BERNOULLI SERVICE SCHEDULES AND MULTIPLE ADAPTIVE VACATIONS FOR BULK ARRIVAL AND GENERAL SERVICE QUEUE WITH CONTROL POLICY ON DEMAND FOR RE-SERVICE

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ABSTRACT

The Bernoulli service schedule and re-service policy in queueing theory has been always a new and hot research field. ABulk queueing system with Poisson arrival rate λ and the general service random variable is considered here. In this paper, the server implements the notion of a binomial vacation schedule. In this policy, if there is no customer at a service completion epoch then the server takes multiple vacations. The number of vacations follows a binomial distribution with parameter$p$, otherwise the server gives service with probability $1-p$if any new customers arrive in the queue. At the completion of an essential service, the batch of customers may request for a re-service with probabilityπ. However, the re-service is rendered only when the number of customer waiting in the queue is less than one. If no request for re-service after an essential service and no customers in the queue, then the server avail a vacation of a random length with probability $(1-\pi)$. When the server returns from vacation and if there is no customer in the queue then he avails another vacation and so on until the server finds single customers in the queue. After completing of an essential service and the number of customers in the queue becomes one then the server will continue the service with general service rule.

Keyword: Bulk queue, Bernoulli schedule, Regular service, Re-service, Multiple adaptive vacations.

INTRODUCTION

Researchers in Queueing system, network transportation, computer science and signal system have taken keen interest in the problem of congestion in signal transmission, telecommunication, production, quality control in recent years. In operating system where server uses their stipulated idle period to utilize some additional job are called multiple adaptive vacations.

Queueing systems with general bulk service and vacations have been studied extensively by many authors. Doshi [6] have made comprehensive survey of queueing systems with vacations. This includes the studies of Medhi [15] on bulk queueing models. Jeyakumar et al. [7] has developed a procedure to find the system size probabilities for a server vacation bulk arrival queueing model. Jeyakumar et al. [8] analyzed a batch arrival queue with N-policy with multiple adaptive vacations. Yi-jun Zhu, Bin Zhuang [18] have analyzed on a batch arrival queue with different arrival rates and N- policy

In the literature of bulk queueing systems, it is observed that some contributions are made to a queueing system in which both arrival and service are done in bulk. A single server non markovian bulk queue with multiple vacations and closedown times, request for re-servicewas analyzed by Arumuganathan and Jeyakumar [1,2]. They derived the PGF of queue size at an arbitrary time epoch and obtained some important performance measures too. Control policy of a hysteretic bulk queueing system was discussed by Tadj and Ke [12]. Tadj et al. [13] analyzed a hysteretic bulk quorum queue with choice of service and optional re-service by embedded markov chain and semi-generative techniques. Rameshkumar et al. have analysed the model and cost analysis of a bulk queue with heterogeneous service, multiple adaptive vacations, setup and closedown times,

Jeyakumar et al. [9,10] considered an M/G/1 queueing model, in which the server performs first essential service to all arriving customers. As soon as the first service is over, they may leave the system with the probability (1-p) and second optional services provided with probability p and generalized the model by incorporating server vacations. Choudhry [3] generalized the result of Madan [11]. Choudhry and Paul [4] analyzed a batch arrival and single service queue with an additional service channel under N-policy. Choudhry has done considerable amount of work in two phase batch arrival queueing system see Choudhry and Paul [4], Choudhry and Madan [3,5]. Madan et al. [11] consider a bulk arrival queue with optional re-service. In their system, before a service starts customer has the option to choose either type of service, after completion of which the customer may leave the service or may opt for re-service of the service taken by him. Wang [14] studied an M/G/1 queue with second optional service and server breakdowns by considering reliability factor and obtained various performance measures. Recently, Y.H.Tang, X.W. Tang [17] analyzed a bulk arrival queue length distribution with single vacation. They derived a search procedure to find optimal threshold value that will minimized the total expected cost function and an extensive numerical study was done. Yinghui Tang and Yong Mao developed the model for M/G/1 queue with 'p' entering discipline during server vacation without numerical result [19].
Lotfi Tadj, Paul Yoon. K, found out various numerical observation and sensitivity analysis of Binomial Schedule for an M/G/1 type queueing system with an unreliable server under N- policy [20].

