Prime Edge Domination number of Graphs

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Abstract: In this paper, we introduce a new concept of edge domination graph named while relatively prime edge dominating set of a graph G. Let G (V, E) be a graph. For a set $D' \subseteq E(G)$ is said to be relatively prime edge dominating set, if it is an edge dominating set with some two elements and for each pair of edge (e, f) $\in D'(G)$ such that (deg e, deg f) = 1 (where degree of edge d(e) = deg u + deg v-2, e(=uv) in E(G)) . The minimum cardinality of a relatively prime edge dominating set (rped-set) is called relatively prime domination number and it is denoted by γ'_{rped}(G). In this paper, we compute about an edge dominating graph of relatively prime.

Keywords: Edge Dominating set, relatively prime dominating set

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1. Introduction

Let G = (V, E) be a graph, we represent a finite undirected graph. The order and size of G are denoted by p and q respectively. For graph notional terms, we refer to Harary [5] and for conditions related to domination we refer to T.W.Haynes [1,2 ,6].

The line graph of a graph G, denoted by $L(G)$, is a graph whose vertices are the edges of G and two vertices of $L(G)$ are adjacent anywhere the corresponding edges of G are incident to a common vertex; see [4 ,11]. (V(L(G)=q=E(G)).

Mitchell and Hedetniemi [10] introduced the concept of edge domination. A X ⊆E(G) is called an edge dominating set of G if for each edge not in X is adjacent to some edge in X. The edge domination number γ(G) is the minimum cardinality taken over all edge dominating sets of G. An edge dominating set X is called an independent edge dominating set if no two edges in X are adjacent. The independent edge domination number (G) of G is the minimum cardinality taken over all independent edge dominating sets of G. The edge independence number (G) is defined to be the number of edges in a maximum independent set of edges. Derived many results related to edge domination.

A subset S of V is said to be a dominating set in G if every vertex in V- S is adjacent to atleast one vertex in S. The domination number γ(G) is the minimum cardinality of a dominating set in G.

Berge and Ore [1,8] formulated the concept of domination in graphs. It was further extended to define many other domination related parameters in graphs.

The distance d (u, v) between two vertices u and v in a connected graph G is the length of a shortest u-v path in G. The diameter of a connected graph G is the maximum distance between two vertices of G and is denoted by diam(G). Many other domination parameters in domination theory were studied in [3,5, 7].

The graph related to the prime elements of *G* has been discussed since 1981. Williams was the first person who introduced the prime graph of a group where the vertices are the primes dividing the order of *G* and two vertices *p* and *q* are joined by an edge if and only if *G* contains an element of order *pq*. The significance of the prime graphs of finite groups can be found in Iiyori and Yamaki (1993) and Williams (1981). [9]

In this paper we define relatively prime edge dominating set γ'_{rped}(G). We obtain the some bounds and some exact value for γ'_{rped}(G). for various classes of graphs.

2. Main Results

Definition and example

Definition 2.1

A set D′ ⊆E(G) is said to be relatively prime edge dominating set, if it is a dominating set with at least two elements and for each pair of edge (e, f) ∈ $D'(G)$ such that (deg e, deg f) = 1 . The minimum cardinality of a relatively prime edge dominating set is called relatively prime edge domination number and it is denoted by γ'_{rped}(G).

The following figure explain the above definition

Example 2.2.

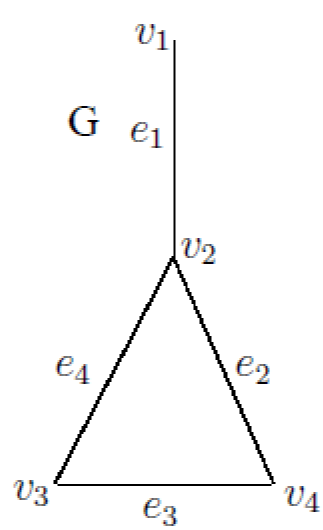

Figure: 1

Let D′ = $\{e_1, e_4\}$ is an edge dominating set and (d(e_1), d(e_4)) = (2,3) = 1, By definition, $\{e_1, e_4\}$ is a relatively prime edge dominating set and hence γ'_{rped}(G) = 2. Also γ'(G) = γ'_{rped}(G) = 2.

Theorem 2.1.

A rped-set D′ (G) is minimal for every edges e∈D′ (G) , if one of the following three conditions hold.

i) There exists an edge f ∈ E − D′ , such that N'(e) ∩ D′ = f.
ii) f is an isolated edges in < D′ >.
iii) (E − D′) ∪ f is connected.

Proof.

Suppose D′ is minimal and there exists an edges e ∈ D′, f ∈ E − D′ such that one of the above condition is true. Then by conditions (i) and (ii), D_1' = D′ − e is a rped-set of G. Further by (iii), < E − D′ > is disconnected, which implies D′ is a rpsd-set of G, which is a contradiction.

