AN ALGORITHM FOR SOLVING HESITANT FUZZY QUADRATIC FRACTIONAL TRANSPORTATION PROBLEM

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Abstract :
Hesitant fuzzy set plays a vital role in group decision making problems when there are several possible memberships for an element to a set. This paper presents the algorithm to solve the hesitant fuzzy quadratic fractional transportation problem. The coefficients of the quadratic fractional transportation problem are considered as hesitant fuzzy elements. The proposed algorithm determines the optimal solution to the hesitant fuzzy quadratic fractional transportation problem. The numerical problem is solved to show the efficiency of the proposed approach.
Keywords: Transportation problem , Nonlinear programming , Fractional programming , Hesitant fuzzy set

1 Introduction :

The transportation problem is one of the special types of linear programming problem where the objective is to be minimized the cost of transportation from a number of sources to a number of destinations. Each source has a limited supply while each destination has a demand to be satisfied.

In real life situations, there may be a situation in which objective function of the transportation problem can be quadratic function. This type of the transportation problem may be defined as quadratic transportation problem. Kaltinska (1995) have studied the features of quadratic transportation problems Arora (2012) have solved the indefinite quadratic transportation. Several authors have developed the algorithms to solve quadratic transportation problem.

The traditional transportation problem have become fractional transportation problem when a decision-maker deals with optimization model with fractional objective function such as profit/cost (financial and corporate planning) and inventory/sales (production planning marketing) . Aim of the fractional transportation problem is to minimize the ratio of the objective function. Several algorithms have been established by different authors, Charnes and Cooper(1962), Birtan(1973) ,Cravan(1988), Schaible (1995). Changkong(1983),Borza(2012), Chakraborty(2002) ,Abouzar (2018) solved the fractional transportation problem with multi objectives. Dangwal (2012) have developed the algorithm for MOFTP by using Taylor series. Many researchers, Nidhi (2020) have developed the algorithm for quadratic fractional transportation problem.

The concept of fuzzy has been introduced by Zadeh (1965). The classical fuzzy set determines a single membership grade for an element, whereas hesitant fuzzy set deals the possible membership grades for single element. Hesitant fuzzy sets are very useful to deal with group decision making problems when experts have a hesitation among several possible memberships for an element to a set. During the evaluating process in practice, however, these possible memberships may be not only crisp values in [0, 1], but also interval values. In this study, we extend hesitant fuzzy sets by intuitionistic fuzzy sets and refer to them as generalized hesitant fuzzy sets. Zadeh's fuzzy sets, intuitionistic fuzzy sets and hesitant fuzzy sets are special cases of the new fuzzy sets. We redefine some basic operations of generalized hesitant fuzzy sets, which are consistent with those of hesitant fuzzy sets. Torra and Narukawa have introduced the concept of a Hesitant Fuzzy Set (HFS) in 2009. A proper definition of a HFS was given by Torra (2010) . In this paper, a new score function is proposed for a hesitant fuzzy elements.

This paper presents the algorithm to solve the hesitant fuzzy quadratic fractional transportation problem . The coefficients of the quadratic fractional transportation problem are considered as hesitant fuzzy elements. The proposed algorithm determines the optimal solution to the hesitant fuzzy quadratic fractional transportation problem. The numerical problem is solved to show the efficiency of the proposed approach.

2. Preliminaries :

2.1 Hesitant fuzzy set :

A hesitant fuzzy set H on Y is defined in terms of a functions h(y) that returns a subset of values in the interval [0,1] once it is applied on Y i.e an element of power set of Y

$$h : Y \rightarrow \rho([0,1])$$

Mathematically it can be stated that $H = \{ (y_i , h(y_i)) : y_i \in Y \}$ where $h(y_i)$ is a set of several values in [0,1].In general each member of $h(y_i)$ is called a hesitant fuzzy element denoted by h_i .

2.2 Example :

Consider Y = { a,b,c } . Define a hesitant fuzzy set H on Y as

H = { (a , 0.5 , 0.7, 0.78), (b, 0.8, 0.9), (c, 0.15, 0.24, 0.3,0.4)}

2.3 Score function of hesitant fuzzy sets:

Let DH= { $(y_i, h(y_i),)$: $y_i \in Y$ } be a hesitant fuzzy set where $\{y_1, y_2, \dots \dots y_n\}$ be hesitant fuzzy element. The score function s_d of dual hesitant fuzzy set is defined as

$$s_d = y_i - \frac{1}{k}\sum_{i=1}^{k} g_d(y_i)$$

Let d_1 *and* d_2 be any two hesitant fuzzy sets. Then using the score function the order relations are defined as follows :

