INTEGRATED THREE ECHELON INVENTORY MODELWITH FINITE MANUFACTURING AND REMANUFACTURINGRATES ALONG WITH THE TRANSPORTATION COST AND PACKAGING COST

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ABSTRACT

Supply chain means is a working togetherness of each individual's profit. In this proposed model we discusses allow the three echelon inventory model along with the transportation cost and packaging cost. Mathematical model and the numerical example are delivered for the best knowingness.

Keywords:Three Echelon, Manufacturing, Remanufacturing, Transportation cost, Packaging cost

1. INTRODUCTION

Providing satisfactory service especially that meets the demands of the customersisthemostexcitingpartofalltheadministrations.Inventorycontrolaims at minimizing inventory expenditure ensuring nonstop service throughout the supply chain of production process. Quantity of the availability of particular goods as demanded by the customer is defined as supply chain. Hence we are proving the importance of inventory in supplychain the following proposed model.Supplier,manufacturer,remanufacturerandtheretailercomeunderthethree echelon closed supply chain which is taken for analysis in this paper. The manufacturer gets the raw material from the supplier. While the newly produced products are bought from the manufacturer to the retailers. Insufficient process control, work inadequacy due to deviating instructions and improper maintenance are the result of imperfect production. Henceforth, the retailer gives the defective product to the remanufacture. The newly produced and remanufactured products fulfill the demands of the retailer in steady rate where the production rate is predictable. Similarly the steady demand of the manufacturer and remanufacturer are carried out by thesupplier.One of the primary branches of the supply chain is packaging. Packaging is essential one for careful supervision, accessibility and for a deliberate. It guarantees simple treatment of things during transportation giving creation from mechanical harm expanding timeframe of realistic usability of item.Every organization are exceeding focused in quantifying their cost. This paperisextendstheworkofBimalKumarMawandiya,J.K.Jha&JiteshThakkar's"Production-Inventory Model For Two-Echelon Closed Loop Supply Chain With Finite ManufacturingAndRemanufacturing Rates" with the concepts of three echelon inventory model along with transportation and packaging cost.Remainingofthispaperextendedinthefollowingway,Chapter2dealswith the review of the literature, at the same time explain the notations and assumption used in the mathematical model. Chapter 3 demonstrates the mathematical model. Chapter 4 derives the numerical example for the above mathematical model,finally Chapter 5 proposes the conclusion of the proposed paper.

2. LITERATUREREVIEW

The optimization of the first inventory was introduced by Harris in the year 1913. Following him many researcher or academicians improved their inventory model with fresh philosophies. Konstantaras&Skouri 2010, Jaber& EL Saadany 2011arediscussedsingleechelonsupplychainbutmultiechelonclosedloopsupply chain issues stated in a simple manner. Arrow et al. (1950) mainly concentrated on multi echelon inventory problem. J.F. Burns & B.D. Sivazlian, "Dynamic analysis of multi echelon supply systems" presented examined the forceful reaction of a multi echelon supply chain to several demand hired upon the organization by the end user. M.C. Van der Heijden derived a simple rule in inventory control under the evaluation short of lot sizing for multilevel supplymethod.

B. Pal, S. S. Sana, & K. Chaudhuri, built up a three layer combined creation stock model considering wild quality happens in supplier and remanufacture arrange. M. K. Salameh& M. Y. Jaber, stated that products are classified into two ways (i.e.) good product and defective products. And at the same time they also established if the amount of imperfect items rise then automatically the economic order quantity will rise.

Teunter (2004) made Koh et al (2002) work widespread and proposed a formula through the rate of limited production and reproduction. Jabber & EL Saadany 2011; Tgai 2012 presented a paper "An economic production and remanufacturing model with learning effects". In this paper, the processes of manufacturing and remanufacturing properties are inspected. The ideal manufacturing and reproduction policies are expressed by Chang et al (2008) to optimize the gain of the total system (consisting of supplier, manufacturer, remanufacturer and the retailer).
With the basis of the above inventory model, now,weestablishthe paper under the concept of three echelon inventory model with following twocases.

Case-1: Each procurement lot size of raw material will be used for n_1(k_1= 1/n_1) number of production batches of the manufacturer, where $n_1$is a positive integer.
Hence, the procurement lot size of raw material (Q_{f1}) in this case will be$\frac{n_1 m_1 Q_1}{f}$.

