

METHOD OF IMPLEMENTING ANALYSIS OF VARIANCE (ANOVA) IN HEALTH CARE RESEARCH

Prasanta Kumar Brahma¹, Anita Behera², Pritam Chhotray³

¹Department of Community Medicine, IMS and SUM Hospital, Siksha 'O' Anusandhan Deemed to be University, K8, Kalinga Nagar, Bhubaneswar-751003, Odisha, INDIA

²Department of Anesthesia, IMS and SUM Hospital, Siksha 'O' Anusandhan Deemed to be University, K8, Kalinga Nagar, Bhubaneswar-751003, Odisha, INDIA

³Department of Pulmonary medicine IMS and SUM Hospital, Siksha 'O' Anusandhan Deemed to be University,

Corresponding Author : Dr. Prasanta Kumar Brahma, PhD

Department of Community Medicine,

IMS and SUM Hospital, Siksha 'O' Anusandhan Deemed to be University,
K8, Kalinga Nagar, Bhubaneswar-751003, Odisha, INDIA

pkbrahma@soa.ac.in

Mob. No:9861126520

Abstract - Health care Sector search for general clarifications for the disparities in procession of treatments in response to a disease. An increasingly accepted explanation is the important statistical technique analysis of variance (ANOVA). This technique is intended to analyze the discrepancy in data in order to meet the disparity among population means. At this point I have clarified the idea of one way ANOVA and its relevance with two examples of for one factor ANOVA and two factors ANOVA.

Keywords - one-way ANOVA test, multiple line of treatments, disease, one factor ANOVA and two factors ANOVA

Introduction

The design of analysis of variance (ANOVA) was established by the British geneticist and statistician Sir R. A. Fisher in 1918 and formally available in his book "statistical methods for workers" in 1925. The technique was developed to create existing statistical procedures for test of significance for a number of group means. ANOVA can be in theory viewed as an expansion

of the two independent samples t-test to multiple samples t-test, but outcome in less type 1 error and as a result appropriate a broad range of practical problems. Earlier, this design was normally used for agricultural conducting tests, but is at present the nearly all commonly higher studies method in business, economic, medical and social science disciplines. Like many other parametric statistical techniques, ANOVA is based on the following statistical assumptions:

- a) Homogeneity of variance.
- b) Normality of data.
- c) Independence of observations.

Basic ideas of one way ANOVA test.

A one-way analysis of variance is applied while the data are divided into groups according to only one factor. Assume with the intention of the data $y_{11}, y_{12}, y_{13}, \dots, y_{1n_1}$ are out comes of 1st treatment , $y_{21}, y_{22}, y_{23}, \dots, y_{2n_2}$ are out comes of 2nd treatment,..... , $y_{k1}, y_{k2}, y_{k3}, \dots, y_{kn_k}$ are out comes of k^{th} treatment. Let y_{ij} denote the data from the i^{th} group (level) and j^{th} observation.

I include principles of independent normal random variables y_{ij} , $i= 1, 2, 3, \dots, k$ and $J = 1, 2, 3, \dots, n_i$ with mean μ_i and constant standard deviation σ , $y_{ij} \sim N(\mu_i, \sigma)$ on the other hand, each $y_{ij} = \mu_i + \varepsilon_{ij}$ where ε_{ij} are normally distributed independent random errors, $\varepsilon_{ij} \sim N(0, \sigma)$. Let $N = n_1 + n_2 + n_3 + \dots + n_k$ is the total number of observations (the total sample size across all groups), where n_i is sample size for the i^{th} group.

The constraints of this model are the population means $\mu_1, \mu_2, \dots, \mu_k$ and the common standard deviation σ .

Using many disconnect two-sample t-tests to compare many pairs of means are an poor idea because we don't get a p-value or a confidence level for the total set of similarities together.

I will be anxious in testing the null hypothesis

$$H_0 : \mu_1 = \mu_2 = \dots = \mu_k \quad (1)$$

$$\text{In opposition to the alternative hypothesis } H_1: \mu_1 \neq \mu_2 \neq \dots \neq \mu_k \quad (2)$$

(There is at least one pair with unequal means).

Let y_i represent the mean sample of i^{th} group ($i = 1, 2, 3, \dots, k$):

$$\bar{y}_i = \frac{1}{n_i} \sum_{j=1}^{n_i} y_{ij} \quad (3)$$

\bar{y} stand for the grand mean, the mean of all the data points:

$$\bar{y} = \frac{1}{N} \sum_{i=1}^k \sum_{j=1}^{n_i} y_{ij} \quad (4)$$

S_i^2 stand for the sample variance of i^{th} group:

$$S_i^2 = \frac{1}{n_i - 1} \sum_{j=1}^{n_i} (y_{ij} - \bar{y}_i)^2 \quad (5)$$

And $S^2 = MSE$ is an estimate of the variance σ^2 common to all k populations,

$$S^2 = \frac{1}{n-k} \sum_{i=1}^k (n_i - 1) S_i^2 \quad (6)$$

ANOVA is centered on the thought to assess the variation stuck between groups (levels) and the difference within samples by analyzing their variances.

