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Study Of Deterministic Inventory Model For Deteriorating Items With Stock-Dependent Demand Rate Under Inflation

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Abstract
Deterministic inventory model for deteriorating items with stock dependent demand is developed. In this model the shortages are allowed and partially backlogged and the effect of inflation rate and delay in payments are discussed. This paper establishes an inventory model for the deteriorating items and stock dependent demand rate under inflation when the supplier offers a permissible delay to the purchaser. A sensitivity analysis of the optimal solution with respect to the parameters of the system is carried out.

Key words : Inventory model, shortages

1. INTRODUCTION

Both in deterministic and probabilistic inventory models of classical type, it is observed that payments are made to the supplier immediately after receiving the items. In practice, the supplier will offer the retailer a delay period in payment for the amount of purchase to increase the demand known as trade credit period. Inventory plays a vital role in business to ensure smooth efficient running of its operation. Large number of research papers/Articles has been presented by many authors for controlling the inventory of deteriorating items such as volatile liquids, blood banks, medicines, fashion goods and non-deteriorating items such as wheat, rice, dry fruits etc. The control and the maintenance of inventories of deteriorating items with shortages have received much attention of several researchers in the recent years because most physical goods deteriorate overtime. In practice, the deterioration of items is a common phenomenon. Hence the impact of product deterioration should not be neglected in the decision process. The retailer must pay off as soon as the items are received. It is tacitly assumed in classical economic order quantity inventory model, a supplier frequently offers his retailers a delay of payment for settling the amount due. The permissible delay in payment is an effective method of attracting new customers and increasing sales. It may be applied as alternative to price discount because it does not provoke the competitors to reduce their prices and thus introduce lasting price reductions. Datta *et al.* (1991) made an attempt to investigate the effects of inflation and time-value of money on an inventory model with linear time-dependent demand rate without shortages. Pal, S., Goswami, A., Chaudhuri, K.S. (1993) studied a deterministic inventory model for deteriorating items with stock-dependent demand rate. Urban, T.L. (1995) considered an inventory model with the demand rate dependent on stock and shortages levels. Hariga, M.A. (1995) deals an effect of inflation and time value of money on an inventory model with time-varying demand rate. Shortages are also permitted in this model. Padmanabhan and Vrat (1995) further presented inventory models for perishable items with stock-dependent selling rate. The selling rate is assumed to be a function of current inventory level. Ray *et al.* (2012) developed a finite time-horizon deterministic economic order quantity (EOQ) inventory model with shortages under inflation, where the demand rate at any instant depends on the on-hand inventory (stock level) at that instant.

2. ASSUMPTIONS AND NOTATIONS

The mathematical models of an inventory problem are based on the following assumptions and notations:

- The consumption rate D(t) at any time t is assumed to be $\alpha + \beta\ I(t) + \gamma\ t\text{-}ds$, where α is a positive constant, β is the stock-dependent demand rate parameter, $0 \le \beta,\ \gamma\ \le 1$, and I($t$) is the inventory level at time t.
- The replenishment rate is infinite and lead time is zero.
- The planning horizon is finite.
- Shortages are backlogged at the rate of $e^{-\delta t}$ where $0 < \delta < 1$ and t is the waiting time for next replenishment.
- The deterioration rate is $a + b\ t$; $a, b > 0$.
- Product transactions are followed by instantaneous cash flow.

Notations:

r discount rate, representing the time value of money
i inflation rate
R $r-i$, representing the net discount rate of inflation is constant
H planning horizon
T replenishment cycle
m the number of replenishments during the planning horizon n = H/T
T_j the total time that is elapsed up to and including the jth replenishment cycle (j=1,2,…..n) where $T_0 = 0$, $T_1 = T$, and $T_n = H$.
t_j the time at which the inventory level in the jth replenishment cycle drops to zero (j=1,2,…..n)
$T_j–t_j$ time period when shortages occur (j=1,2,…..n)
Q the 2nd, 3rd,…..nth replenishment lot size.
I_m maximum inventory level
A ordering cost per replenishment
C per unit cost of the item
C_h holding cost per unit per unit time
C_s shortage cost per unit per unit time
C_o opportunity cost per unit per unit time due to lost sale.

