Modified Zagreb Indices of Product ofGraphs

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Abstract

In this paper, we obtain the oremson modified Zagrebindex of Cartesian product, Strong product and Tensor product of graphs. Graph slike path and cycle are considered in this work.
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1 Introduction

In this article, we are concerned with simple graphs, that is finite and undirected graphs without loops or multiple edges. Let G be such a graph and V(G) and E(G) be its vertex set and edge set respectively. An edge of G, connecting the vertices u and v will be denoted by *uv*. The degree d(v) of a vertex v϶V(G) is the number of vertex of G adjacent to v. The most elementary constituents of a (molecular) graph are vertices, edges, vertex-degrees, walks and paths[7]. They are the basis of many graph-theoretical invariants referred to as topological index, which have found considerable use in Zagreb index. The Modified first Index is denoted by
${}^{m}M_1(G)$, and defined as ${}^{m}M_1(G) = \sum_{v\in V(G)} \frac{1}{d(v)^2}$.These have been conceived in the 1970s and found considerable applications in chemistry[2,5,6]. The Zagreb indices were subject to a large number of mathematical studies, of which we mention only a few nearest [3,4].

The**Cartesian Product** G□H is a graph such that, the vertex set of G□H is the Cartesian product V(G)×V(H) and two vertices (u,u′) and (v,v′) are adjacent in G□H if and only if either u=v and u′ is adjacent to v′ and u is adjacent to v in G. The Strong Product G⊠H is the Cartesian product V(G)×V(H) and distinct vertices (u,u′) and (v,v′) are adjacent in G⊠H if and only if either u=v and u′ is adjacent to v′ or u′=v′ and u is adjacent to v or u is adjacent to v or u is adjacent to v and u′ is adjacent to v′. The **Tensor Product** G×H of graphs G and H is a graph such that, the vertex set of G×H is the Cartesian product V(G)×V(H) and distinct vertices (u,u′) and (v,v′) are adjacent in G×H if and only if u is adjacent to v and u′ is adjacent to v′.
All the definitions and notations in graphs and digraphs, which are not mentioned in this paper, one may refer[1].

2MAIN RESULTS

In this section, we obtain results on modified first Zagreb indices of Cartesian, strong and Tensor product of paths and cycles.

Theorem 1:

The Modified first Zagreb index G of a Cartesian product of two path p_n and p_m

$${}^mM_1(G) = \frac{9nm + 14n + 14m + 52}{144}$$

Proof:

The Cartesian Product of two path p_n and p_m has 4 vertices of degree 2,2n-4 vertices degree 3, 2m-4 vertices degree and 3(nm-2n-2m+4) vertices of degree 4, then the Modified first Zagreb index G is ${}^mM_1(G) = \sum_{v\in V(G)} \frac{1}{d(v)^2}$.

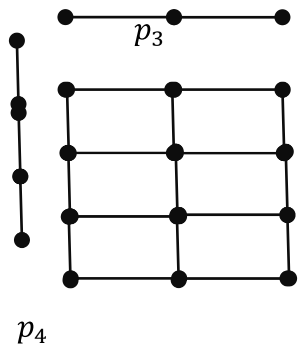
$$=\frac{4}{2^2} + \frac{2n-4}{3^2} + \frac{2m-4}{3^2} + \frac{nm-2n-2m+4}{4^2}$$

$$=\frac{4}{4} + \frac{2n-4}{9} + \frac{2m-4}{9} + \frac{nm-2n-2m+4}{16}$$

$$=\frac{144+(2n-4)(16)+(2m-4)(16)+(nm-2n-2m+4)(9)}{144}$$

$$=\frac{144+32n-64+32m-64+9nm8n-18m+36}{144}$$

$$=\frac{9nm+14n+14m+52}{144}$$



G□H

Figure 1

Theorem 2:

The Modified first Zagreb index G of a strong product of two path p_n and p_m ${}^mM_1(G) - \frac{225nm+702n+702m+2692}{14400}$

Proof:

The strong product of two path p_n and p_m has 4 vertices of degree 3,2n-4 vertices of degree 5, 2m-4 vertices of degree 5 and (nm-2n-2m+4) vertices of 8, then the Modified first Zagreb index

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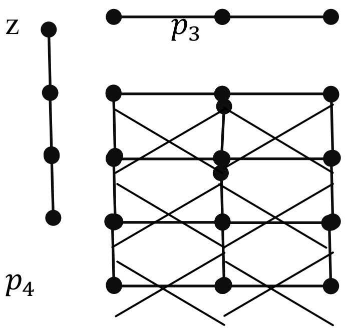
G is ${}^{m}M_1(G) = \sum_{v \in V(G)} \frac{1}{d(v)^2}$.