Name the total sum of squares SST , sum of squares due to error (or within groups) SSE , and the sum of squares for treatments (or between groups) SSC :

$$SST = \sum_{i=1}^k \sum_{j=1}^{n_i} (y_{ij} - \bar{y})^2 \quad (7)$$

$$SSE = \sum_{i=1}^k \sum_{j=1}^{n_i} (y_{ij} - \bar{y}_i)^2 = \sum_{i=1}^k (n_i - 1) S_i^2 \quad (8)$$

$$SSC = \sum_{i=1}^k \sum_{j=1}^{n_i} (\bar{y}_i - \bar{y})^2 = \sum_{i=1}^k n_i (\bar{y}_i - \bar{y})^2 \quad (9)$$

Imagine about the difference from an observation to the grand mean written in the following way:

$$y_{ij} - \bar{y} = (y_{ij} - \bar{y}_i) + (\bar{y}_i - \bar{y}) \quad (10)$$

Study that the left side is at the heart of *SST*, and the right side has the similar pieces of *SSE* and *SSC*. It actually works out that:

$$SST = SSE + SSC. \quad (11)$$

The total mean sum of squares *MST*, the mean sums of squares for error *MSE*, and the mean sums of squares for treatment *MSC* are:

$$MST = \frac{SST}{df_{SST}} = \frac{SST}{N-1} \quad (12)$$

$$MSE = \frac{SSE}{df_{SSE}} = \frac{SSE}{N-K} \quad (13)$$

$$MSC = \frac{SSC}{df_{SSC}} = \frac{SSC}{K-1} \quad (14)$$

The one-way ANOVA, presuming the test conditions are satisfied, uses the following test statistic:

$$F = \frac{MSC}{MSE} \quad (15)$$

Under H_0 this statistic has Fisher's distribution $F(k-1, N-k)$. In case it holds for the test criteria

$$F > F_{1-\alpha, k-1, N-k}, \tag{16}$$

Where $F_{1-\alpha, k-1, N-k}$ is $(1-\alpha)$ quintile of F distribution with $k-1$ and $N-k$ degrees of freedom, then hypothesis

H_0 is rejected on significance level α .

The outcomes of the estimates that direct to the F statistic are existing in an ANOVA table, the form of which is shown in the table 1.

This p -value says the probability of rejecting the null hypothesis in case the null hypothesis holds. In case $P < \alpha$, where α is chosen significance level, the null hypothesis is rejected with probability greater than $(1-\alpha)$ 100 probability.

Table 1. Basic one way ANOVA table.

Source of Variation	Sum Of Squares <i>SS</i>	Degrees of freedom <i>df</i>	Mean square	<i>F</i> - Statistic	Tail area above <i>F</i>
Between group	SSC	k-1	MSC	MSC/MSE	P value
Within	SSE	N-k	MSE		
Total	SST	N-1			

Methodology

Intended for this study suppose you have four lines of management you encompass to capture four groups of patients of harmonized state. The sample for this study is depending on the

convenience of patients. Data must gather from the target population. Data analysis was by means of the support of inferential statistics (one way ANOVA). Independent variable for the study was the amount of some determining parameter i.e. Hemoglobin level, blood sugar level etc.. The significance test for the between treatment effect was the researcher's statistical evidence of the result of the treatment on the determining parameter.

Test for normality and homogeneity of the data.

To start ANOVA test, one have to corroborate the validity of the normality and homogeneity postulations of the data beneath study. These tests were based on Kolmogorov –Siminov and levene's statistic in that order.

Test of significance for the treatment effect.

Next the tests for the supposition of normality and equality of variance (Homoscedesticity), the subsequent thing is to set up the significant effect of the independent variable. The significance of the treatment is based on F distribution, suppose the test discovered that the probability of the Fisher distribution F was less than the level of significance of 0.05 (i.e, $P < 0.05$). The null hypothesis that there is no significant dissimilarity between the treatments or also there is a significant difference between the treatments.

7. Conclusion

In several statistical functions in agriculture, business administration, psychology, social science, and the natural sciences we require to compare more than two groups. For hypothesis testing more than two population means, scientists have expanded ANOVA method. The ANOVA test method assesses the difference in observations between samples (sum of squares for groups, SSC) to the difference within samples (sum of squares for error, SSE). The ANOVA F test rejects

the null hypothesis that the mean reactions are not equal in all groups if SSC is large relative to SSE . The analysis of variance presumes that the observations are normally and independently distributed with the same variance for each treatment or factor level.

