Bianchi Universe-I Anisotropic Dark Energy Cosmological Models

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Abstract:Required solutions of the field equations of Einstein are achieved by supposing that Λ is proportionate to m^{th} power of R, here m is fixed and R is scale factor. The deceleration parameter in the model is found to be time dependent. Lambda, the cosmological term approaches to be zero when t is approaching infinite. For this objective, we also apply an association between metric potential. Dark energy cosmological models have become very important in modern era which is smallest interrelating has dynamical energy and anisotropic equation of state parameter density to describe the accelerated growth of the cosmos. Certain significant structures of the models, hence attained, have been conversed.

Keywords: ***Cosmological model, Field Equations, Anisotropic dark energy***

I. INTRODUCTION

The Einstein's field equations are system of simultaneous nonlinear differential equations. We discover corporeal results to the field equations in the applications of cosmology and astrophysics. In order to solve the field equations, we normally propose a form of the matter content or propose that property of space requires that the metric of corresponding space must have Killing Kramer and Schmutzer [1].

The Invention of cosmic acceleration was the most important finding in modern cosmology. The basis of cosmic acceleration is still unknown. As per General Relativity, the slowing rate of expansion should be led by gravity, if the Universe consists of ordinary matter or radiation, then this expansion of powerful explosion is known as "Big Bang". But, if we observe the chronology of the Universe, after the beginning of cosmic inflation, the Universe still expands in an accelerating rate. After that, Researchers originated three explanations, which were "cosmological constant", "Dark Energy" and "Dark Matter." A long-discarded version of Einstein's theory of gravity was "Cosmological constant." "Dark Energy" was tied to Einstein's cosmological constant, explaining the unrealistic expansion of the Universe, nevertheless the correct explanation was not known to theorists even then they gave the solution a name, the dark energy. Gravity, being the weakest fundamental force of the Universe, couldn't have alone played the role. Therefore, a hypothetical form of matter, known as "Dark Matter" was introduced. Hence, the origin of dark forces from the present epoch of the Big Bang was known.

Pavon,D.[2] assumed the scale factor of principle of divergence generates the solution to field equations. The matter of several investigation for the solution of field equations of Einstein is specified by cosmological scale variable R (t). Pavon, D. [2], Holye, F. et al [3], Olson, T.S. et al. [4], Maia, M.D. et al. [5], Silveria, V. et al. [6, 7], Torres, L.F.B. et al. [8], Chen and Wu [9] investigated cosmological models with cosmological term that are equivalent to scale factor. According to the prior history, they examined lambda as varying $(-2)^{th}$ power of R which is denoted as the scale factor. By deriving the value of lambda, Carvalho, J. C. et al. [10] considers the value of $\Lambda = \alpha R^{-2} + \beta H^2$, in the metric of Robertson

–Walker, R is indicated as the scale factor, H is signified as the Hubble parameter and amendable dimensionless parameters are signified as α, β.
In the structure of general relativity, Dirac [11] offered the proposal of variable gravitational constant G and also Lau [12] suggested the variation connecting the variation of G with Λ. By applying this method, number of authors A.-M. M. Abdel-Rahman [13], Berman, M. S. et al. [14], Sister´o, R. F [15], Kalligas, D. et al. [16], Vishwakarma, R. G. et al. [17-19], Pradhan, A., & Otarod, S [20], Singh, C. P., Kumar, S. et al. [21], Singh, J. P. et al. [22] explored Bianchies models. The latest papers of Borges, H. A., and Carneiro, S [23] shows Bianchi form-I model with changing gravitational constant and lambda, as an isotropic and uniform smooth universe occupied with substance and a cosmological term that is equivalent to Hubble parameter H.