Model Formulation

Suppose the planning horizon H is divided into n equal intervals of length $T = H/n$. Hence the reorder times over the planning horizon H are $T_j = jT$ (j=1, 2,…..n). The period for which there is no shortage in each interval $[jT, (j+1)T]$ is a fraction of the scheduling period T and is equal to kT, $(0 < k < 1)$. Shortages occur at time $t_j = (k+j-1)T$, (j=1,2,…..n) and are accumulated until time $t=jT$ (j=1,2,…..n)and shortages are backlogged exponentially.

The first replenishment lot size of I_m is replenished at $T_0=0$. During the time interval $[0, t_1]$ the inventory level decreases due to stock-dependent demand rate and deterioration and falls to zero at $t = t_1$, now shortages start during the time interval $[t_1, T]$ and accumulated until $t = T$ The differential equations governing the instantaneous state of inventory level at any time t are given by

$I'(t) + (a+bt)\, I(t) = -[+\beta I(t)+\gamma t\text{-}ds]$ with $I(t_1)=0$ 0 t t_1 …(1)

$I'(t) = -(\alpha + \gamma\, t\text{-}ds)\, e^{-\delta t}$ t_1 t T …(2)

The respective solutions of the above differential equations are

$$I(t) = (\alpha - ds)\exp\left(-(a+\beta)t - \frac{bt^2}{2}\right)\int_t^{t_1}(\alpha - ds + \gamma x)\exp\left((a-ds+\beta)x + \frac{bx^2}{2}\right)dx \quad 0 \text{ t } t_1$$

…(3)

And $$I(t) = -\frac{1}{\delta^2}\left[(\delta\alpha - ds + \gamma)\left(e^{-\delta t_1} - e^{-\delta t}\right) + \delta\gamma\left(t_1 e^{-\delta t_1} - te^{-\delta t}\right)\right] \quad t_1 \text{ t T}$$ …(4)

The maximum inventory level during first replenishment cycle is

$$I(0) = I_m = \int_0^{t_1}(\alpha - ds + \gamma x)e^{\left((a-ds+\beta)t + \frac{bt^2}{2}\right)}dt$$ …(5)

And the maximum shortage quantity during the first replenishment, which is backlogged

$$I_b = \frac{1}{\delta^2}\left[(\delta\alpha - ds + \gamma)\left(e^{-\delta t_1} - e^{-\delta T}\right) + \delta\gamma\left(t_1 e^{-\delta t_1} - Te^{-\delta T}\right)\right]$$ …(6)

The present value of the ordering cost during first replenishment cycle is A, as the replenishment is done at the start of each cycle.

The present value of the holding cost of inventory during first replenishment cycle is

$$H.\,C. = C_h\int_0^{t_1} I(t)e^{-Rt}dt$$ (7) …

The present value of the shortage cost during first replenishment cycle is

$$S.\,C. = C_s\int_{t_1}^{T}\frac{1}{\delta^2}\left[(\delta\alpha - ds + \gamma)\left(e^{-\delta t_1} - e^{-\delta t}\right) + \delta\gamma\left(t_1 e^{-\delta t_1} - te^{-\delta t}\right)\right]e^{-Rt}dt$$

…(8)

Replenishment items are consumed by demand as well as deterioration during $[0, t_1]$. The present value of material cost during the first replenishment cycle is

$$C_p = CI_m + \frac{Ce^{-RT}}{\delta^2}\left[(\delta\alpha - ds + \gamma)\left(e^{-\delta t_1} - e^{-\delta T}\right) + \delta\gamma\left(t_1 e^{-\delta t_1} - Te^{-\delta T}\right)\right]$$ …(9)

Opportunity cost due to lost sale

$$O.\,C. = C_o\int_{t_1}^{T}(\alpha - ds + \gamma t)\left(1 - e^{-\delta t}\right)e^{-Rt}dt$$ …(10)

Consequently, the present value of total cost of the system during the first replenishment cycle is

$$TRC = A + H.C. + S.C. + C_p + O.C.$$ …(11)