Example

One-Factor ANOVA

Calcium is a necessary mineral so as to manages the heart, is very important for blood clotting plus for building healthy bones. The National Osteoporosis Foundation suggests a daily calcium eating of 1000-1200 mg/day for adult men and women. As calcium is included in some foodstuffs, the majority adults do not acquire sufficient calcium in their diets and take additions. Be disappointed some of the supplements have side effects such as gastric suffering, making them difficult for some patients to take on a regular basis.

A research is intended to test whether there is a variation in mean every day calcium intake in adults with normal bone density, adults with osteopenia (a low bone density which may lead to osteoporosis) and adults with osteoporosis. Adults 60 years of age with normal bone density, osteopenia and osteoporosis are chosen at random from hospital documents and requested to participate in the study. Each participant's daily calcium intake is measured based on reported food eating and supplements. The data are shown below.

Normal Bone Density	Osteopenia	Osteoporosis
1200	1000	890
1000	1100	650
980	700	1100
900	800	900
750	500	400
800	700	350

Is there a statistically significant difference in mean calcium eating in patients with normal bone density as compared to patients with osteopenia and osteoporosis? We will run the ANOVA using the five-step approach.

- **Step 1.** Set up hypotheses and settle on level of significance

$H_0: \mu_1 = \mu_2 = \mu_3$ H_1 : Means are not all equal $\alpha=0.05$

- **Step 2.** Select the appropriate test statistic.

The test statistic is the F statistic for ANOVA, $F = MSB/MSE$.

- **Step 3.** Set up decision rule.

In order to determine the critical value of F we need degrees of freedom, $df_1 = k - 1$ and $df_2 = N - k$. In this example, $df_1 = k - 1 = 3 - 1 = 2$ and $df_2 = N - k = 18 - 3 = 15$. The critical value is 3.68 and the decision rule is as follows: Reject H_0 if $F \geq 3.68$.

- **Step 4.** Compute the test statistic.

To systematize our estimates we will complete the ANOVA table. In order to compute the sums of squares we have to first compute the sample means for each group and the overall mean.

Normal Density	Bone	Osteopenia	Osteoporosis
$n_1 = 6$		$n_2 = 6$	$n_3 = 6$
$\bar{X}_1 = 938.3$		$\bar{X}_2 = 800$	$\bar{X}_3 = 715$

If we pool all $N = 18$ observations, the overall mean is $\bar{X} = 817.8$.

$$SSC = \sum n_j (\bar{X}_j - \bar{X})^2$$

Substituting:

$$SSC = 6(938.33 - 817.8)^2 + 6(800 - 817.8)^2 + 6(715 - 817.8)^2$$

Finally,

$$SSC = 87201.77 + 63379.66 + 1896.3 = 152477.7$$

SSE requires calculating the squared differences between each observation and its group mean.

We will calculate SSE in parts. For the participants with normal bone density:

Normal Density	Bone	(X - 938.3)	(X - 938.3333)²
1200		261.6667	68,486.9
1000		61.6667	3,806.9
980		41.6667	1,738.9

900	-38.3333	1,466.9
750	-188.333	35,456.9
800	-138.333	19,126.9
Total	0	130,083.3

Thus, $\sum(X_{ij} - \bar{X}_1)^2 = 130083.3$

For participants with osteopenia:

Osteopenia	(X - 800.0)	(X - 800.0)²
1000	200	40,000
1100	300	90,000
700	-100	10,000
800	0	0
500	-300	90,000
700	-100	10,000
Total	0	240,000

Thus, $\sum(X_{ij} - \bar{X}_2)^2 = 240000$

For participants with osteoporosis:

Osteoporosis	(X - 715.0)	(X - 715.0)²
890	175	30,625
650	-65	4,225
1100	385	148,225
900	185	34,225
400	-315	99,225
350	-365	133,225
Total	0	449,750

Thus, $\sum(X_{ij} - \bar{X}_3)^2 = 449750$

$SSE = \sum \sum (X_{ij} - \bar{X}_j)^2 = 130083.3 + 240000 + 449750 = 819833.3$

We can now construct the ANOVA table.

Source of Variation	Sums of Squares (SS)	Degrees of freedom (df)	Mean Squares (MS)	F
Between Treatments	152,477.7	2	76,238.6	1.395
Error or Residual	819,833.3	15	54,655.5	
Total	972,311.0	17		

- **Step 5. Conclusion.**

We do not reject H_0 because the computed value 1.395 is less than the tabulated value 3.68 with degree of freedom (2, 15). We do not have statistically significant evidence at $\alpha = 0.05$ to show that there is a variation in mean calcium eating in patients with normal bone density as compared to osteopenia and osterporosis. Are the variations in mean calcium intake clinically meaningful? If so, what might account for the lack of statistical significance?