Case-2: Each production lot size of the manufacturer will use n_2(k_2=n_2) number of lots of raw material, where $n2$ is a positive integer. Hence, procurement lot size of raw material (Q_{f2}) in this case will be m_2Q_2/n_2f. The expression for the joint total cost of the system in both the cases will be different

ISSN 2515-8260 Volume 07, Issue 09, 2020

Notations:

Develop the mathematical model the following notations and assumptions are used.

Suppliers:

P_s : Purchasing cost of the supplier perunit
A_s : Ordering cost of the supplier perunit
h_s : Holding cost of the supplier perunit
F_s : Freight cost of the supplier perunit
PC_S : Packaging cost of the supplier peritem

Manufacturer:

P : Production rate of themanufacturer
A_2 : Set up cost of the manufacturer perproduction
A_4 : Ordering cost of the manufacturer for rawmaterial
h_2 : Inventory holding cost of the manufacturer for finished product per unit time
h_4 :Inventoryholdingcostofthemanufacturerforrawmaterialperunit time
f : transformation factor of finished product from the rawmaterial
Q_{fi} : Number of procurement of manufacturer's rawmaterial
T_i : Length of the production cycle in case-I (1,2)

Remanufacturer:

P' : Production rate of theremanufacturer
h_3^r : Inventory holding cost of the remanufacturer for raw material perunit time
h_5 :Inventoryholdingcostoftheremanufacturerforfinishedproductperunit time
r : returned part of order (which is lies between 0 and 1)
α : Transformation factor of remanufactured item from the returneditem
F_m : Shipping cost the manufacturer perdelivery
P_m : Purchasing cost of manufacturer and remanufacturer perunit
PC_m : Packaging cost of manufacturer and remanufacturer peritem

Retailer

D : Demand of the retailer per unittime
A_1 : Ordering cost of the retailer for the manufacturedproduct
A_2 : Ordering cost of the retailer for the remanufacturedproduct
h_1 : Holding cost of the product per unit in theretailer
Q_i :Productsuppliedbytheretailerfromthemanufacturer(decisionvariable) (i=1,2)
m_i :Withinoneproductioncyclenumberofshipmentsmadebythemanufacture to the retailer (i=1,2) here $m_1 and m_2$are positiveinteger
Q_{ri} : Product supplied by the retailer from theremanufacturer
l_i : Within one production cycle number of shipments made by the remanufacture to the retailer (i= 1, 2) here $l_1 and l_2$are positiveinteger
F_r : Transportation cost of theretailer
P_c : selling cost of used product perunit M

	: Substantial price perunit
L	: Employer cost perunit
P_r	: Purchasing cost of retailer perunit
P_{Cr}	: Packaging cost of the retailer per item

2 .2 Assumption

1. The quality of the new product and the remanufactured product issame.
2. Constant demand is faced by theretailer.
3. Returned rate of the defective product is alsoconstant.
4. Tocomparewiththeholdingcostofthenewproductandremanufacturedproduct of the manufacturer and the remanufacturer while retailer faces highest holding cost of inventory.
5. Holding cost of raw material to produce per unit of new product is less than holding cost of newly produced product produced by themanufacturer.
6. Holding cost of returned product is less than the holding cost of the remanufactured product produced by theremanufacturer.

3. MATHEMATICAL MODEL

Here the profit of the supplier, manufacturer, remanufacturer and the retailer are depend upon the sales revenue, ordering cost of both manufactured and remanufactured product, holding cost of both manufactured and remanufactured product, transportation cost and the packaging cost.

The cost functions are defined as follows.

Supplier

Sales revenue: $P_m D$

Ordering cost: $\frac{A_s D}{n Q_1}$

Purchasing cost: $P_s D$

Holding cost: $\frac{h_s D Q_1 m_1}{2P(1-\alpha r)}$

Packaging cost of the raw material: $PC_s \frac{D}{Q}$

Transportation cost for the raw materials: $\frac{F_s D}{Q_1}$

Manufacturer

Sales revenue of manufacturer and remanufacturer $= P_r D$

Set up cost per production: $\frac{(1-\alpha r) D A_2}{m_1 Q_1}$

Inventory holding cost for finished product:

$$(1-\alpha r)\left\{(m_1-1)-(m_1-2)\frac{D}{P}\right\}\frac{Q_1 h_2}{2}$$

Ordering cost of raw material for case (1): $\frac{(1-\alpha r)DA_4}{n_1 m_1 Q_1}$

