy\ cycle\ with\ 6\ or\ more\ vertices. Then\ Gc\ will\ not\ be\ complete\ fuzzy\ cycle.$

6. Conclusion

We have defined and studied two types of contractions, namely edge and neighborhood contraction in fuzzy graphs. We have discussed few basic results on the same and investigated these new topics on some special classes of fuzzy graphs. We have applied the big idea of domination to the networks which has been explained through an example. Our future work is to further extend this concept of contraction to other variants of domination and also to apply it for different types of K_{2n+1} Fuzzy graphs.

7. REFERENCES

[1] J.N. Mordeson, P.S. Nair, Fuzzy graphs and fuzzy hyper graphs, Physica – veriag, 2000.

[2] G.Nirmala and M.Vijaya, Fuzzy graphs on composition, Tensor and Normal products, International journal of scientific and research publications, vol 2, Issue 6, 2012.

[3] L.A. Zadeh, Fuzzy sets, information control 8 (1965) 338- 353. www.ijsrp.org

[4] G.Nirmala and M.Vijaya, Application of fuzzy back propagation network, The PMU journal of humanities and sciences, vol 2, no 2, (jul I– Idec I2011) Ipp I99-107. I

[5] Bhattacharya,p Some remarks on fuzzy graphs, Pattern Recognition Lett 6(1987) 297-302.

[6] G.Nirmala and M.Vijaya, Fuzzy Number – Imprecise aspects of real world problems, Advances in Mathematics Scientific Developments and Engineering Applications. Narosa Publishing House.

[7] Frank Harary, Graph theory, Narosa /Addison Wesley. Indian student edition 1988.

[8] A. NagoorGani and V.T. Chandrasekaran, A First Look at Fuzzy Graph Theory, Allied Publishers, 2010.

[9] G. Nirmala and M. Vijaya, Strong Fuzzy Graphs on Composition, Tensor and Normal products, Accepcted and published in, Arabhatta Journal of Mathematics and Informatics, Vol 4, No.2, Nov 2012

[10] A. Nagoor Gani and K. Radha, On Regular Fuzzy Graphs, Journal of physical sciences, Vol. 1233-40 (2010).

[11] Klas Markstrom, Even cycle decompositions of 4- Regular graphs and line graphs

[12] V. BV.N. Prasad, Dr.T.Ramarao, T S Rao, T.Nageswara Rao, K.Prasad, "Some Basic Principles on Posets, Hasse Diagrams and Lattices", Test Engineering and Management Volume 83 Page No. 10771– 10775.

[13] K. Prasad, S. Venu Madhava Sarma , D.V. Ramalinga Reddy , Y. Prabhakara Rao, P. Rangaswamy, "A Note on Anti-Fuzzy Ideals in Subtraction Semi Groups" International Journal of Advanced Science and Technology Vol. 29, No. 6, (2020), pp. 8557-8562

[14] K. Prasad, S. Venu Madhava Sarma , P. Rangaswamy, "ON SUBTRACTION ALGEBRAS" Journal of Xidian University VOLUME 14, ISSUE 6, 2020 Page No.2070-2074.

[15] K Prasad, V B V N Prasad, T S Rao, M Gnanakiran, Mudda Ramesh, "ANTI FUZZYGAMMANEAR-ALGEBRASOVERANTIFUZZYFIELDS" Journal of Critical Reviews Vol 7, Issue 9, 2020 page.no 313-315.

[16] K. Prasad, V.B.V. N Prasad, T.S. Rao, G.Balaji Prakash , Rama Devi Burri, "A Note On Stone Spaces Of Advanced Distributive Lattices" International Journal of Advanced Science and Technology Vol. 29, No. 5, (2020), pp. 8494-8500

[17] K. Prasad , V. B. V. N. Prasad , K. Pushpalatha , M. Gnanakiran, " a1 and a2 NEAR SUBTRACTION SEMIGROUP" Journal of Critical Reviews Vol 7, Issue 9, 2020 pp.727-730.