AN ALGORITHM FOR MULTI OBJECTIVE FUZZY FRACTIONAL TRANSPORTATION PROBLEM

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Abstract

In this paper a new algorithm is proposed to find the optimal solution of the multi objective fuzzy fractional transportation problem. The proposed algorithm is very simple and easy to understand. This algorithm gives the better solution in both crisp environment and fuzzy environment. The numerical example is solved to explain the algorithm. The solution of the problem is compared with the several existing problem. The proposed algorithm gives the better solution than the existing one.

Key Words: Transportation Problem , Multi Objective Transportation Problem, Fractional Transportation Problem

1. Introduction

Transport plays a vital role in economic growth and globalization. The transportation problem is a distribution-type problem, the main goal of transportation problem is to decide how to transfer goods from various sending locations to various receiving locations with minimal costs or maximum profit. The transportation problem was first studied by Hitchcock in 1941.

The classical transportation problem involves only one objective at a time but in general there are many situations involving more objectives other that total cost. This leads to the concept of multi objective transportation problem (MOTP). To solve the multi objective transportation problem goal programming method was introduced by Lee(1973). Zeleny (1974) solved the multi objective transportation problem by generating non dominated basic feasible solution. Diaz(1978) developed the algorithm to obtain all non dominated solutions for MOTP. Also many authors, Aneja (1979), Gupta(1983), Nomini(2017) have developed the various solution procedures to solve the MOTP.

The generalization of linear programming problem is fractional programming problem (FPP) in which the objectives are ratio of two functions. The aim of this is to obtain optimization of the ratio of the cost functions. FPP is very applicable in many real life situations such as ratio between the profit and time, profit and cost , minimizing the inventory and sales etc. Several algorithms have been established by different authors, Charnes and Cooper(1962), Birtan(1973) ,Cravan(1988), Schaible (1995). Changkong(1983),Borza(2012), Chakraborty(2002) ,Abouzar (2018) solved the fractional transportation problem with multi objectives. Dangwal (2012) have developed the algorithm for MOFTP by using taylor series.

Due to shortage of information, insufficient data, lack of evidence, and so forth, the data for a transportation system such as availabilities, demands and conveyance capacities are not always exact but can be fuzzy or arbitrary or both. Fuzzy set was first introduced by Zadeh(1965).

The fractional transportation problem was formulated with fuzzy parameters by Liu(2016). Pop(2007) have extended the goal programming algorithm for multi objective fractional transportation problem. Sheema(2017) have developed the algorithm for solving multi objective transportation problem in fuzzy environment.

In this paper a new algorithm is proposed to find the optimal solution of the multi objective fuzzy fractional transportation problem. The proposed algorithm is very simple and easy to understand. The numerical example is solved to explain the algorithm. The solution of the problem is compared with the existing methods of various authors. The proposed algorithm gives the better solution than the existing one.

2. Mathematical formulation multi-objective fractional fuzzy transportation problem (MOFFTP) :

Mathematical formulation of multi-objective fractional fuzzy transportation problem is defined as follows:

$$\min(\max)q^1 = \frac{\sum_{l=1}^{x}\sum_{p=1}^{y}\widetilde{s_{lp1}}t_{lp}}{\sum_{l=1}^{x}\sum_{p=1}^{y}\widetilde{u_{lp1}}t_{lp}}$$

$$\min(\max)q^2 = \frac{\sum_{l=1}^{x}\sum_{p=1}^{y}\widetilde{s_{lp2}}t_{lp}}{\sum_{l=1}^{x}\sum_{p=1}^{y}\widetilde{u_{lj2}}t_{lp}}$$

….

$$\min(\max)q^r = \frac{\sum_{l=1}^{x}\sum_{p=1}^{y}\widetilde{s_{lpr}}t_{lp}}{\sum_{l=1}^{x}\sum_{p=1}^{y}\widetilde{u_{lpr}}t_{lp}}$$

Subject to

$$\sum_{p=1}^{y} t_{lp} = a_l$$

$$\sum_{l=1}^{x} t_{lp} = b_p \;;$$

$$t_{lp} \geq 0 \text{ for every } l = 1\ to\ x\ , p = 1\ to\ y\ and\ k = 1\ to\ r$$

where $\widetilde{s_{lpr}}$ and $\widetilde{u_{lpr}}$ are fuzzy elements of the r^{th} objective function.

3. Procedure to solve multi objective fractional transportation problem:

Step 1 **:**Find the crisp value for every fuzzy number using any ranking number.