Ordering cost of raw material for case (2): $\frac{(1-\alpha r)DA_4 n_2}{m_2 Q_2}$

Inventory holding cost of raw material for case (1)

$$:m_1\left\{(n_1-1)+(1-\alpha r)\frac{D}{P}\right\}\frac{Q_1 h_4/f}{2}$$

Inventory holding cost of raw material for case (2): $\frac{m_2(1-\alpha r)Q_2 h_4 D}{2n_2 Pf}$

Packaging cost of the manufactured and remanufactured product: $PC_m\frac{D}{Q}$

Transportation cost manufactured and remanufactured product: $\frac{F_m D}{n_1 Q_1}$

Remanufacturer

Production set up cost: $(1-\alpha r)\frac{A_3 D}{m_1 Q_1}$

Holding cost of the remanufactured product: $\frac{m_1\alpha r}{l_1}\left(\frac{\alpha r}{(1-\alpha r)}\right)\begin{Bmatrix}(l_1-1)-\\(l_1-2)\frac{D}{P_r}\end{Bmatrix}\frac{Q_1 h_5}{2}$

Holding cost of the returned product: $\frac{m_1}{\alpha}\left(\frac{\alpha r}{1-\alpha r}\right)\left(1-\frac{\alpha r D}{P_r}\right)\frac{Q_1 h_3}{2}$

Retailer

Sales revenue: $P_c D$

Ordering cost of new product and remanufactured product: $(1-\alpha r)\left(A_1+\frac{l_1 A_5}{m_1}\right)\frac{D}{Q_1}$

Holding cost of new product and remanufactured product:

$$(1-\alpha r)\left\{1+\frac{m_1}{l_1}\left(\frac{\alpha r}{1-\alpha r}\right)^2\right\}\frac{Q_1 h_1}{2}$$

Packaging cost: $PC_r\frac{D}{Q}$

Transportation cost: $\frac{F_r D}{n_1 Q_1}$

In this paper, we concentrate on the growth of revenue of the supplier, manufacturer, remanufacturer and the retailer.

The predictable joint total profit per unit time is determined by adding the total profit of the supplier, manufacturer, remanufacturer and the retailer.

The joint total annual profit (JTP) = Supplier's total profit +
manufacturer and remanufacturer's total profit +
retailer's total profit

The joint total annual profit (JTP_1) =

Supplier's sales revenue – [supplier's purchasing cost + supplier's ordering cost + supplier's holding cost + supplier packaging cost +supplier transportation cost] + sales revenue of manufacturer and remanufacturer – [manufacture's inventory holding cost of finished product + manufacturer's inventory holding cost of raw material + manufacturer's ordering cost of raw material + manufacturer's production set up cost + remanufacturer inventory holding cost of returned product + remanufacturer production set up cost +packaging cost + transportation cost] + retailer's sales revenue – [retailer's ordering cost of new product and remanufactured product + retailers holding cost of new product and remanufactured product + retailer's transportation cost + retailer packaging cost +retailer transportation cost]

$$\text{JT}P_1 = P_mD - \left[P_sD + \frac{A_sD}{n_1Q_1} + \frac{h_sDQ_1m_1}{2P(1-\alpha r)} + \frac{F_r\,D}{Q_1} + \frac{PC_sD}{Q_1}\right] +$$

$$P_rD - \Bigg[(1-\alpha r)\left\{(m_1-1)-(m_1-2)\frac{D}{P}\right\}\frac{Q_1h_2}{2} + m_1\left\{(n_1-1) + (1-\alpha r)\frac{D}{P}\right\}\frac{Q_{1h_4}/f}{2} +$$

$$\frac{(1-\alpha r)DA_4}{n_1m_1Q_1} + \frac{(1-\alpha r)DA_2}{m_1Q_1} + \frac{m_1\alpha r}{l_1}\left(\frac{\alpha r}{(1-\alpha r)}\right)\left\{(l_1-1) - (l_1-2)\frac{D}{P'_r}\right\}\frac{Q_{1h_5}}{2} + \frac{m_1}{\alpha}\left(\frac{\alpha r}{1-\alpha r}\right)\left(1-\right.$$