In this paper we investigate homogeneous Bianchi type –I space time with variables G and Λ containing matter in the form of perfect fluid. We get solutions of the field equations supposing that cosmological term is proportional to $(-m)^{th}$ power of R, here m is fixed and R is scale factor. We have claimed about the conduct of the anisotropy of the dark energy and the geometrical characteristics of the models. It is noticed that these anisotropic and isotropic dark energy cosmological models constantly signifies an accelerated universe and are fixed with the current interpretations.
The Prime purpose of proposed work is to explored in this paper we explore, a dark energy model in the existence of general relativity for the context of symmetric Bianchi form-I cosmological models. The manuscript leads by the series of sections as mentioned: In section 2, the basic definitions of anisotropic models are mentioned. In the division 3, field equation's results are obtained in the existence of general relativity for the context of Bianchi form-I cosmological models, in section 4, the conclusion drawn from the results.

II. METRIC AND FIELD EQUATIONS

Line element equation of the Bianchi form - I space time is given by

$$ds^2 = -dt^2 + R_1^2\,(t)\,dx^2 + R_2^2\,(t)\,dy^2 + R_3^2\,(t)\,dz^2$$

(1)

here the R_1, R_2, R_3 are metric potentials, that denotes cosmic time functions t.

The cosmic matter represented by a perfect fluid of the energy momentum tensor

$$T_{ij} = (\rho + p)\, v_i v_j + p\, g_{ij}$$

(2)

"p" specifies its pressure, The four velocity vector is denoted by v_i, therefore $v_i v^i = 1$,energy density of the cosmic matter is denoted by ρ.

The equation of state is defined as $p = \omega\rho$, $0 \le \omega \le 1$

(3)

The field equations of Einstein is identified as given below with time dependent G and lambda with time dependent G and lambda , the field equations of Einstein is specified by Weinberg [24] as

$$R_{ij} - \frac{1}{2} R\, g_{ij} = -T_{ij}\,(8\pi\, G(t)) + \Lambda(t)\, g_{ij}$$

(4)

In co-moving co-ordinate system, by using equation of (1) and equation (2), the field equation of (4) gives as

$$\frac{\ddot{R_2}}{R_2} + \frac{\ddot{R_3}}{R_3} + \frac{\dot{R_2}\dot{R_3}}{R_2 R_3} = -8\pi G p + \Lambda$$

(5)

$$\frac{\ddot{R_1}}{R_1} + \frac{\ddot{R_3}}{R_3} + \frac{\dot{R_1}\dot{R_3}}{R_1 R_3} = -8\pi G p + \Lambda$$

(6)

$$\frac{\ddot{R_1}}{R_1}+\frac{\ddot{R_2}}{R_2}+\frac{\dot{R_1}\dot{R_2}}{R_1R_2} = -8\pi G p + \Lambda$$
(7)
$$\frac{\dot{R_1}\dot{R_2}}{R_1R_2} + \frac{\dot{R_2}\dot{R_3}}{R_2R_3} + \frac{\dot{R_1}\dot{R_3}}{R_1R_3} = 8\pi G\rho + \Lambda$$
(8)
" . " , denotes ordinary differentiation, with respect to the cosmic time t.
Einstein tensor's equation, with respect to disappearing of divergence is given as
$$8\pi\dot{G}\rho + 8\pi G\left[\dot{\rho} + (\rho+p)\left[\frac{\dot{R_1}}{R_1}+\frac{\dot{R_2}}{R_2}+\frac{\dot{R_3}}{R_3}\right]\right] = -\dot{\Lambda}$$
(9)
The equation of energy conservation gives $(T^{j}_{i,j} = 0)$
$$\dot{\rho} + (\rho+p)\left[\frac{\dot{R_1}}{R_1}+\frac{\dot{R_2}}{R_2}+\frac{\dot{R_3}}{R_3}\right] = 0$$
(10)
By substituting the value of equation (10) in the equation (9), we get G and Λ coupled field specified by $\dot{\Lambda} = -8\pi\dot{G}\rho$
(11)
The equation (11) representing as lambda (Λ) is a constant when G is a constant.By substituting the value of equation (3) in (10) equation, also by integrating,
$$\rho = \frac{k}{R^{3(\omega+1)}}$$
(12)
here k signifies as the constant of integration that is $k > 0$
Volume of this model is specified by
$$V(t) = R^3 = [R_1R_2R_3]$$
(13)
We define Average scale factor R is $[R_1R_2R_3]^{1/3}$ of Bianchi type$-I$ universe.
From the equations (5), (6) and (7)
$$\frac{\dot{R_1}}{R_1}-\frac{\dot{R_2}}{R_2}=\frac{k_1}{R^3}$$
(14)
$$\frac{\dot{R_2}}{R_2}-\frac{\dot{R_3}}{R_3}=\frac{k_2}{R^3}$$
(15)
The Constant of integration is denoted by k_1. The Hubble-parameter is denoted by 'H', and deceleration parameter is denoted by q , σ is shear and θ is the volume expansion.
$$\theta = 3H = \frac{3\dot{R}}{R},$$
$$H = \frac{\dot{R}}{R} = \frac{1}{3}\frac{\dot{V}}{V} = \frac{1}{3}\left[\frac{\dot{R_1}}{R_1}+\frac{\dot{R_2}}{R_2}+\frac{\dot{R_3}}{R_3}\right]$$
$$\sigma = \frac{K}{\sqrt{3}\,R^3},$$
$$q = -1-\frac{\dot{H}}{H^2} = -\frac{-R\ddot{R}}{[\dot{R}]^2}$$
In terms of H, σ and q, the equations (5) to (7) and also equation (9) can be specified by
$$(2q-1)H^2 - \sigma^2 = 8\pi pG - \Lambda ,$$
(16)
$$3H^2 - \sigma^2 = 8\pi\rho G + \Lambda ,$$
(17)
$$\dot{\rho} + 3(\rho+p)\frac{\dot{R}}{R} = 0$$
(18)