Step 2 : Find the fractional values of the each cell as shown below:

	W$_1$	**W$_2$**	**….**	**W$_y$**	**Supply**
U$_1$	$\frac{s_{111}}{u_{111}}$ $\frac{s_{112}}{u_{112}}$ …. $\frac{s_{11r}}{u_{11r}}$	$\frac{s_{121}}{u_{121}}$ $\frac{s_{122}}{u_{122}}$ …. $\frac{s_{12r}}{u_{12r}}$	….	$\frac{s_{1y1}}{u_{1y1}}$ $\frac{s_{1y2}}{u_{1y2}}$ …. $\frac{s_{1yr}}{u_{1yr}}$	a_1
U$_2$	$\frac{s_{211}}{u_{211}}$ $\frac{s_{212}}{u_{212}}$ …. $\frac{s_{21r}}{u_{21r}}$	$\frac{s_{221}}{u_{221}}$ $\frac{s_{222}}{u_{222}}$ …. $\frac{s_{22r}}{u_{22r}}$	….	$\frac{s_{2y1}}{u_{2y1}}$ $\frac{s_{2y2}}{u_{2y2}}$ …. $\frac{s_{2yr}}{u_{2yr}}$	a_2
….					
U$_x$	$\frac{s_{x11}}{u_{x11}}$ $\frac{s_{x12}}{u_{x12}}$ …. $\frac{s_{x1r}}{u_{x1r}}$	$\frac{s_{x21}}{u_{x21}}$ $\frac{s_{x22}}{u_{x22}}$ …. $\frac{s_{x2r}}{u_{x2r}}$	….	$\frac{s_{xy1}}{u_{xy1}}$ $\frac{s_{xy2}}{u_{xy2}}$ …. $\frac{s_{xyr}}{u_{xyr}}$	a_x
Demand	b_1	b_2	…..	b_y	

Step 3 : If all the objectives are maximization then it can be converted into minimization type by subtracting the greatest element from all the fractional values.

Step 4 : Find the maximum ratio of the each row γ_{lk} and each column δ_{pk} and fix as given below:

	W₁	**W₂**	**….**	**W_y**	**Supply**	
U₁	$\frac{s_{111}}{u_{111}}$ $\frac{s_{112}}{u_{112}}$ …. $\frac{s_{11r}}{u_{11r}}$	$\frac{s_{121}}{u_{121}}$ $\frac{s_{122}}{u_{122}}$ …. $\frac{s_{12r}}{u_{12r}}$	….	$\frac{s_{1y1}}{u_{1y1}}$ $\frac{s_{1y2}}{u_{1y2}}$ …. $\frac{s_{1yr}}{u_{1yr}}$	a_1	γ_{11} γ_{12} …. γ_{1r}
U₂	$\frac{s_{211}}{u_{211}}$ $\frac{s_{212}}{u_{212}}$ …. $\frac{s_{21r}}{u_{21r}}$	$\frac{s_{221}}{u_{221}}$ $\frac{s_{222}}{u_{222}}$ …. $\frac{s_{22r}}{u_{22r}}$	….	$\frac{s_{2y1}}{u_{2y1}}$ $\frac{s_{2y2}}{u_{2y2}}$ …. $\frac{s_{2yr}}{u_{2yr}}$	a_2	γ_{21} γ_{22} … γ_{2r}
….	….	….	….	…	…	…
Uₓ	$\frac{s_{x11}}{u_{x11}}$ $\frac{s_{x12}}{u_{x12}}$ …. $\frac{s_{x1r}}{u_{x1r}}$	$\frac{s_{x21}}{u_{x21}}$ $\frac{s_{x22}}{u_{x22}}$ …. $\frac{s_{x2r}}{u_{x2r}}$	….	$\frac{s_{xy1}}{u_{xy1}}$ $\frac{s_{xy2}}{u_{xy2}}$ …. $\frac{s_{xyr}}{u_{xyr}}$	a_x	γ_{2r} γ_{2r} … γ_{xr}
Demand	b_1	b_2	…..	b_y		

	δ_{11} δ_{12} $\cdots$ δ_{1r}	δ_{21} δ_{22} $\cdots$ δ_{2r}	$\cdots$ $\cdots\cdot$ $\cdots$	δ_{y1} δ_{y2} $\cdots$ δ_{yr}		

Step 5 : Choose L = max { γ_{lr}, δ_{pr} } for every $l = 1\ to\ x\ , p = 1\ to\ y\ and\ k = 1\ to\ r$ **.**

Step 6: Select the call having L as one of its ratio. Suppose there are more than one cell choose the cell which has maximum ration for other fractional objectives.

Step 7: Choose the cell containing min$\left\{\sum_{l=1}^{x} \frac{s_{lpk}}{u_{lpk}}\ for\ fixed\ p\right\}$. If there is a tie then select one to which maximum allocation.

Step 8: Do the procedure of step 4 to step 6 until supply and demand requirement are not met.

