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Global Rebellion Chromatic Number Applications and Some Interesting Graphs

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Abstract
To begin, the fundamental concepts of coloring, rebellion, and global rebellion are presented. The terms rebellion coloring, global rebellion coloring, rebellion chromatic number, and global rebellion chromatic number are defined. Applications of rebellion chromatic number and global rebellion chromatic number are being investigated. For some interesting graphs, the tight limits of rebellion chromatic number and global rebellion chromatic number are obtained.

Keywords: Chromatic number, Rebellion coloring, Global rebellion coloring, Rebellion chromatic number, Global rebellion chromatic number.

AMS subject classification: 05C05, 05C69

I.INTRODUCTION

Vertex coloring is a fundamental idea in graph theory that is employed in a wide range ofreal-world application. The initial graph coloring results are almost entirely concerned with planar graphs in the form of map coloring. Francis Guthrie proposed the four colour conjecture while attempting to colour a map of England's counties, finding that four colours were sufficient to colour the map so that no parts sharing a shared boundary received the samecolor. Kenneth Appel and Wolfgang Haken proved the four colour theorem in 1977[1]. George David Birkhoff [4] proposed the chromatic polynomial to analyse coloring problems in 1912, which Tutte expanded to the Tutte polynomial, fundamental structures in algebraic graph theory. Kempe drew attention to the broader, non-planar case in 1879, and several studies on expansions of planar graph coloring to higher-order surfaces followed in the early twentieth century.

P.Kristiansen, S.K. Hedetniemi, and S.T.Hedetniemi proposed the concept of alliance in graphs. T.W.Hanyes, S.T.Hedetniemi, and M.A.Henning investigated it further [5]. By swapping out the discrepancies in the alliance set, the auothers had recently introduced the rebellion concept in [8]. For simple graphs, we defined rebellion colouring, global rebellion colouring, rebellion chromatic number, and global rebellion chromatic number in this study. We also established and defined its tight boundaries for various standard graphs.

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II. BASIC CONCEPTS AND DEFINITION

A. Vertex Colorings

Vertex colouring is the process of assigning colours to the vertices of a graph G in such a way that no two adjacent vertices have the same colour.

B. Proper Vertex colorings

A proper vertex in a graph is one with vertex colouring in which the end points of each edge are coloured differently. If a graph has proper vertex K-coloring, it is said to be vertex K-colorable.

C. Chromatic Number

G's vertex chromatic number is the minimum number of distinct colours required for proper vertex coloring..

D. Rebellion Number

A set $R \subseteq V$of a graph G is said to be a rebellion set R,$(i)|N_R(v)| \leq |N_{V\setminus R}(v)|, \forall v \in R$, $(ii)|R| \geq |V\setminus R|$. An rebellion number $r(G)$ is the minimum cardinality of any rebellion set in G. A rebellion set with cardinality $r(G)$ is denoted by $r - set$.

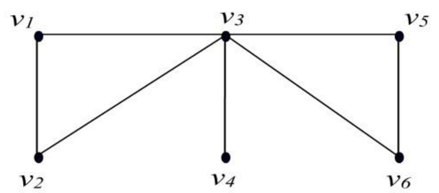
E. Global Rebellion Number

A rebellion set R of a graph G is said to be a global rebellion set GR, if R is also a dominating set of G. The global rebellion number $gr(G)$ is the minimum cardinality of any global rebellion sert in G. A global rebellion set with cardinality $gr(G)$ is denoted by $gr - set$.

F. Rebellion Coloring

If no two vertices in a Rebellion set $R \subseteq V$ of a graph G have the same colour, the set is said to have a Rebellion vertex colouring set RVC. The minimum cardinality of each rebellion vertex colouring set in G is represented by the rebellion vertex colouring number $rc(G)$. rc −set denotes a rebellion vertex colouring set with cardinality $rc(G)$.