\Theorem 2.2

If H is a connected spanning sub graph of G, then

$$\gamma'_{rped}(H) \leq \gamma'_{rped}(G)$$

Proof:

Since every relatively prime edge dominating set of G is a relatively prime edge dominating set of H. that is $\gamma'_{rped}(H) \leq \gamma'_{rped}(G)$

Observation: 2.3

For any graph $\gamma'_{rped}(P_n) = \begin{cases} 2 & \text{if } 4 \leq n \leq 6 \\ 3 & \text{if } n = 7,8 \\ 0 & \text{otherwise} \end{cases}$

Observation: 2.4

For any graph $\gamma'_{rped}(\overline{P_n}) = \begin{cases} 2 & \text{if } n \geq 4 \\ & . \\ 0 & \text{otherwise} \end{cases}$

3. BOUNDS ON γ'_{rped}

Theorem: 3.1

Let G be a graph.Then

$$\frac{q}{\Delta'(G)+\ 1} \leq \gamma'_{rped}\ (L(G) \leq q\text{-}\delta_e'(G).$$

Proof:

Let e ∈ E(G) with deg (e) = $\delta_e'(G)$. Now without loss of generality by definition .of line graph e=uv in V(L(G)) and Let R be a relatively prime edge dominating set of L(G). Such that $|R| = \gamma'_{rped}$ (L(G). If $\delta_e'(G) \leq 2$. Then

$$\gamma'_{rped}\ (L(G) \leq q\text{-}2,$$

$$\gamma'_{rped}\ (L(G) \leq q - \delta_e'(G). \quad \text{----------------------(A)}$$

If $\delta_e'(G) > 2$, Then for some edge f in N'(e) and by definition of L(G), f = w ϵ N'(u), that is

$$R \subseteq (V(L(G)) - N'(u)) \cup \{w\}.$$

Hence

$$\gamma'_{rped}(L(G) \leq q - (\delta_e'(G)+1)+1$$
$$\leq q - \delta_e'(G).$$

Now for the lower bound $|V(L(G)\text{-} D'| \leq \sum \deg V_i$ and the fact that for some edge e ϵ E(G) and deg(e) $\leq \Delta'(e)$. We have

$$q - \gamma'_{rped}(L(G) = |V(L(G)\text{-} D'|$$

$$q - \gamma'_{rped}(L(G) \leq \sum \deg V_i$$

$$\leq \gamma'_{rped}\ (L(G)\ (\Delta'(e))$$

$$q \leq \gamma'_{rped}(L(G)\ (\Delta'(e)) + \gamma'_{rped}\ (L(G)$$

$$\leq \gamma'_{rped}(L(G)(\ (\Delta'(e)+1)$$

$$\frac{q}{\Delta'(G)+\ 1} \leq \gamma'_{rped}(L(G) \quad \text{---------(B)}$$

Hence $$\frac{q}{\Delta'(G)+\ 1} \leq \gamma'_{rped}\ (L(G) \leq q\text{-}\delta_e'(G)$$

Theorem :3.2

For any graph G,

$$\lambda_0(G) \leq \gamma'_{rped}\ (G)$$

where $\lambda_0(G)$ is the edge independent number of G.

Proof :

Let D′ be a γ'_{rped} (G) –set of G and T be an independent set of edges in G .Then either H ⊊ D′ or H contains at most one edge from E- D′ and at most ⎮T⎮- 1 edge from D′. This implies λ_0(G) ≤ γ'_{rped}(G).

Theorem 3.3:

If diam(G) ≤ 3, Then

γ'_{rped}(G) ≤ q –m+1, where m is the number of cut edge of G.

Proof:

If G has no cut edges, then the result is trivial. Let M be the set of all cut edges with ⎮M⎮= m. Let (e,f) ∈ M , Suppose e and f are not adjacent. Since two edges e_1 and f_1 since such that e_1 is adjacent to e and f_1 is adjacent to f. it implies d(e,f) ≥ 2 , which is a contradiction.

Hence each two edges in M are not adjacent and each edge in M is adjacent to at least one edge in (E-M)∪{e} is a relatively prime edge dominating set of G. Thus,

$$\gamma'_{rped}(G) \leq (E\text{-}M)\cup\{e\}$$

Therefore γ'_{rped}(G) ≤ q –m+1.

Theorem: 3.4

For a graph G without isolated edges□ γ'_{rped}(G) = □□(G) if and only if there exists a relatively prime edge dominating set of D′ such that E(G)- D′ is independent.

Proof:

Let E(G) – D′ be an independent edge set. Then D′ is an edge cover for G and

γ'_{rped} ≥ | D′ | . Hence □□□□≤ | D′ | =□ γ'_{rped}(G) , Now, □□ =□ γ'_{rped}(G) ,
it follows from□ γ'_{rped}(G) = □□(G).

Conversely, Let □□ =□ γ'_{rped}(G) be true and D′ be an any minimum edges cover for G. Then E(G) – D′ is independent and D′ is an edge dominating set of G. Also since □ D′ □ = □□=□ γ'_{rped} (G). Hence D′ is a relatively prime edge dominating set of G.

Conclusion

In this paper, we found results on relatively prime edge dominating sets of graphs. These results determine means relationships between the relatively prime numbers and the Edge dominating sets in graphs. We also extend the results for P_n and complement of P_n

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