1. If $s_{d_1} > s_{d_2}$, then d_1 is said to be superior to d_2 and it is denoted by $d_1 > d_2$
2. If $s_{d_1} < s_{d_2}$, then d_1 is said to be inferior to d_2 and it is denoted by $d_1 < d_2$
3. If $s_{d_1} = s_{d_2}$, then d_1 is said to be equivalent to d_2 and it is denoted by $d_1 = d_2$

3 . Mathematical Formulation of Hesitant Fuzzy Quadratic Fractional Transportation Problem:

Mathematical formulation of hesitant fuzzy quadratic fractional transportation problem is defined as follows:

$$\min(\max)\, p = \frac{(\sum_{l=1}^{x}\sum_{p=1}^{y} \widetilde{s_{lp}} t_{lp})^2}{(\sum_{l=1}^{x}\sum_{p=1}^{y} \widetilde{u_{lp}} t_{lp})^2}$$

Subject to $\sum_{p=1}^{y} t_{lp} = \tilde{a}_l$

$$\sum_{l=1}^{x} t_{lp} = \widetilde{b_p}\,;$$

$$t_{lp} \geq 0 \text{ for every } l = 1\ to\ x\,, p = 1\ to\ y\ and\ k = 1\ to\ r$$

where $\widetilde{s_{lpr}}$, $\widetilde{u_{lpr}}$, $\tilde{a}_l$ and $\widetilde{b_p}$ are hesitant fuzzy elements of the r^{th} objective function.

4. Procedure to solve Hesitant Fuzzy Quadratic Fractional Transportation Problem:

This section presents the procedure for finding optimal solution of hesitant fuzzy quadratic fractional transportation problem.

Step 1: Find the score value for the given hesitant fuzzy element using the score function discussed in section 2.

Step 2: Use any method to find the initial basic feasible solution to the problem .

Step 3: If the number of occupied cells is not equal to x + y -1 then add some ∈ and proceed.

Step 4: Determine the dual variables $a_l , b_p , a_l^{'} , b_p^{'}$ from the basic cells

$$a_l + b_p = s_{lp}$$
$$a_l^{'} + b_p^{'} = u_{lp}$$

Step 5: For all non basic cells calculate $s_{lp}^{'}$ *and* $u_{lp}^{'}$

$$s_{lp}^{'} = s_{lp} - a_l - b_p$$
$$u_{lp}^{'} = u_{lp} - a_l^{'} - b_p^{'}$$

Step 6: Determine

$$\nabla_{lp} = 2A\,(B \times s_{lp}^{'} - A \times u_{lp}^{'})$$

Where A = $\sum_{l=1}^{x}\sum_{p=1}^{y} \widetilde{s_{lp}} t_{lp}$ and B = $\sum_{l=1}^{x}\sum_{p=1}^{y} \widetilde{u_{lp}} t_{lp}$

Step 7: If all $\nabla_{lp} \geq 0$, then current basic feasible solution is optimal, and stop else go to next step.

Step 8:Select most negative ∇_{lp} to make a closed loop and improve the solution and go to Step 2.

5. Numerical Example :

Consider the following quadratic fractional hesitant fuzzy transportation problem with three sources and three destinations. All the coefficients of the problem are hesitant fuzzy elements.

Table 1:

	I	II	III	Demand
1	$\frac{(9; 0.8, 0.7)}{(3 ; 0.4, 0.5)}$	$\frac{(6 ; 0.5, 0.7)}{(2 ; 0.7, 0.6)}$	$\frac{(8 ; 0.6, 0.7, 0.8)}{(1 ; 0.8, 0.6)}$	10
2	$\frac{(5 ; 0.8, 0.7)}{(2 ; 0.6, 0.7)}$	$\frac{(4 ; 0.9, 0.8)}{(1 ; 0.7, 0.6)}$	$\frac{(5 ; 0.9, 0.8)}{(4 ; 1, 0.9)}$	8
3	$\frac{(6 ; 0.5 , 0.4)}{(3 ; 0.7, 0.6)}$	$\frac{(2 ; 0.9, 0.8)}{(3 ; 0.8, 0.7)}$	$\frac{(9 ; 0.8, 0.7)}{(3 ; 0.6, 0.5)}$	6
Supply	4	15	5	

Step 1:

The score value for given problem is found as follows

Table 2:

	I	II	III	Demand
1	$\frac{(8.25)}{(2.55)}$	$\frac{(5.55)}{(1.35)}$	$\frac{(7.3)}{(0.45)}$	10