$$\left.\frac{\alpha rD}{P'_r}\right)\frac{Q_1h_3}{2} + (1-\alpha r)\frac{A_3D}{m_1Q_1} + \frac{F_m\,D}{n_1Q_1} + \frac{PC_mD}{Q_1}\Bigg] +$$

$$P_cD - \left[(1-\alpha r)\left(A_1 + \frac{l_1A_5}{m_1}\right)\frac{D}{Q_1} + \left\{(1-\alpha r)\left\{1+\frac{m_1}{l_1}\left(\frac{\alpha r}{1-\alpha r}\right)^2\right\}\frac{m_1}{l_1}\left(\frac{\alpha r}{1-\alpha r}\right)^2\right\}\frac{Q_1h_1}{2} + \frac{F_r\,D}{Q_1} + \frac{PC_rD}{Q_1}\right]$$

$$= P_mD - P_sD + P_rD + P_cD - \frac{F_m\,D}{n_1Q_1} - \frac{A_sD}{n_1Q_1} - \frac{PC_sD}{Q_1} - \frac{PC_mD}{Q_1} - \frac{PC_rD}{Q_1} - \frac{F_r\,D}{Q_1} + \frac{F_m\,D}{Q_1} - \frac{h_sDQ_1m_1}{2P(1-\alpha r)} -$$

$$(1-\alpha r)\left(A_1 + \frac{l_1A_5}{m_1}\right)\frac{D}{Q_1} - \frac{(1-\alpha r)DA_2}{m_1Q_1} - (1-\alpha r)\frac{A_3D}{m_1Q_1} - \frac{(1-\alpha r)DA_4}{n_1m_1Q_1} - (1-\alpha r)\left\{1+\frac{m_1}{l_1}\left(\frac{\alpha r}{1-\alpha r}\right)^2\right\}\frac{Q_1h_1}{2} -$$

$$(1-\alpha r)\left\{(m_1-1) - (m_1-2)\frac{D}{P}\right\}\frac{Q_1h_2}{2} - \frac{m_1}{\alpha}\left(\frac{\alpha r}{1-\alpha r}\right)\left(1-\frac{\alpha rD}{P'_r}\right)\frac{Q_1h_3}{2} -$$

$$m_1\left\{(n_1-1) + (1-\alpha r)\frac{D}{P}\right\}\frac{\frac{Q_{1h_4}}{f}}{2} - \frac{m_1\alpha r}{l_1}\left(\frac{\alpha r}{(1-\alpha r)}\right)\left\{(l_1-1) - (l_1-2)\frac{D}{P'_r}\right\}\frac{Q_{1h_5}}{2}$$

$$\text{JTP}_1 = (P_m - P_s + P_r + P_c)D - \left[\frac{A_s}{n_1} + F_r + F_s + PC_s + PC_m + PC_r + \frac{F_m}{n_1}\right]\frac{D}{Q_1} - \frac{h_sDQ_1m_1}{2P(1-\alpha r)} - (1-\alpha r)\left[A_1 + \frac{A_2}{m_1} + \frac{A_3}{m_1} + \frac{A_4}{n_1m_1} + \frac{l_1A_5}{m_1}\right]\frac{D}{Q_1} - \frac{Q_1}{2}\left[\begin{array}{c}(1-\alpha r)\left\{1+\frac{m_1}{l_1}\left(\frac{\alpha r}{1-\alpha r}\right)^2\right\}h_1 + \\ (1-\alpha r)\left\{(m_1-1) - (m_1-2)\frac{D}{P}\right\}h_2 + \\ \frac{m_1}{\alpha}\left(\frac{\alpha r}{1-\alpha r}\right)\left(1-\frac{\alpha rD}{P'_r}\right)h_3 + m_1\left\{(n_1-1) + (1-\alpha r)\frac{D}{P}\right\}h_4 + \\ \frac{m_1\alpha r}{l_1}\left(\frac{\alpha r}{(1-\alpha r)}\right)\left\{(l_1-1) - (l_1-2)\frac{D}{P'_r}\right\}h_5\end{array}\right]$$

….(1)

