Effect Of Inclination On The Growth Rate Of LL And Surfactant Mode Instabilities

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Abstract: Linear stability analysis of two liquid layers with different viscosities flowing down on rigid surface in presence of insoluble surfactant at LL interface and in absence of shear stress at interfaces is studied. This problem is studied for two different viscosity ratio (μ_r = 0.4, 50) and has shown the effect of angle (θ) on the growth rate of LL perturbations and surfactant perturbations. Here, we find three interfacial modes like gas-liquid (GL), surfactant (or Marangoni) and liquid-liquid (LL) modes in which the effect of an angle (θ) on two modes (surfactant and LL) are studied. The effect of angle on the growth rate of surfactant mode for viscosity ratio μ_r = 0.4 is found unstable. It means the growth rate of unstable surfactant mode enhances exponentially with time. On the other hand, the growth rate of LL perturbations for high viscosity ratio (μ_r = 50) are decaying exponentially with time. It can be said that we can use or apply this research as per our requirement. It can be applied to produce either stable or unstable configurations. For solving linear governing equation, we used MATLAB for initial guesses and C coding for producing final results.

Key words: Instability, surfactant mode, LL mode, growth rate, perturbations, angle, rigid surface, viscosity ratio.

Introduction

Linear stability of two Newtonian liquid layered configurations is widely studied problem due to its large number of applications in lubrications, heat transfer [1, 2] and in coatings [3]. It also has the applications in biological [4, 5] flows like blood flows in arteries and veins.

Many research papers related to the important effect of soluble or insoluble surfactant on the instability of various type of interfacial flow configurations have been studied [6-11]. The numbers of research paper have been published on interfacial flow on inclined and vertical rigid surfaces. Flow in microfluidic devices and lung airways [12-16] have also been studied. In lung airways, researcher found the effect of natural surfactant and liquid layer lining on the liquid flow. The application of lung-airways is in disease like asthma attack. Time of asthma attack can be delayed by using up to certain percentages by using inherently present surfactant on the lung airways. Pozrikidis et al [4] also studied the problem concerning the fluid flow on inclined solid surfaces in presence of insoluble surfactant and they found the effect of surfactant is stabilizing on the marangoni mode as well as on GL mode perturbations. Wei [17] studied the problem of single liquid layer flowing down an inclined rigid surface in presence of insoluble surfactant on GL interface and in presence of shear stress at GL interface as well. Our research problem based on basic above study with different configuration and effects [18]. In this article, we have shown the effect of angle on the growth rate of perturbations of surfactant and LL mode for various viscosity ratios.

Problem Formulation

We consider two Newtonian and incompressible liquid layers falling down a rigid surface. The viscosity of layer A and B is μ_a and μ_b and the density ρ_a and ρ_b respectively. The angle of inclination of rigid plane from horizontal is θ. The surfactant which is introduced at LL interface has a concentration Γ^* (x, t) and surface tension is denoted by γ^*. This flow is governed by the equation of continuity and equation of motions and one constitutive equation for monolayer of insoluble surfactant which is present at LL interface.

$$\nabla^* \cdot \mathbf{v}^* = 0, \tag{1}$$

$$\rho[\partial_t^* \mathbf{v}^* + \mathbf{v}^* \cdot \nabla^* \mathbf{v}^*] = \nabla^* \cdot \mathbf{T}^* + \rho \mathbf{g}. \tag{2}$$

where $\mathbf{v}^*$ and p^* are the velocity and pressure fields in the liquid layer; $\mathbf{T}^* = -p^*\mathbf{I} + \mu[\nabla^*\mathbf{v}^* + (\nabla^*\mathbf{v}^*)^T]$ is the total stress tensor for the liquid layer.

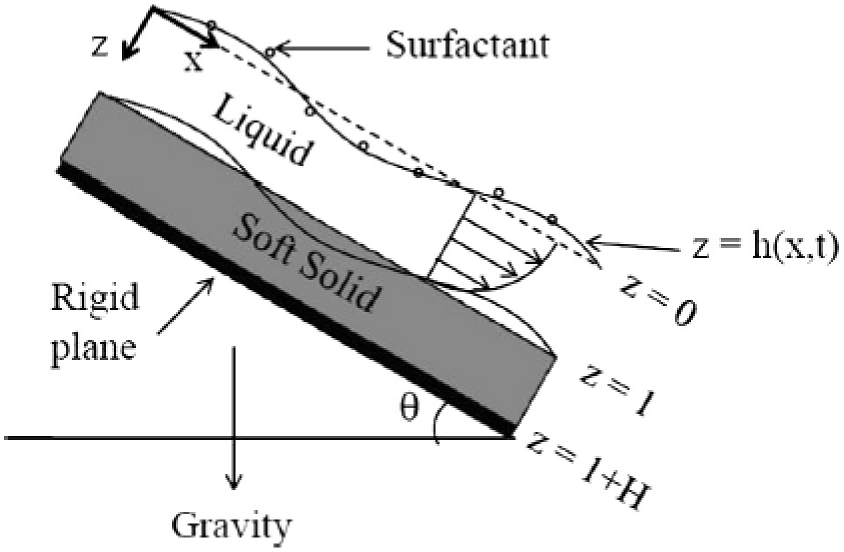

Fig. 1. Schematic of the configuration considered in the present work: Surfactant-laden Newtonian liquid film falling down an inclined plane coated with a soft, deformable neo-Hookean solid layer.

Result and Discussion

In this research paper, we study the effect of inclination of rigid surface or angle (θ) on the growth rate of various modes like liquid-liquid and Marangoni (or surfactant). Here, we have verified and validated our code with Wei [17-19] and some other research paper as well. Results of this article are also matching with the analytical solution (not shown here) or long wave solution. Two liquid layers with different viscosities are flowing down an inclined surface in presence of interfacial insoluble surfactant and in absence of imposed shear stress on interface.