The objective of this paper is to study on $M^x/G/1$ type queueing system where the server implements the concept of a binomial vacation schedule and re-service options for the request of customers who not satisfy the actual service. Under this policy, at a service completion and before serving the next customer, the server takes multiple adaptive vacations, if the customers not required the optional re-service and no customers in the essential service.

The paper is organized as follows. Mathematical model is discussed in section 2. System size distribution at an arbitrary time epoch is derived in section3. Some important performance measures are obtained in section 4. The cost model is discussed in section 5. Finally, the numerical aspects of the model and conclusions are presented respectively in sections 6 and 7.

MATHEMATICAL MODEL

In this paper, the server implements the notion of a binomial vacation schedule. Under this policy, there is no customer at a service completion epoch the server takes multiple vacations. The Number of vacations follows a binomial distribution with parameter p, otherwise the server gives service with probability $1-p$ if any new customers arrive in the queue. At the completion of an essential service, the batch of customers may request for a re-service with probability π. However, the re-service is rendered only when the number of customer waiting in the queue is less than one. If no request for re-service after an essential service and no customers in the queue, then the server avail a vacation of a random length with probability $(1-\pi)$.

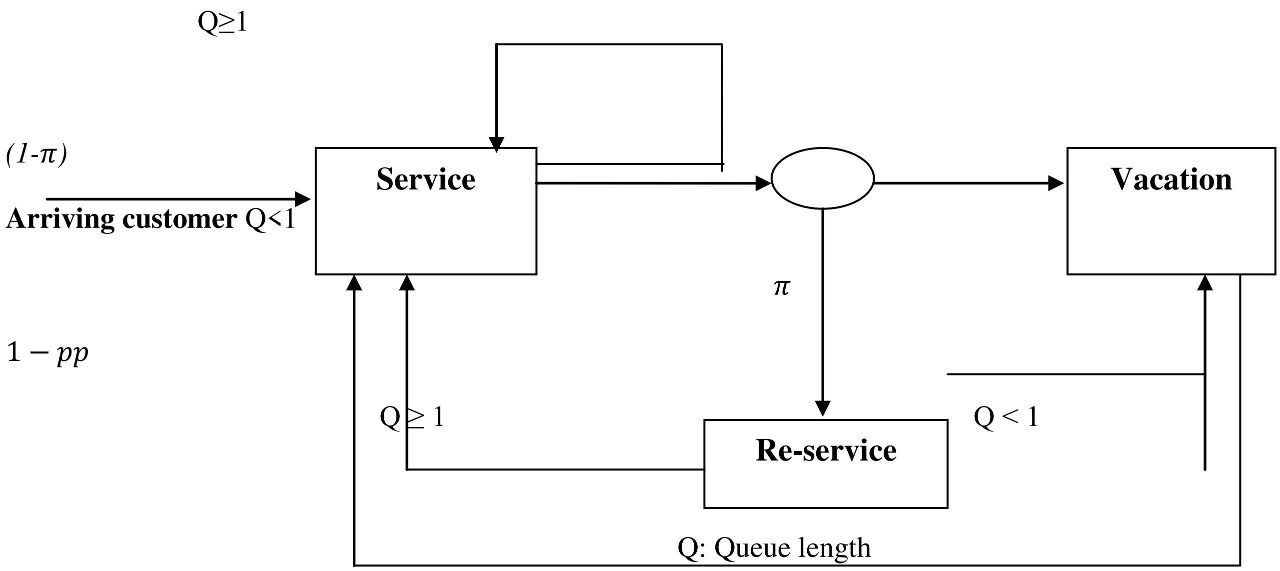

Figure 1: The semantic representation of the model.

NOTATIONS

The following notations are used in this paper.

λ - Arrival rate

X - Group size random variable.

P(X=k)- g_k

X (z) - Probability generating function of X.

S(.), V(.), and G(.) represent the cumulative distribution function (cdf) of services time, vacation time, and re-service time, their corresponding probability density functions are s(x), v(x), and g(x) respectively.

$S^0(t), V^0(t),\ and G^0(t)$ represent the remaining service time of a batch, vacation time, and re-service time at 't' respectively.

$\tilde{S}(\theta), \tilde{V}(\theta), and \tilde{G}(\theta)$ represent the Laplace - Stieltje's transforms of S, V and G respectively.