According to Overduin, & Cooperstock, [25]

$$\rho_c = \frac{3H^2}{8\pi G} \quad \text{consider as critical density}$$
(19)

$$\rho_v = \frac{\Lambda}{8\pi G} \quad \text{specifies as vacuum density}$$
(20)

$$\Omega = \frac{\rho}{\rho_c} = \frac{8\pi G\rho}{3H^2} \quad \text{consider as density parameter}$$
(21)

By substituting these values in the equation (16), we acquire

$$\frac{3\sigma^2}{\theta^2} = 1 - \frac{(24\pi G\rho)}{H^2} - \frac{3\Lambda}{\theta^2}$$

Signifies that for $\Lambda \geq 0$

$$0 \leq \frac{\sigma^2}{\theta^2} \leq \frac{1}{3},$$

$$0 \leq \frac{8\pi G\rho}{\theta^2} \leq \frac{1}{3}$$

Therefore the existence of negative lambda gives more for anisotropy whereas positive lambda lowers the upper limit of anisotropy

From (16) and (17), we have

$$\frac{d\theta}{dt} = -12\pi pG - \frac{3}{2}\sigma^2 + \frac{3}{2}\Lambda - \frac{\theta^2}{2}$$
$$= -12\pi G(\rho + p) - 3\sigma^2$$

That indicates, during the evolution of time, the rate of volume extension reduces and therefore the existence of positive lambda implies that the universe is slowing down its rate of decay whereas a negative lambda would support it.

$$\dot{\sigma} = -3\sigma\frac{\dot{R}}{R}$$

It indicates that in the growing universe, the value of σ reduces and for infinitely value of R. the value of σ is insignificant.

III. SOLUTION OF THE FIELD EQUATIONS

Five equations in six unknowns (R_1, R_2, ρ, p, G and Λ) are provided by the system of equations (3), (5) – (7) and (10). For completely solving the system, an extra equation is required.

$$\Lambda = \frac{a}{R^m}, \text{ signifies a decaying vacuum energy density.} \quad ....$$
(22)

By substituting the equations (11) and (20) in the equation (10), then

$$G = \left[\frac{R^{3\omega+3-m}}{3\omega+3-m}\right]\left[\frac{a\,m}{8\pi k}\right] \quad ....$$
(23)