Now, partially differentiate equation (1) with respect to Q_1. We

$$\frac{\partial JTP_1(Q_1, m_1, n_1, l_1)}{\partial Q_1} = \left[\begin{array}{c}\frac{A_s}{n_1} + F_r + F_s + PC_s + PC_m \\ +PC_r + \frac{F_m}{n_1}\end{array}\right]\frac{D}{Q_1^2} - \frac{h_sDm_1}{2P(1-\alpha r)} +$$

$$(1-\alpha r)\left[A_1 + \frac{A_2}{m_1} + \frac{A_3}{m_1} + \frac{A_4}{n_1m_1} + \frac{l_1A_5}{m_1}\right]\frac{D}{Q_1^2} -$$

$$\frac{1}{2}\begin{bmatrix} (1-\alpha r)\left\{1+\frac{m_1}{l_1}\left(\frac{\alpha r}{1-\alpha r}\right)^2\right\}h_1 + \\ (1-\alpha r)\left\{(m_1-1)-(m_1-2)\frac{D}{P}\right\}h_2 + \\ \frac{m_1}{\alpha}\left(\frac{\alpha r}{1-\alpha r}\right)\left(1-\frac{\alpha r D}{P_r'}\right)h_3 + m_1\left\{(n_1-1)+(1-\alpha r)\frac{D}{P}\right\}h_4 + \\ \frac{m_1 \alpha r}{l_1}\left(\frac{\alpha r}{(1-\alpha r)}\right)\left\{(l_1-1)-(l_1-2)\frac{D}{P_r'}\right\}h_5 \end{bmatrix}$$

….. (2)

$$Q_1 = \sqrt{\frac{2PD(1-\alpha r)\begin{bmatrix} \frac{A_s}{n_1}+F_r+F_s+PC_s+PC_m+PC_r+ \\ \frac{F_m}{n_1}+(1-\alpha r)\left[A_1+\frac{A_2}{m_1}+\frac{A_3}{m_1}+\frac{A_4}{n_1 m_1}+\frac{l_1 A_5}{m_1}\right]\end{bmatrix}}{\begin{bmatrix} h_s D m_1 + P(1-\alpha r) \\ \begin{bmatrix} (1-\alpha r)\left\{1+\frac{m_1}{l_1}\left(\frac{\alpha r}{1-\alpha r}\right)^2\right\}h_1 + \\ (1-\alpha r)\left\{(m_1-1)-(m_1-2)\frac{D}{P}\right\}h_2 + \\ \frac{m_1}{\alpha}\left(\frac{\alpha r}{1-\alpha r}\right)\left(1-\frac{\alpha r D}{P_r'}\right)h_3 + m_1\left\{(n_1-1)+(1-\alpha r)\frac{D}{P}\right\}h_4 + \\ \frac{m_1 \alpha r}{l_1}\left(\frac{\alpha r}{(1-\alpha r)}\right)\left\{(l_1-1)-(l_1-2)\frac{D}{P_r'}\right\}h_5 \end{bmatrix}\end{bmatrix}}}$$

….. (3)

Equation (3) proposes the corresponding optimal order quantity of case (1) per unit time.

The predictable joint total profit per unit time for case -2 written as follows

$$\text{JT}P_2 = P_m D - \left[P_s D + \frac{A_s D}{n_2 Q_2} + \frac{F_s D}{Q_2} + \frac{h_s D Q_2 m_2}{2P(1-\alpha r)} + \frac{PC_s D}{Q_2}\right] +$$
$$P_r D - \left[(1-\alpha r)\left\{(m_2-1)-(m_2-2)\frac{D}{P}\right\}\frac{Q_2 h_2}{2} + \frac{m_2}{n_2}\left\{(1-\alpha r)\frac{D}{P}\right\}\frac{Q_{2h_4}/f}{2} + \frac{(1-\alpha r)n_2 D A_4}{m_2 Q_2} + \right.$$
$$\frac{(1-\alpha r)DA_2}{m_2 Q_2} + \frac{m_2 \alpha r}{l_2}\left(\frac{\alpha r}{(1-\alpha r)}\right)\left\{(l_2-1)-(l_2-2)\frac{D}{P_r'}\right\}\frac{Q_{2h_5}}{2} + \frac{m_2}{\alpha}\left(\frac{\alpha r}{1-\alpha r}\right)\left(1-\frac{\alpha r D}{P_r'}\right)\frac{Q_2 h_3}{2} +$$
$$\left.(1-\alpha r)\frac{A_3 D}{m_2 Q_2} + \frac{F_m D}{n_2 Q_2} + \frac{PC_m D}{Q_2}\right] +$$
$$P_c D - \left[(1-\alpha r)\left(A_1 + \frac{l_2 A_5}{m_2}\right)\frac{D}{Q_2} + (1-\alpha r)\left\{1+\frac{m_2}{l_2}\left(\frac{\alpha r}{1-\alpha r}\right)^2\right\}\frac{Q_2 h_1}{2} + \frac{PC_r D}{Q_2} + \frac{F_r D}{Q_2}\right]$$