Example:



G

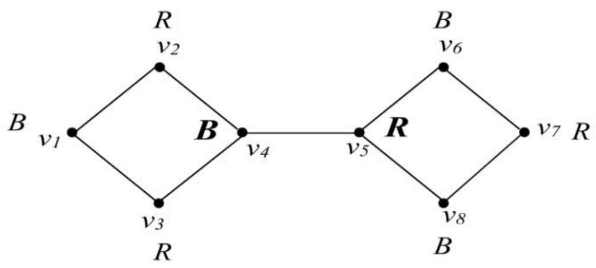
Figure 1

The rebellion set $R = \{v_2, v_4, v_6\}$ is a minimum rebellion coloring set and hence $rc(G) = 1$.
The rebellion set $R = \{v_1, v_3, v_5\}$ is a minimum rebellion coloring set and hence $rc(G) = 2$.

G. Global Rebellion Coloring

A global rebellion colouring set GRC is a Rebellion colouring set RC of a graph G if RC is also a dominating set of G. The minimum cardinality of each global rebellion colouring set in G is represented by the global rebellion colouring number $grc(G)$. $grc -$ set denotes a global rebellion colouring set with cardinality $grc(G)$.

Example:



G

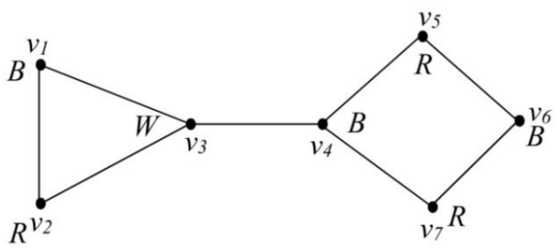
Figure 2

The rebellion set $R = \{v_1, v_4, v_6, v_8\}$ is a global rebellion coloring set. Since, R has also a dominating set and hence $rc(G) = 1$. The rebellion set $R = \{v_2, v_3, v_6, v_8\}$ is a global rebellion coloring set. Since, R has also a dominating set and hence $rc(G) = 2$.

H. Rebellion Chromatic Number

The minimum number of various colours required for proper vertex colouring of G is said to be rebellion chromatic number $R\chi(G)$. $r\chi\ (G)$ - set denotes a global rebellion chromatic set.

Example:



G

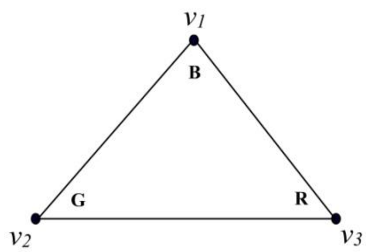
Figure 3

The rebellion coloring set $RC = \{v_2, v_3, v_5, v_7\}$ is a rebellion chromatic set and hencerχ(G) =2.

I. Global Rebellion Chromatic Number

The minimum number of various colours required for a proper vertex colouring of G is said to be the rebellion chromatic number $GR\chi(G)$. The symbol $gr\chi(G)$ - set represents a global rebellion chromatic set.

Example:



G

Figure 4

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The global rebellion coloring set $GR = \{v_1, v_2\}$ is a global rebellion chromatic set. Since, R has also a dominating set and hence $gr\chi(G) = 2$.

III. SOME INTERESTING GRAPH AND REBELLION &GLOBAL REBELLION CHROMATIC NUMBER OF GRAPHS

Using $R\chi(G)$ and $GR\chi(G)$, some interesting graphs are described in the following.

3.1 Bipartite graph

If V can be divided into the subsets V_1 *and* V_2, and each edge in the graph $G = (V, E)$ joins a vertex in V_1 and a vertex in V_2, then the graph is bipartite.

3.2 Theorem

The graph is bipartite if and only if the global rebellion chromatic number is atmost 1.

Proof:

If the graph G has two portions, V_1 *and* V_2, it is possible to colour G by giving each vertex in V_1 red and each vertex in V_2 green. No two adjacent vertices can have the same colour since there isn't an edge connecting any vertex in V_1 to any other vertex in V_2.

Case (i): If $m = n, then\ GR\chi(G) = 1$. The result is obvious.

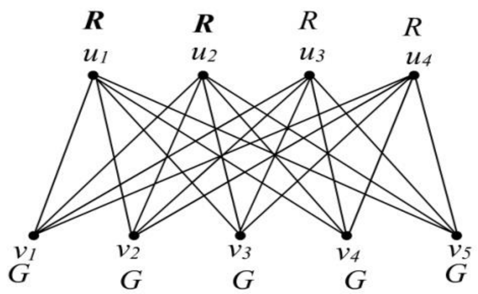
Case (ii): If $m > n,\ n > m\ , then\ GR\chi(G) = 1$.