The present value of total cost of the system over a finite planning horizon H is

$$TC\,(\text{m}, k) = \sum_{j=1}^{m-1} TRC\ e^{-RjT} - Ae^{-RH} = TRC\left(\frac{1-e^{-RH}}{1-e^{-RH/m}}\right) - Ae^{-RH}$$ …(12)

The present value of total cost TC (n, k) is a function of two variables n and k, where n is a discrete variable and k is a continuous variable. For a given value of n, the necessary condition for $TC\,(m, k)$ to be minimized is $\frac{dTC(m,k)}{dk} = 0$ which gives

$$\frac{C_hH}{m}\left(\alpha-ds+\frac{\gamma kH}{m}\right)\exp\left((a-ds+\beta)\frac{kH}{m}+\frac{b}{2}\left(\frac{kH}{m}\right)^2\right)\int_0^{\frac{kH}{m}}\exp\left(-(a-ds+\beta)t-\frac{bt^2}{2}\right)e^{-Rt}dt$$

$$+\frac{C_sH}{\delta Rm}\left[(\delta\alpha-ds+\gamma)\left\{\exp\left(\frac{(\delta k+R)H}{m}\right)-\exp\left(\frac{(\delta+R)kH}{m}\right)\right\}-\gamma\left(1-\frac{\delta kH}{m}\right)\right.$$

$$\left.\times\exp\left(-\frac{(\delta k+R)H}{m}\right)+\frac{\gamma}{(\delta+R)}\left\{\left\{\delta-\left(\frac{\delta(\delta+R)kH}{m}-R\right)\right\}\exp\left(-\frac{(\delta+R)kH}{m}\right)\right]$$

$$+\frac{CH}{m}\left(\alpha-ds+\frac{\gamma kH}{m}\right)\left[\exp\left((a-ds+\beta)\frac{kH}{m}+\frac{b}{2}\left(\frac{kH}{m}\right)^2\right)-\exp\left(\frac{(\delta k+R)H}{m}\right)\right]$$

$$\frac{C_oH}{m}\left(\alpha-ds+\frac{\gamma kH}{m}\right)\left[\exp\left(-\frac{(\delta+R)kH}{m}\right)-\exp\left(-\frac{RkH}{m}\right)\right]=0 \qquad \ldots(13)$$

Provided the condition $\frac{d^2TC(m,k)}{dk^2}>0$ is satisfied.

We follow the optimal solution procedure proposed by Montgomery (1982), we let (m^*, k^*) denote the optimal solution of $TC(m, k)$ and let (m, $k(m)$)denote the optimal solution to $TC(m, k)$ when m is given. If $\tilde{m}$ is the smallest integer such that $TC(\tilde{m},k(\tilde{m}))$is less than each value of $TC(\tilde{m},k(\tilde{m}))$ in the interval $\tilde{m}+1 < \tilde{m} < \tilde{m}+10$. Then we take $(\tilde{m},k(\tilde{m}))$ as the optimal solution to $TC(m, k(m))$. Hence $(\tilde{m},k(\tilde{m})) = (m^*,k^*)$. Using the optimal procedure described above, we can find the maximum inventory level and optimal order quantity to be

$$I_m=\int_0^{\frac{k^*H}{m^*}}(\alpha-ds+\gamma x)\exp\left((a-ds+\beta)x+\frac{bx^2}{2}\right)dx \qquad \ldots(15)$$

And $$Q^*=\int_0^{\frac{k^*H}{m^*}}(\alpha-ds+\gamma x)\exp\left((a-ds+\beta)x+\frac{bx^2}{2}\right)dx$$

$$+\frac{1}{\delta^2}\left[(\delta\alpha-ds+\gamma)\left(e^{-\delta k^*H/m^*}-e^{-\delta H/m^*}\right)+\delta\gamma\left(\frac{k^*H}{m^*}e^{-\delta k^*H/m^*}-\frac{H}{m^*}e^{-\delta H/m^*}\right)\right] \qquad \ldots(16)$$

3. CONCLUSION

To make our study more suitable to present-day market, we have done our research in an inflationary environment. Even till now, most of the researchers have been either completely ignoring the decay factor or are considering a constant rate of deterioration which is not practical. Therefore, we have taken the time dependent decay factor. The problem has been formulated analytically and has been used to arrive at the optimal solution.

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