4. Numerical Example:

To show the effectiveness of the proposed algorithm numerical example is solved by using proposed algorithm. Consider the following multi objective fuzzy fractional transportation problem. Here three objectives are considered, first objective is concerned with transportation cost which is the ratio of actual cost and preferred cost. Second objective is about time of transportation which is the ratio of actual transportation time and preferred transportation time. Third objective is related to damage cost which is the ratio of actual cost and preferred cost. All the parameters of the problem are described as fuzzy triangular number. This fuzzy number can be converted to crisp number by using Maleke Ranking function.

Table 1 : Transportation Cost

	A	**B**	**C**	**Supply**
D	$\frac{(3,5,7)}{(1,3,5)}$	$\frac{(4,7,8)}{(1,4,5)}$	$\frac{(14,15,17)}{(11,13,15)}$	(10,12,14)
E	$\frac{(6,8,10)}{(11,12,15)}$	$\frac{(14,17,18)}{(11,13,15)}$	$\frac{(11,12,13)}{(5,7,8)}$	(13,15,17)

F	$\frac{(12,14,16)}{(12,15,16)}$	$\frac{(7,10,11)}{(4,6,8)}$	$\frac{(11,13,14)}{(6,8,9)}$	(18,20,22)
Demand	(7,9,11)	(11,13,15)	(19,21,23)	

Table 2 : Transportation Time

	A	**B**	**C**	**Supply**
D	$\frac{(15,17,18)}{(7,9,12)}$	$\frac{(3,5,6)}{(1,2,4)}$	$\frac{(8,10,12)}{(0,3,4)}$	(10,12,14)
E	$\frac{(0,1,4)}{(1,2,5)}$	$\frac{(8,11,12)}{(3,4,5)}$	$\frac{(4,6,7)}{(3,5,6)}$	(13,15,17)
F	$\frac{(10,13,14)}{(6,8,10)}$	$\frac{(15,16,17)}{(10,12,13)}$	$\frac{(9,10,13)}{(9,11,14)}$	(18,20,22)
Demand	(7,9,11)	(11,13,15)	(19,21,23)	

Table 3: Damage Cost

	A	**B**	**C**	**Supply**
D	$\frac{(10,13,14)}{(6,8,11)}$	$\frac{(13,12,16)}{(7,9,10)}$	$\frac{(6,8,9)}{(9,11,12)}$	(10,12,14)
E	$\frac{(13,15,16)}{(9,11,12)}$	$\frac{(13,14,15)}{(4,6,8)}$	$\frac{(15,19,21)}{(5,7,8)}$	(13,15,17)
F	$\frac{(4,7,8)}{(6,9,10)}$	$\frac{(13,15,16)}{(4,6,7)}$	$\frac{(15,17,18)}{(6,7,9)}$	(18,20,22)
Demand	(7,9,11)	(11,13,15)	(19,21,23)	

Step 1 :

To convert the given fuzzy number to crisp number , Maleke ranking function has been used.

Maleke Ranking Function:

Ranking function of the triangular fuzzy number $\breve{M} = (l_1, l_2, l_3)$ is defined as

$$R(\widetilde{M}) = 2l_2 + \frac{1}{2} (l_3 - 2l_2 + l_1)$$

By using the above ranking function the given fuzzy parameters of the multi objective transportation problem can be written as follows:

Table 4: Transportation Cost, Transportation Time and Damage Cost

	A	B	C	Supply
D	$\frac{10}{6}$ $\frac{13.5}{18.5}$ $\frac{25}{16.5}$	$\frac{13}{7}$ $\frac{9.5}{4.5}$ $\frac{29.5}{15.5}$	$\frac{30.5}{26}$ $\frac{20}{5}$ $\frac{15.5}{21.5}$	12
E	$\frac{16}{25}$ $\frac{3}{5}$ $\frac{29.5}{21.5}$	$\frac{33}{28}$ $\frac{21}{8}$ $\frac{28}{12}$	$\frac{24}{35}$ $\frac{11.5}{9.5}$ $\frac{37}{13.5}$	15
F	$\frac{28}{29}$ $\frac{25}{16}$ $\frac{13}{17}$	$\frac{19}{12}$ $\frac{33.5}{23.5}$ $\frac{29.5}{11.5}$	$\frac{25.5}{15.5}$ $\frac{21}{22.5}$ $\frac{33.5}{15}$	20
Demand	9	13	21	

Total supply = 47
Total demand = 43.
Therefore dummy column can be included with the 0 cost and 4 units demand.