$$= P_m D - P_s D + P_r D + P_c D - \frac{A_s D}{n_{2aa} Q_2} - \frac{F_r D}{Q_2} - \frac{F_s D}{Q_2} - \frac{F_m D}{n_2 Q_2} - \frac{h_s D Q_2 m_2}{2P(1-\alpha r)} - \frac{PC_s D}{Q_2} - \frac{PC_m D}{Q_2} - \frac{PC_r D}{Q_2} -$$
$$(1-\alpha r)\left(A_1 + \frac{l_2 A_5}{m_2}\right)\frac{D}{Q_2} - \frac{(1-\alpha r)DA_2}{m_2 Q_2} - (1-\alpha r)\frac{A_3 D}{m_2 Q_2} - \frac{(1-\alpha r)D n_2 A_4}{m_2 Q_2} -$$
$$(1-\alpha r)\left\{1+\frac{m_2}{l_2}\left(\frac{\alpha r}{1-\alpha r}\right)^2\right\}\frac{Q_2 h_2}{2} - (1-\alpha r)\left\{(m_2-1)-(m_2-2)\frac{D}{P}\right\}\frac{Q_2 h_2}{2} - \frac{m_1}{\alpha}\left(\frac{\alpha r}{1-\alpha r}\right)\left(1-\right.$$
$$\left.\frac{\alpha r D}{P_r'}\right)\frac{Q_2 h_3}{2} - \frac{m_2}{n_2}\left\{(1-\alpha r)\frac{D}{P}\right\}\frac{h_4 Q_2}{2f} - \frac{m_2 \alpha r}{l_2}\left(\frac{\alpha r}{(1-\alpha r)}\right)\left\{(l_2-1) - (l_2-2)\frac{D}{P_r'}\right\}\frac{Q_{2h_5}}{2}$$

$$\text{JTP}_2 = (P_m - P_s + P_r + P_c)D - \left[\frac{A_s}{n_2} + F_r + F_s + PC_s + PC_m + PC_r + \frac{F_m}{n_2}\right]\frac{D}{Q_2} - \frac{h_s D Q_2 m_2}{2P(1-\alpha r)} -$$
$$(1-\alpha r)\left[A_1 + \frac{A_2}{m_2} + \frac{A_3}{m_2} + \frac{n_2 A_4}{m_2} + \frac{l_2 A_5}{m_2}\right]\frac{D}{Q_2}$$

$$-\frac{Q_2}{2}\begin{bmatrix} (1-\alpha r)\left\{1+\frac{m_2}{l_2}\left(\frac{\alpha r}{1-\alpha r}\right)^2\right\}h_1 + \\ (1-\alpha r)\left\{(m_2-1)-(m_2-2)\frac{D}{P}\right\}h_2 + \\ \frac{m_2}{\alpha}\left(\frac{\alpha r}{1-\alpha r}\right)\left(1-\frac{\alpha r D}{P_r'}\right)h_3 + \frac{m_2}{n_2}\left\{(1-\alpha r)\frac{D}{P}\right\}\frac{h_4}{f} \\ + \frac{m_2 \alpha r}{l_2}\left(\frac{\alpha r}{(1-\alpha r)}\right)\left\{(l_2-1)-(l_2-2)\frac{D}{P_r'}\right\}h_5 \end{bmatrix} \quad \text{….. (4)}$$