$= \frac{4}{3^2} + \frac{2n-4}{5^2} + \frac{2m-4}{5^2} + \frac{(nm-2n-2m+4)}{8^2}$

$= \frac{4}{9} + \frac{2n-4}{25} + \frac{2m-4}{25} + \frac{(nm-2n-2m+4)}{64}$

$= \frac{6400+(2n-4)576+(2m-4)+(nm-2n-2m+4)225}{14400}$

$= \frac{225nm+702n+702m+2692}{14400}$



G⊠H

Figure 2

Theorem 3:

The Modified first Zagreb index G of a Tensor productof two pathp_n *and* p_m *is* ${}^{m}M_1(G)$=$\frac{nm+6n+6m+36}{16}$

Proof:

The Tensor product of two path p_n and p_m has 4 vertices of degree 1, 2n-4 vertices of degree 2,2m-4 vertices of degree 2 and (nm-2n-2m+4) vertices of degree 4, then the Modified first Zagreb index G is ${}^{m}M_1(G) = \sum_{v \in V(G)} \frac{1}{d(v)^2}$.

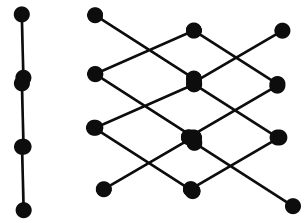
$= \frac{4}{1^2} + \frac{2n-4}{2^2} + \frac{2m-4}{2^2} + \frac{nm-2n-2m+4}{4^2}$

$= \frac{4}{1} + \frac{2n-4}{4} + \frac{2m-4}{4} + \frac{nm-2n-2m+4}{16}$

$= \frac{64+(2n-4)4+(2m-4)4+(nm-2n-2m+4)}{16}$

$= \frac{nm+6n+6m+36}{16}$

p_3



Theorem 4:

The Modified first Zagreb index G of a Cartesian product of two cycle c_n and c_m is ${}^{m}M_1(G)=\frac{nm}{16}$

Proof:

The Cartesian product of two cycle c_n and c_m has nm vertices of degree 4, then the Modified first Zagreb index G is

$$ {}^{m}M_1(G) = \sum_{v\in V(G)} \frac{1}{d(v)^2} $$

$=\frac{nm}{4^2}$

$=\frac{nm}{16}$.

Theorem 5:

The Modified first Zagreb index G of a strong product of two cycle c_n and $c_m$is ${}^{m}M_1(G)=\frac{225nm+350n+350m+900}{14400}$

Proof:

The stong product of two cycle c_n and c_m has 4 vertices of degree 5, 2n-4 vertices of degree 6, 2m-4 vertices of degree 6 and (nm-2n-2m+4) vertices of degree 8,b then the Modifies first Zagreb index G is

$$ {}^{m}M_1(G) = \sum_{v\in V(G)} \frac{1}{d(v)^2} $$

$=\frac{4}{5^2}+\frac{2n-4}{6^2}+\frac{2m-4}{6^2}+\frac{nm-2n-2m+4}{8^2}$

$=\frac{4}{25}+\frac{2n-4}{36}+\frac{2m-4}{36}+\frac{nm-2n-2m+4}{64}$

$=\frac{2304+(2n-4)400+(2m-4)400+(nm-2n-2m+4)225}{14400}$

$=\frac{2304+800n-1600+800-1600+225nm-450n-450m+1800}{14400}$

$=\frac{225nm+350n+350m+900}{14400}$

Theorem 6:

The modified first Zagreb index G of a tensor product of two cycle c_n and c_m is ${}^{m}M_1(G)=\frac{nm+6n+6m+36}{16}$