In this case, maximum {vertex in V_1 , vertex inV_2} is an global rebellion chromatic set of G. Each vertex set assign the different color. Hence the global rebellion chromatic number is 1.

Conversely, if a graph G has one global rebellion vertex colour, let V_1 represent the set of coloured vertices, and let V_2 represent the set of coloured vertices. There is no edge between any two vertices in the same part since adjacent vertices must receive different colours. The graph is thus bipartite.

3.3 Example

The following$K_{4,5}$ graph, the global rebellion coloring set $GR = \{v_1, v_2, v_3, v_4, v_5\}$ is a global rebellion chromatic set. Since, R has also a dominating set and hence $gr\chi(G) = 1$.



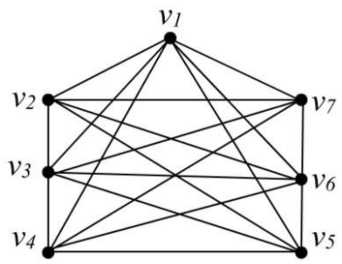
$K_{4,5}$

Figure 5

3.4 Complete Graph

The complete graph on n vertices for $n \geq 1$, is a graph with n vertices and an edge connecting each unique pair of vertices. This graph is denoted as K_n.

3.5 Example



K_7
Figure 6

3.6 Example

We can determine K_n's chromatic number. By assigning a different colour to each vertices, K_n can be coloured using n colours. Since every pair of vertices in this graph is adjacent, no two vertices may be given the same colour. Hence, $K_n = n$ has chromatic number n.

3.7 Theorem

For a complete graph $K_n, n \geq 2,\ GR\chi(G) = \begin{cases} \frac{n}{2}, n \equiv 0\ mod(2) \\ \left\lceil\frac{n}{2}\right\rceil, n \equiv 1\ mod(2) \end{cases}$

Proof:

Let G be the complete graph $K_n$with atleast three vertices and $V(G) = \{v_1, v_2, \dots, v_n\}$.

Case (i):

In this case, the set $R = \{v_{2i} \setminus i = 1,2, \dots, .\frac{n}{2}\}$ is a global rebellion chromatic set of G. Since R satisfies all the conditions of $GR\chi$ −set,

we have $GR\chi(G) \leq R = \frac{n}{2}$ (1)

Let R be an $GR\chi$ −set of G, since $|R| \geq |V \setminus R|$ and $\langle R \rangle$ has only minimum coloring vertices gives R must contain atleast $\frac{n}{2}$vertices. Hence $GR\chi(G) \geq R = \frac{n}{2}$ (2)

The result follows from (1) and (2).

Case (ii):

In this case, the set $R = \{v_{2i+1} \setminus i = 0,1,2, \dots, \left\lceil\frac{n}{2}\right\rceil\}$ is a global rebellion chromatic set of G. Since R satisfies all the conditions of $GR\chi$ −set,

we have $GR\chi(G) \leq R = \left\lceil\frac{n}{2}\right\rceil$ (3)

Let R be an $GR\chi$ −set of G, since $|R| \geq |V \setminus R|$ and $\langle R \rangle$ has only minimum coloring vertices gives R must contain atleast $\left\lceil\frac{n}{2}\right\rceil$vertices. Hence $GR\chi(G) \geq R = \left\lceil\frac{n}{2}\right\rceil$ (4)

The result follows from (3) and (4).