Now partially differentiate equation (5) with respect to Q_2. We get,

$$\frac{\partial JTP_2(Q_2,m_2,n_2,l_2)}{\partial Q_2} = \begin{bmatrix} \frac{A_s}{n_2} + F_r + PC_s + F_s + \\ PC_r + PC_m + F_s + \frac{F_m}{n_2} \end{bmatrix}\frac{D}{Q_2^2} - \frac{h_s D m_2}{2P(1-\alpha r)} + (1-\alpha r)\left[A_1 + \frac{A_2}{m_2} + \frac{A_3}{m_2} + \frac{n_2 A_4}{m_2} + \frac{l_2 A_5}{m_2}\right]\frac{D}{Q_2^2}$$

$$-\frac{1}{2}\begin{bmatrix} (1-\alpha r)\left\{1+\frac{m_2}{l_2}\left(\frac{\alpha r}{1-\alpha r}\right)^2\right\}h_1 + \\ (1-\alpha r)\left\{(m_2-1)-(m_2-2)\frac{D}{P}\right\}h_2 + \\ \frac{m_2}{\alpha}\left(\frac{\alpha r}{1-\alpha r}\right)\left(1-\frac{\alpha r D}{P_r'}\right)h_3 + \frac{m_2}{n_2}\left\{(1-\alpha r)\frac{D}{P}\right\}\frac{h_4}{f} + \\ \frac{m_2 \alpha r}{l_2}\left(\frac{\alpha r}{(1-\alpha r)}\right)\left\{(l_2-1)-(l_2-2)\frac{D}{P_r'}\right\}h_5 \end{bmatrix} \quad \text{…. (5)}$$

$$Q_2 = \sqrt{\frac{2PD(1-\alpha r)\begin{bmatrix} \frac{A_s}{n_2} + F_r + PC_r + PC_s + PC_m + \\ F_s + \frac{F_m}{n_2} + (1-\alpha r)\left[A_1 + \frac{A_2}{m_2} + \frac{A_3}{m_2} + \frac{n_2 A_4}{m_2} + \frac{l_2 A_5}{m_2}\right] \end{bmatrix}}{\begin{bmatrix} h_s D m_2 + \\ P(1-\alpha r)\begin{bmatrix} (1-\alpha r)\left\{1+\frac{m_2}{l_2}\left(\frac{\alpha r}{1-\alpha r}\right)^2\right\}h_1 + \\ (1-\alpha r)\left\{(m_2-1)-(m_2-2)\frac{D}{P}\right\}h_2 + \\ \frac{m_2}{\alpha}\left(\frac{\alpha r}{1-\alpha r}\right)\left(1-\frac{\alpha r D}{P_r'}\right)h_3 + \\ \frac{m_2}{n_2}\left\{(1-\alpha r)\frac{D}{P}\right\}\frac{h_4}{f} \\ + \frac{m_2 \alpha r}{l_2}\left(\frac{\alpha r}{(1-\alpha r)}\right)\left\{(l_2-1)-(l_2-2)\frac{D}{P_r'}\right\}h_5 \end{bmatrix} \end{bmatrix}}} \quad \text{…. (6)}$$

Equation (6) proposes the corresponding optimal order quantity per unit time for case (2)\

4. NUMERICAL EXAMPLE

D = 1000, P = 9400, P_s = 20, A_s= 50, h_s= 2,F_s = 50,A_1=100, A_2 =400, A_3 =200, A_4 = 250, A_5 =75,F_m = 50,P_m = 35, h_1=40, h_2 =20, h_3 =8, h_4 =12, h_5 =15, P_r'=12000, P_r = 50, F_r = 65, P_c =70, m_1=12.43, l_1=3.726, n_1=1,m_2=15.13, l_2=4.53, n_2=4.83, α =0.9, f =0.8, r = 0.25 PC_r = 25, PC_m=15, PC_s= 8

SOLUTION:

Equation (3) and (6) gives the optimal order quantity. Now, substitute the above values in this equations we get Q_1=57, Q_2= 50 for case 1 and case 2 respectively.

The joint total profit with respect to case 1 and case 2 are **1, 20,570** and **1, 21,224** respectively.

5. CONCLUSION:

Here the mathematical model has been developed for the maximization of the total profit of the three echelon closed loop supply chain consisting of the supplier,manufacturer,remanufacturerandtheretailerwhenthedemandisconsider as constant, at the same time returned rate from the retailer is also consider as constant. The numerical example provide the better considerate to the suggested

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