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2	$\frac{(4.25)}{(1.35)}$	$\frac{(3.15)}{(0.35)}$	$\frac{(4.15)}{(3.05)}$	8
3	$\frac{(5.55)}{(2.35)}$	$\frac{(1.15)}{(2.25)}$	$\frac{(8.25)}{(2.45)}$	6
Supply	4	15	5	

Step 2:

The initial basic feasible solution is obtained by VAM

Table 3:

	I	II	III	Demand
1	$\frac{(8.25)}{(2.55)}$ 1	$\frac{(5.55)}{(1.35)}$ 9	$\frac{(7.3)}{(0.45)}$	10
2	$\frac{(4.25)}{(1.35)}$ 3	$\frac{(3.15)}{(0.35)}$	$\frac{(4.15)}{(3.05)}$ 5	8
3	$\frac{(5.55)}{(2.35)}$	$\frac{(1.15)}{(2.25)}$ 6	$\frac{(8.25)}{(2.45)}$	6
Supply	4	15	5	

Step 4:

Computing the dual variables

Table 4:

	I	II	III	Demand	
1	$\frac{(8.25)}{(2.55)}$ 1	$\frac{(5.55)}{(1.35)}$ 9	$\frac{(7.3)}{(0.45)}$	10	$a_1 = 0$ $a_1' = 0$
2	$\frac{(4.25)}{(1.35)}$ 3	$\frac{(3.15)}{(0.35)}$	$\frac{(4.15)}{(3.05)}$ 5	8	$a_2 = -4$ $a_2' = -1.2$

3	$\frac{(5.55)}{(2.35)}$	$\frac{(1.15)}{(2.25)}$ 6	$\frac{(8.25)}{(2.45)}$	6	$a_3 = -4.35$ $a_3^{'} = 4.25$
Supply	4	15	5		
	$b_1 = 8.25$ $b_1^{'} = 2.55$	$b_2 = 5.5$ $b_2^{'} = 1.35$	$b_3 = 8.15$ $b_3^{'} = 4.25$		

Step 5:

For all non basic cells (1,3), (2,2) (3,1) and (3,3) calculate $s_{lp}^{'}$ *and* $u_{lp}^{'}$

Step 6 :

Determination of ∇_{lp} for the cells (1,3), (2,2) (3,1) and (3,3) is shown below:

Table 5:

	I	II	III	Demand
1	$\frac{(8.25)}{(2.55)}$ **1**	$\frac{(5.55)}{(1.35)}$ **9**	$\frac{(7.3)}{(0.45)}$ $\nabla_{13}= 65858$	10
2	$\frac{(4.25)}{(1.35)}$ **3**	$\frac{(3.15)}{(0.35)}$ $\nabla_{22}= 11566.77$	$\frac{(4.15)}{(3.05)}$ **5**	8
3	$\frac{(5.55)}{(2.35)}$ $\nabla_{31}= 36843.86$	$\frac{(1.15)}{(2.25)}$ **6**	$\frac{(8.25)}{(2.45)}$ $\nabla_{33}= 94086.22$	6
Supply	4	15	5	

Step 7:

Here all $\nabla_{lp} \geq 0$, hence obtained solution is optimum.

The optimized objective value is $\frac{98.6}{47.5} = 2.08$. Hence the optimal solution of the given problem is given in following table:

Table 6:

	I	II	III	Demand
1	$\frac{(9; 0.8, 0.7)}{(3; 0.4, 0.5)}$	$\frac{(6; 0.5, 0.7)}{(2; 0.7, 0.6)}$	$\frac{(8; 0.6, 0.7, 0.8)}{(1; 0.8, 0.6)}$	10

	1	**9**		
2	$\frac{(5\,;0.8,0.7)}{(2\,;0.6,0.7)}$ **3**	$\frac{(4\,;0.9,0.8)}{(1\,;0.7,0.6)}$	$\frac{(5\,;0.9,0.8)}{(4\,;1,0.9)}$ **5**	8
3	$\frac{(6\,;0.5\,,0.4)}{(3\,;0.7,0.6)}$	$\frac{(2\,;0.9,0.8)}{(3\,;0.8,0.7)}$ **6**	$\frac{(9\,;0.8,0.7)}{(3\,;0.6,0.5)}$	6
Supply	4	15	5	

6: Conclusion

In this paper the mathematical model of hesitant fuzzy quadratic fractional transportation problem is presented. A new score function is defined to find the score value of the hesitant fuzzy element. The proposed algorithm is more efficient to obtain optimal feasible solution of hesitant fuzzy quadratic fractional transportation problem. The numerical problem is solved to show the efficiency of the proposed approach

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