Proof:

The Tensor product of two cycle c_n and c_m has 4 vertices of degree 1, 2n-4 vertices of degree 2,2m-4 vertices of degree 2 and (nm-2n-2m+4) vertices of degree 4, then the modified first Zagreb index G is ${}^{m}M_1(G) = \sum_{v\in V(G)}\frac{1}{d(v)^2}$

$=\frac{4}{1^2}+\frac{2n-4}{2^2}+\frac{2m-4}{2^2}+\frac{nm-2n-2m+4}{4^2}$

$=\frac{4}{1}+\frac{2n-4}{4}+\frac{2m-4}{4}+\frac{nm-2n-2m+4}{16}$

$=\frac{64+(2n-4)4+(2m-4)4+(nm-2n-2m+4)4}{16}$

$=\frac{64+8n-16+8m-16+nm-2n-2m+4}{16}$

$=\frac{nm+6n+6m+36}{16}$

Theorem 7:

The Modified first Zagreb index G of a Cartesian product of path p_n and cycle $c_m$is ${}^{m}M_1(G)=\frac{9nm+14m}{144}$

Proof:

The Cartesian product of of path p_n and cycle $c_m$has 2m vertices of degree 3 and (nm-2m) vertices of degree 4, then the Modified first Zagreb index G is${}^{m}M_1(G) = \sum_{v\in V(G)}\frac{1}{d(v)^2}$

$=\frac{2m}{3^2}+\frac{nm-2m}{4^2}$

$=\frac{2m}{9}+\frac{nm-2m}{16}$

$=\frac{32m+(nm-2m)9}{144}$

$=\frac{32m+9m-18m}{144}$

$=\frac{9nm-14m}{144}$

Theorem 8:

The Modified first Zagreb index G of a strong product of path p_n and cycle $c_m$is ${}^{m}M_1(G)=\frac{225nm+350n+720m+596}{14400}$

Proof:

The Strong Product of path p_n and cycle c_m has 4 vertices of degree 4,(2n-4) vertices of degree 6, (2m-4) vertices of degree 5 and (nm-2n-2m+4) vertices of degree 8, then the Modified first Zagreb index G is

$^{m}M_1(G) = \sum_{v\in V(G)} \frac{1}{d(v)^2}$

$= \frac{4}{2^2} + \frac{2n-4}{6^2} + \frac{2m-4}{5^2} + \frac{(nm-2n-2m+4)}{8^2}$

$= \frac{4}{16} + \frac{2n-4}{36} + \frac{2m-4}{25} + \frac{(nm-2n-2m+4)}{64}$

$= \frac{4(900)+(2n-4)400+(2m-4)576+(nm-2n-2m+4)225}{14400}$

$= \frac{3600+800n-1600+1152m-2304+225nm-450n+450m+900}{14400}$

$= \frac{225nm+350n+702m+596}{14400}$

Theorem 9:
The Modified first Zagreb index G of a Tensor product of path p_n *and cycle* c_m *is* $^{m}M_1(G) = \frac{nm+6n+6m+36}{16}$

Proof:

The Tensor product of path p_n and cycle c_m has 4 vertices of degree 1,2n-4 vertices of degree 2,2m-4 vertices of degree 2 and (nm-2n-2m+4) vertices of degree 4, then the Modified first Zagreb index G is

$^{m}M_1(G) = \sum_{v\in V(G)} \frac{1}{d(v)^2}$

$= \frac{4}{1^2} + \frac{2n-4}{2^2} + \frac{2m-4}{2^2} + \frac{(nm-2n-2m+4)}{4^2}$

$= \frac{4}{1} + \frac{2n-4}{4} + \frac{2m-4}{4} + \frac{(nm-2n-2m+4)}{16}$

$= \frac{64+(2n-4)4+(2m-4)4+(nm-2n-2m+4)}{16}$

$= \frac{64+8n-16+8m-16+nm-2n-2m+4}{16}$

$= \frac{nm+6n+6m+36}{16}$

3 CONCLUSION

In this paper,modified first Zagreb index of product of graphs are obtained. This index can be used as a numerical description in comparision with chemical,physical and biological parameters to study about its relationships.

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