Step 2 :

Finding the fractional value of every value in the above table , reduced table as follows :

Table 5 :

	A	B	C	D1	Supply
D	1.67 1.81 1.51	1.86 2.11 1.69	1.17 4 0.72	0 0 0	12
E	0.64 0.6 1.37	1.18 2.625 2.33	1.78 1.21 2.74	0 0 0	15
	0.97	1.58	1.65	0	20

F	1.56 0.76	1.43 2.57	0.93 2.23	0 0	
Demand	9	13	21	4	

Step 3:

Find the maximum value for each row and each column and the maximum value are written in the above table :

Table 6:

	A	**B**	**C**	**D1**	**Supply**	**Max. Value**
D	1.67 1.81 1.51	1.86 2.11 1.69	1.17 **4** 0.72	0 0 0	12	1.86 4 1.69
E	0.64 0.6 1.37	1.18 2.625 2.33	1.78 1.21 2.74	0 0 0	15	1.78 2.625 2.74
F	0.97 1.56 0.76	1.58 1.43 2.57	1.65 0.93 2.23	0 0 0	20	1.65 1.56 2.57
Demand	9	13	21	4		
Max.Value	1.67 1.81 1.51	1.86 2.625 2.57	1.78 4 2.74	0 0 0		

Step 4: Choose the maximum value (L) among the largest value of each row and column. That is

L = Max{ 1.86 , 4,1.69,1.78,2.625,2.74,1.65,1.56,2.57, 1.67,2.81,1.51, 1.86, 2.625, 2.57, 1.7,

4, 2.74 }.

L = 4

Step 5:

The cell allocation of 4 is (1,3) of second objective. Find the summation of all objective values of 3^{rd} column.

That is , 1.78 + 1.21 + 2.74 = 5.73

$1.65 + 0.93 + 2.23 = 4.81$

Among them choose the minimum value . Here the minimum value is 4.18 which lies in the cell (2,3). Allocate min{20,21} in the cell (1, 4).

Repeating the procedure till all the optimum solution is obtained. The optimum solution is given as follows:

Table 7:

	A	B	C	D1	Supply
D	1.67 1.81 1.51	1.86 2.11 1.69 **12**	1.17 4 0.72	0 0 0	12
E	0.64 0.6 1.37 **9**	1.18 2.625 2.33 **1**	1.78 1.21 2.74 **1**	0 0 0 **4**	15
F	0.97 1.56 0.76	1.58 1.43 2.57	1.65 0.93 2.23 **20**	0 0 0	20
Demand	9	13	21	4	

The objective values are

$$Z_1 = \frac{156+144+33+24+510}{84+225+28+135+310} = \frac{864}{782} = 1.104$$

$$Z_2 = \frac{114+27+21+11.5+420}{54+45+8+905+450} = \frac{593.5}{566.5} = 1.048$$

$$Z_3 = \frac{354+265.5+28+37+670}{210+193.5+12+13.5+300} = \frac{1354.5}{729} = 1.633$$

The numerical problem has taken from the article SHEEMA SADIA (2017). Using the developed algorithm the numerical problem solved. The obtained solution have compared with solution with Sheema (2017). The following comparison table is given the effectiveness of the proposed method.

Table 8:

Objective values	**Proposed Method**	**Existing method**
Z_1 (Transportation Cost)	1.104	1.187
Z_2 (Transportation Time)	1.048	1.486
Z_3 (Damage Cost	1.6	1.6

Also the numerical problem from the article Sheema (2017), Abouzar(2018) and Vishwas(2020) solved by proposed algorithm. The obtained solution have compared with the existing solution The following comparison table is given the effectiveness of the proposed method.

Table 9:

Authors	**Proposed Method**	**Existing method**
Sheema et.al	Z_1 =0.660 $Z_2 = 1.6$ $Z_3 = 1.399$	Z_1 =0.928 $Z_2 = 1.6$ $Z_3 = 1.45$
Abouzar et. Al	$Z_1 = \frac{140}{288} = 0.486$ $Z_2 = \frac{662}{632} = 1.047$	$Z_1 = \frac{199}{245} = 0.812$ $Z_2 = \frac{704}{543} = 1.296$
Vishwas Deep Joshi et.al	Z_1 =0.671 $Z_2 = 0.645$ $Z_3 = 0.641$	$Z_1 = 0.823$ Z_2 =0.56 $Z_3 = 0.69$

5. Conclusion

In this paper a new algorithm is proposed to find the optimal solution of the multi objective fuzzy transportation problem. The proposed algorithm is very simple and easy to understand. This algorithm gives the better solution in both crisp environment and fuzzy environment. The numerical example is solved to explain the algorithm. The comparison Table 8 and Table 9 has shown the effectiveness of the proposed algorithm. The solution of the problem is compared with the existing method. The proposed algorithm gives the better solution than the existing one.

6. References

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