By using the equations (14), (15), (20) and (21), we acquire

$$\frac{\ddot{R}}{R} + 2\left[\frac{\dot{R}}{R}\right]^2 - \frac{am(1-w)}{2(3\omega+3-m)R^m} - \frac{a}{R^m} = 0 \quad ....$$
(24)

Take the value $\omega = 0$

To get cosmological models for dense matter, this relates to the equation

The value of the equation (24) turn into

$$\frac{\ddot{R}}{R} + 2\left[\frac{\dot{R}}{R}\right]^2 - \frac{a}{R^m}\left[\frac{(m-6)}{2(m-3)}\right] = 0$$
(25)

To acquire the time progression of Hubble parameter, integrate the equation (23)

$$\frac{\dot{R}}{R} = H = \sqrt{\frac{a}{(3-m)}}\left[\frac{m}{2}\sqrt{\left(\frac{a}{(3-m)}\right)t} + t_0\right]^{-1}$$

(26)

here t_0 is a constant,

To acquire the value of scale factor, using the equation (26)

$$R = \left(\frac{m}{2}\sqrt{\left(\frac{a}{(3-m)}\right)t} + t_0\right)^{2/m}$$

(27)

By substituting the equation (25) into the equation (13), the equation of metric (1) exist as

$$ds^2 = -dt^2 + \left(\frac{m}{2}\sqrt{\left(\frac{a}{(3-m)}\right)t} + t_0\right)^{\frac{4}{m}} \times$$

$$\left[m_1^2 \exp 2\left\{\frac{(2k_1+k_2)}{3}\left[2\sqrt{\frac{(3-m)}{a}}\right]\frac{1}{m-6}\left(\frac{m}{2}\sqrt{\left(\frac{a}{(3-m)}\right)t} + t_0\right)^{\frac{m-6}{m}}\right\}dx^2 + \right.$$

$$m_2^2 \exp 2\left\{\frac{(k_2-k_1)}{3}\left[2\sqrt{\frac{(3-m)}{a}}\right]\frac{1}{m-6}\left(\frac{m}{2}\sqrt{\left(\frac{a}{(3-m)}\right)t} + t_0\right)^{\frac{m-6}{m}}\right\}dy^2 +$$

$$m_3^2 \exp 2\left\{\frac{-(k_1+2k_2)}{3}\left[2\sqrt{\frac{(3-m)}{a}}\right]\frac{1}{m-6}\left(\frac{m}{2}\sqrt{\left(\frac{a}{(3-m)}\right)t} + \right.\right.$$

$$\left.\left.t_0\right)^{\frac{m-6}{m}}\right\}dz^2\Bigg] \qquad (28)$$

Here m_1 and m_2 signifies as constants.

From the above model (28), ρ is denoted as matter density, p is denoted as pressure and G signifies as gravitational constant , Λ is denoted as cosmological constant and the spatial volume is denoted as V are specified as

$$V = \left(\frac{m}{2}\sqrt{\left(\frac{a}{(3-m)}\right)t} + t_0\right)^{6/m} ,$$

(29)

$$\rho = \frac{k}{\left[\frac{m}{2}\sqrt{\left(\frac{a}{((3-m)}\right)t} + t_0\right]^{\frac{6}{m}}}$$

(29)'

p = 0

(30)

$$G = \frac{am}{8\pi k\,(3-m)}\left[\frac{m}{2}\sqrt{\left(\frac{a}{(3-m)}\right)t} + t_o\right]^{\frac{2}{m}(3-m)}$$

(31)

$$\Lambda = a\left[\frac{m}{2}\sqrt{\left(\frac{a}{(3-m)}\right)t} + t_o\right]^{-2}$$

(32)

θ is denoted as expansion scalar, σ is signifies by shear

$$\theta = 3\sqrt{\frac{a}{(3-m)}}\left[\frac{m}{2}\sqrt{\left(\frac{a}{(3-m)}\right)t} + t_0\right]^{-1}$$

(33)

$$\sigma = \frac{k}{\sqrt{3}}\left[\frac{m}{2}\sqrt{\left(\frac{a}{(3-m)}\right)t} + t_0\right]^{-6/m}$$

(34)