Two-Factor ANOVA

Assume regarding the clinical trial planed exceeding in which three competing treatments planed for joint pain are evaluated in terms of their mean time to pain release in patients with osteoarthritis. Because researchers hypothesize that there might be a difference in time to pain release in men against women, they arbitrarily give 15 participating men to one of the three competing treatments and arbitrarily give 15 participating women to one of the three competing treatments (i.e., stratified randomization). Participating men and women do not know to which treatment they are given. They are instructed to obtain the assigned medication at the same time as they know-how joint pain and to record the time, in minutes, until the pain drops. The data (times to pain relief) are shown below and are planned by the allotted treatment and sex of the participant.

Table of Time to Pain Relief by Treatment and Sex

Treatment	Male	Female
A	12	21
	15	19

	16	18
	17	24
	14	25
B	14	21
	17	20
	19	23
	20	27
	17	25
C	25	37
	27	34
	29	36
	24	26
	22	29

The analysis in two-factor ANOVA is similar to that revealed above for one-factor ANOVA. The computations are once more systematized in an ANOVA table, but the whole variation is partitioned interested in that due to the main consequence of treatment, the main result of sex and the interaction result. The results of the analysis are shown below (and were generated with a statistical computing package - here we focus on interpretation).

ANOVA Table for Two-Factor ANOVA

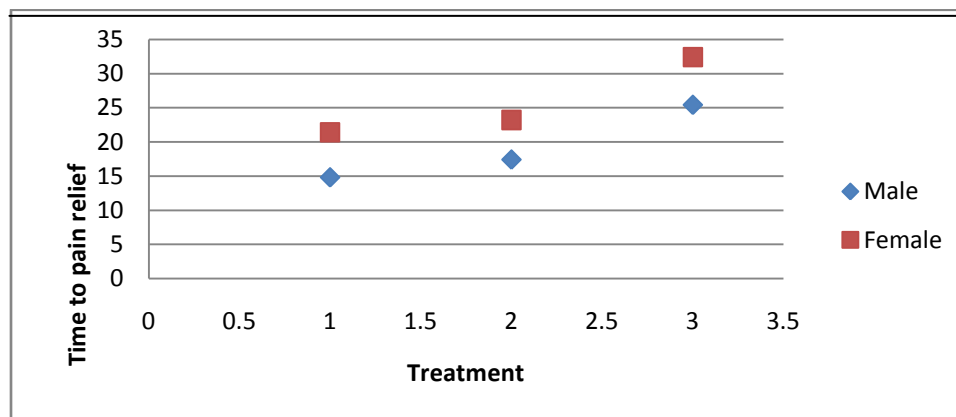
Source of Variation	Sums of Squares (SS)	Degrees of freedom (df)	Mean Squares (MS)	F	P-Value
Model	967.0	5	193.4	20.7	0.0001
Treatment	651.5	2	325.7	34.8	0.0001
Sex	313.6	1	313.6	33.5	0.0001
Treatment * Sex	1.9	2	0.9	0.1	0.9054
Error or Residual	224.4	24	9.4		
Total	1191.4	29			

Here are 4 statistical tests in the ANOVA table above. The first test is an on the complete test to evaluate whether there is a difference amongst the 6 cell means (cells are described by treatment and sex). The F statistic is 20.7 as well as is very statistically significant with $p=0.0001$. At the same time as the on the entire test is significant, center then turns to the marks that may be pouring the significance (in this example, treatment, sex or the interaction between the two). The next three statistical tests evaluate the significance of the main effect of treatment, the main effect of sex and the interaction effect. In this instance, there is a highly significant main effect of treatment ($p=0.0001$) and a highly significant main effect of sex ($p=0.0001$). The interaction between the two does not reach statistical significance ($p=0.91$). The table below contains the mean times to pain release in each of the treatments for men and women (Note that each sample mean is computed on the 5 observations measured under that experimental condition).

Mean Time to Pain Relief by Treatment and Gender

Treatment	Male	Female
A	14.8	21.4
B	17.4	23.2
C	25.4	32.4

Treatment A appears to be the most successful treatment for both men and women. The mean times to release are lower in Treatment A for both men and women and highest in Treatment C for both men and women. In all treatments, women report longer times to pain relief (See below).



Notice that there is the alike pattern of time to pain release across treatments in both men and women (treatment effect). There is also a sex effect - particularly, time to pain relief is longer in women in every treatment.

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