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3.8 Example

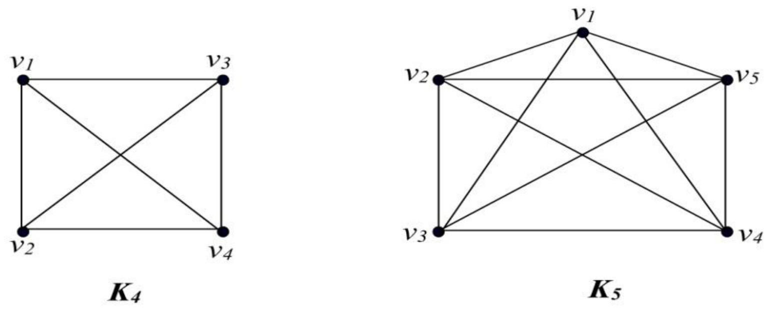


Figure 7

For the graph K_4, the global rebellion coloring set $GR = \{v_1, v_2\}$ is a global rebellion chromatic set. Since, R has also a dominating set and hence grχ(G) $= \frac{n}{2} = \frac{4}{2} = 2$.

For the graphK_5, the global rebellion coloring set $GR = \{v_1, v_2, v_3\}$ is a global rebellion chromatic set. Since, R has also a dominating set and hence grχ(G) $= \left\lceil\frac{n}{2}\right\rceil = \left\lceil\frac{5}{2}\right\rceil = 3$.

3.9 Star Graph

Star graph is a tree on nodes with one node having vertex degree and the other having vertex degree 1.

3.10 Theorem

For the star graph $K_{1,n}, n \geq 2, R\chi(K_{1,n}) = \left\lceil\frac{n}{2}\right\rceil$+1

Proof:

Let G be the star graph $K_{1,n}$ with atleast two vertices.

In this case the set $R = \{v_i \backslash i = 1,2, \ldots, \left\lceil\frac{n}{2}\right\rceil + 1\}$ is an $R\chi$ −set of G, since R satisfies all the conditions of $R\chi$ −set, we have

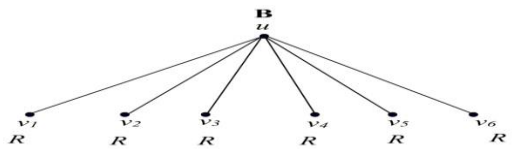
$R\chi(G) \leq |R| = \left\lceil\frac{n}{2}\right\rceil$ +1 (5)

Let R be an $R\chi$ −set of G, since $|R| \geq |V \backslash R|$ and $\langle R \rangle$ has only minimum coloring vertices gives R must contain atleast $\left\lceil\frac{n}{2}\right\rceil + 1$vertices.

Hence $GR\chi(G) \geq |R| = \left\lceil\frac{n}{2}\right\rceil + 1$ (6)

Then from equations (5) and (6) the result follows.

3.11 Example



$\boldsymbol{K_{1,6}}$

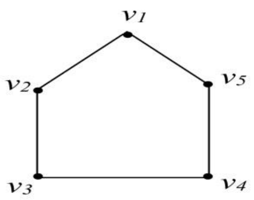
Figure 8

The rebellion set $R = \{v_1, v_2, v_3, v_4\}$ is a rebellion chromatic set and hence $r\chi(G) = 1$.

3.12 Cycle

The n-cycle, for $n \geq 3$ denoted C_n, is the graph with n vertices, $v_1, v_2, \dots, v_n$ and edge set $E = \{v_1v_2, v_2v_3, \dots, v_{n-1}v_n, v_nv_1\}$.

3.13 Example



C_5

Figure 9

3.14 Theorem

For the cycle graph $C_n, n \geq 3, GR\chi(G) = \begin{cases} 1, n \equiv 0\ mod(2) \\ 2, n \equiv 1\ mod(2) \end{cases}$

Proof:

Case (i): $n \equiv 0\ mod(2)$

The red-colored odd-numbered vertices and green-colored even-numbered vertices. The first vertexv_1, is red, and the nth vertex v_n, is green. Thus, the colours of the two vertices are different. Both the colour green and the colour red each have $\frac{n}{2}$vertices. Therefore, when n is even, the global rebellion chromatic number of C_n is 1.

Case (i): $n \equiv 1\ mod(2)$

Let n is odd. Since n is odd, (n-1) even. We simply require two colours, such as red and green, for the vertices from first to $(n-1)^{st}$. The initial and $(n-1)^{st}$vertices, which are of different colours, are located close to the n^{th} vertex. Therefore, we require a third colour for the n^{th} vertex. Thus the global rebellion chromatic number of $C_n$is 2 when n is odd.