$\Omega = \frac{\rho}{\rho_c} = \frac{m}{3}$ is the density parameter

(35)

For acquired model, value of deceleration parameter q specifies as

$$q = \frac{m}{2} - 1$$

(36)

ρ_v is denoted as the vacuum energy , ρ_c is denoted as density critical density are

$$\rho_v = \frac{k\,(3-m)}{m}\left[\frac{m}{2}\sqrt{\left(\frac{a}{(3-m)}\right)t} + t_0\right]^{-6/m}$$

(37)

$$\rho_c = \frac{3k}{m}\left[\frac{m}{2}\sqrt{\left(\frac{a}{(3-m)}\right)t} + t_0\right]^{-6/m}$$

(38)

From the given model (28), it has been noticed that for $0 < m < 3$, If t = t'' then value of the spatial volume $V = 0$ and

Also the value of $t' = \frac{-t_0}{\frac{m}{2}\sqrt{\frac{a}{(3-m)}}}$ and the value of θ which is denoted as expansion scalar is infinite, that shows that with zero volume, our space begins to grow at t=t'' and an infinite rate of development. As *t* increases, however expansion scalar decreases but the spatial volume increases. Thus, the expansion rate falls as time rises. When t approaches to infinity, then the spatial volume become extremely huge. All the parameters $p, \rho, \theta, \rho_v, \rho_c, \Lambda$ reduces with rising time and tend to zero asymptotically. Hence, the model essentially gives an empty universe for large *t*. The proportion $\frac{\sigma}{\theta}$ approaches to zero as t approaches to ∞ that displays that the model tends isotropy for the big values of *t*. At t = t'' the value of *G* (*t*), the gravitational constant is zero and according to Abdel-Rahman [13], Chow[26] , Levitt [27], and Milne[28], it becomes extremely huge as *t* rises, *G* rises, that is identically result in the past era.

When $t \to \infty$ then ratio $\frac{\sigma}{\theta} \to 0, m < 3$, so the model tends to isotropy for a big value of *t*. Therefore, with a big bang start approaching the isotropy at late times, the model signifies a shearing, non-rotating and expanding model of the universe. This has examined when

$$q < 0, for\ 0 < m < 2,\ q = 0, for\ m = 2, q > 0;\ for\ 2 < m < 3$$

So the universe starts with decelerating expansion changes and the expansion changes from a decelerating phase to an accelerating one. This cosmological scenario is in agreement with SNe Ia astronomical observations, this acceleration strongly suggested by Knop et al. [29], Riess et al. [30-31], Spergel et al. [32], Tegmark et al. [33], Perlmutter et al. [34] and it offers a unified depiction of the growth of the universe. Kalligas et al. [35], Berman [36], Berman & Som [37-38], and Bertolami [39-40], follows the cosmological constant $\Lambda(t) \propto 1/t^2$.

IV. CONCLUSION

In this manuscript we have acquired a dark energy model in the existence of general relativity for the context of Bianchi form-I cosmological models. In order to discover a deterministic model of the cosmos and to acquire the particular solutions of the field equations of Einstein, we suppose Hubble parameter which gives the fixed value of deceleration parameter that acquires the required results of the field equations of Einstein. This parameter in the model is observed to be time subordinate which shows the development from initial decelerating period to the present accelerating period of expansion and supplies the biggest value and the high speed at which the universe is expanding, with the interpretations of modern cosmology, this point is in contract. Model offers dynamically anisotropic expansion to the universe which permits for symmetrical Bianchi metrics, therefore the cosmic microwave background anisotropy. The dark energy is slightly interrelating is presumed has dynamical energy density and anisotropic equation of state parameter. The corporal properties of corresponding cosmological models and the behavior of the anisotropic model has been conferred. The universe starts to accelerate and increases exponentially with infinite time t. Hence the space starts through a decelerating extension deviates and the growth deviates through decelerating phase to accelerating one. In the conclusion, in Bianchi type-I space-time with variables *G* and lambda, the results obtained in this manuscript are new and beneficial for a better consideration of the evolution of the universe.

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