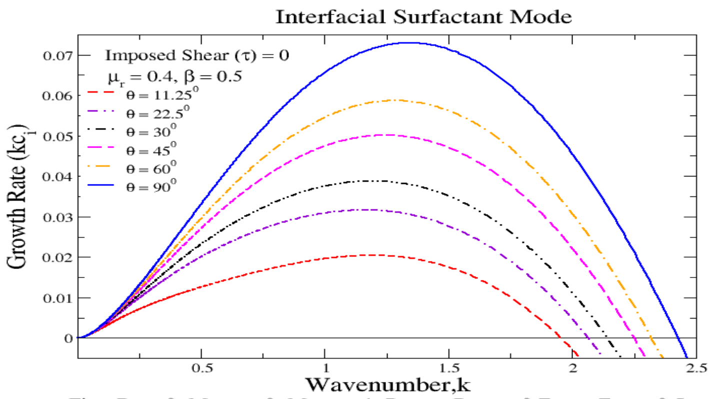

Fig.: Re = 0, Ma_{GL} = 0, Ma_{LL} = 1, Pe_{GL} = Pe_{LL} = 0, Σ_{GL} = Σ_{LL} = 0.5

Figure 1: Effect of angle of inclination (θ) on the growth rate of Surfactant mode.

Figure 1 is plotted between growth rate and wavenumber which is showing the effect of angle (θ) on the growth rate of surfactant mode in absence of stress at gas-liquid interface for viscosity ratio μ_r = 0.4 (viscosity of upper liquid layer to lower liquid layer) and other data are Re = 0, Ma_{GL} = 0, Ma_{LL} = 1, Pe = 0 (for both interface) and surface tension for both interface is constant ($\Sigma_{GL} = \Sigma_{LL}$ = 0.5). Here, it clear that surfactant is present only on LL interface because we have some finite value of Marangoni number (Ma_{LL} = 1). For creeping flow (Re = 0), we have low value of growth rate of Marangoni mode for low value of angle and high value of growth rate for high value of angle. This analysis predicts that the growth rate of Maranagoni mode or surfactant mode increases with increasing the value of angle when plane becomes vertical at θ = 90^0 then we find the higher positive value of growth rate ($kc_i \sim 0.07$). Here, we can predict that system or two layered configuration becomes highly unstable compare to other value of angle (θ). In this case, it is very difficult to make system stable if we need. We need to stabilize or unstabilize the flow, it depends on the requirements and applications for example flow should be stable in coating process so that we can produce a defect free and smooth surface. On the other hand, instability required to create turbulence in flow when we need to high heat transfer or mixing of fluids.

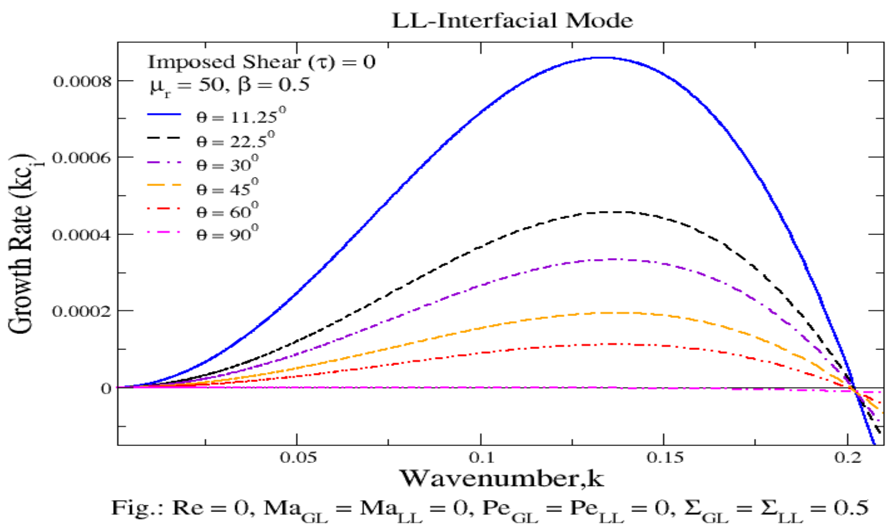

Figure 2: Growth rate versus wavenumber plot for LL mode.

Figure 2 is plotted between growth rate of lquid-liquid interfacial instabilities and wavenumber for different value of inclination angle and viscosity ratio (μ_r) = 50 (upper liquid layer is more viscous than lower) in absence of the shear stress at interfaces. It is clear from figure 2 that the inclination of plane by some angle shows the stabilizing effect because growth rate of LL mode or instabilities decreases with increasing the value of angle. As a result, we can predict that the angle has a stabilizing effect on instabilities because in this case, the growth rates of instabilities of LL mode are suppressed by increasing the value of angle (θ) to 90^0.

Conclusion

We studied the linear stability analysis of gravity-driven two liquid layer on the inclined rigid surface in presence of insoluble surfactant at LL interface and in absence of shear stress at interfaces for different angles. For low value of viscosity ratio (μ_r =0.4), we find that angle shows the destabilizing effect on the growth rate of surfactant mode and it means interface or flow becomes unstable. On the other hand, for high value of viscosity ratio (μ_r = 50), the angle shows the stabilizing effect on the growth rate of LL mode or liquid-liquid interfacial instabilities. It means LL interfacial instabilities can be suppressed by increasing the value of angle. From above discussion, it can be predict that we can use this research for both cases, for producing stable and unstable flow.

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