3.15 Example

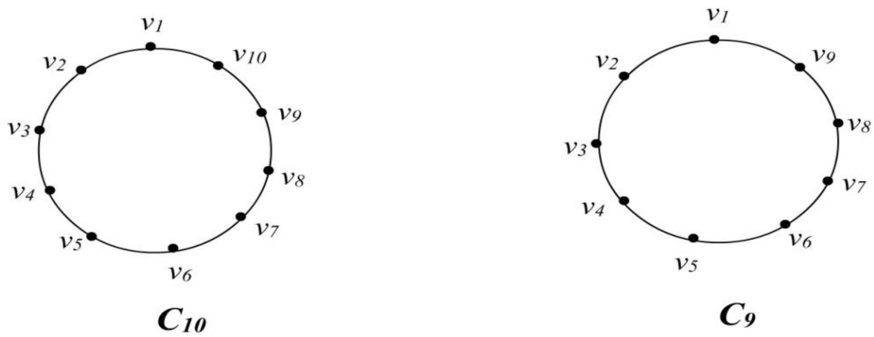


Figure 10

For the graph C_{10}, The global rebellion coloring set $GR = \{v_1, v_{3,} v_{5,} v_{7,} v_9\}$ is a global rebellion chromatic set. Since, R has also a dominating set and hence $gr\chi(G) = 1$.

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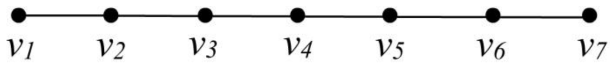
For the graph C_9, The global rebellion coloring set $GR = \{v_1, v_{3,} v_{5,} v_{7,} v_9\}$ is a global rebellion chromatic set. Since, R has also a dominating set and hence $\text{gr}\chi(\text{G}) = 2$.

3.16 Path

A path is a sequence of non-repeated nodes connected through edges present in a graph.

3.17 Example



$\boldsymbol{P_7}$

Figure 11

3.18 Theorem

For the path graph $P_n, n \geq 2, GR\chi(P_n) = 1$.

Proof:

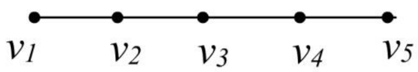
Let G be the path graph P_n,with atleast two vertices.

The odd number of vertices colored with red and even number of vertices colored with green.

Global rebellion chromatic number of P_n= χ{ Max{Odd vertices or Even vertices}}

$$GR\chi(P_n) = 1$$

3.19 Example



$\boldsymbol{P_5}$

Figure 12

For the graph P_5, The global rebellion coloring set $GR = \{v_1, v_{3,} v_5\}$ is a global rebellion chromatic set. Since, R has also a dominating set and hence $\text{gr}\chi(\text{G}) = 1$.

IV. APPLICATIONS OF GLOBAL REBELLION CHROMATIC NUMBER:

Mobile Radio Frequency Assignment: Each tower at the same site must have a distinct frequency given to it when it comes to mobile radio frequencies. How should frequencies be assigned given this restriction? What is the bare minimum required for frequencies? The towers in this issue individually serve as vertices in a graph colouring problem, and any edges connecting them signify that two towers are within striking distance of each other.

Register Allocation: Register allocation is the process of placing several target programme variables onto a limited number of CPU registers during compiler optimization. This issue is a colouring issue as well.

Aircraft Scheduling: Let's say we have to assign K aircrafts to n flights, with the Ith flight taking place during the interval (X_i,Y_i). It is obvious that we cannot assign the same aircraft to two flights that overlap. The flights are represented by the conflict graph's vertices, which are connected when their respective time periods overlap. Since the conflict graph is an interval graph, polynomial time colouring is best for this type of graph.

Making Schedule or Time Table: Now let us suppose we need to create a university exam schedule. We have a list of the many subjects and the students enrolled in each one. There would be common students in many subjects (of same batch, some backlog students, etc). How can the exam be scheduled so that no exam with the same student is scheduled at the same time? How many time slots at a minimum are required to schedule all exams? Every node of the graph used to represent this issue is a subject, and any edges that connect two nodes signify that they share a student. In this case, the minimum number of time slots is equal to the graph's chromatic number, making it a graph colouring problem.

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