For the queueing system, let us define the random variables as:

ξ (t)- (0)[1]{2}, if the server is on (essential service)[vacation]{re-service}.

Z (t) =j, if the server is on j^{th} vacation.

N_q (t) - Number of customers in the queue at time't'.

N_s (t) - Numbers of customers in the service at time't'.

The supplementary variables $S^0(t)$,$V^0(t)$ and $G^0(t)$ are introduced in order to obtain bivariate Markov process {N(t), ξ (t)}, where N(t)= {$N_q(t) \cup N_s(t)$}.

Let us define the following probabilities:

$P_{1,n}(x,t)dt = P\{N_s(t) = 1, N_q(t) = n , x \le S^0(t) \le x+dt, \xi (t)=0\}$, a≤ i ≤b,n≥0,

$Q_{i,n}(x,t)dt = P\{N_q(t) = n, x \le V^0(t) \le x+dt, \xi (t)=1, Z(t)=j\}$,j≥1,n≥0,

$G_n(x,t)dt = P\{ N_q(t)= n , x \le G^0(t) \le x+dt, \xi (t)=2\}$,n≥0.

In steady state, let us define for x>0,

$P_{1,j} = \lim_{t\to\infty} P_{1,j}(x,t)$for j≥0,

$Q_{i,j} = \lim_{t\to\infty} Q_{i,j}(x,t)$for i=1 to M, j≥1 and

$G_n(x) = \lim_{t\to\infty} G_n(x,t)$for n≥0.

STEADY STATE SYSTEM EQUATIONS

Following Cox [7], the equations govern the system under steady state conditions are obtained as follows:

$$-P'_{1,0}(x) = -\lambda P_{1,0}(x) + P_{1,1}(0)s(x) + \pi \sum_{l=1}^{M} Q_{l,i}(0)\, s(x) + G_i(0)s(x)(1-p), \tag{1}$$

$$-P'_{1,j}(x) = -\lambda P_{1,j}(x) + \lambda P_{1,j-1}(x)g_1 + (1-p)G_{1+j}(0)s(x) + P_{1,j+1}(0)s(x) + \pi \sum_{l=1}^{M} Q_{l,1+j}(0)s(x), \quad j \ge 1 \tag{2}$$

$$-Q'_{1,0}(x) = -\lambda Q_{1,0}(x) + (1-\pi)\sum_{m=a}^{b} P_{m,0}(x)v(x) + pG_0(x)v(x) \tag{3}$$

$$-Q'_{1,n}(x) = -\lambda Q_{1,n}(x) + (1-\pi)P_{1,n}(0)v(x) + pG_n(0)v(x) + \sum_{k=1}^{n} Q_{l,n-k}(x)\lambda g_k, \quad n \ge 1 \tag{4}$$

$$-Q'_{j,0}(x) = -\lambda Q_{j,0}(x) + Q_{j-1,0}(0)v(x), \quad 2 \le j \le M \tag{5}$$

$$-Q'_{j,n}(x) = -\lambda Q_{j,n}(x) + Q_{j-1,n}(0)v(x) + \sum_{k=1}^{n} Q_{j,n-k}(x)\lambda g_k, \quad 2 \le j \le M, n \ge 1 \tag{6}$$

$$-G'_0(x) = -\lambda G_0(x) + \pi P_{1,n}(0)g(x) \tag{7}$$

$$-G'_n(x) = -\lambda G_n(x) + \pi P_{1,n}(0)g(x) + \sum_{k=1}^{n} G_{n-k}(x) \quad n \ge 1 \tag{8}$$

$0 = -\lambda T_0(x) + \sum_{l=1}^{M} Q_{l0}(0)$ (9)

The Laplace-Stieltjes transform of $P_{i,n}(x), Q_{j,n}(x) and\ G_n(x)$ are defined asfollows:

$\tilde{P}_{i,n}(\theta) = \int_0^\infty e^{-\theta x} P_{i,n}(x)dx, \tilde{Q}_{j,n}(\theta) = \int_0^\infty e^{-\theta x} Q_{j,n}(x)dx$ and

$\tilde{G}_n(\theta) = \int_0^\infty e^{-\theta x} G_n(x)dx$ (10)

Multiplying Equations (1) to (9) by$e^{-\theta x}$ and integrating with respect to x over 0 to ∞, we get after using LST and (10)

$\theta\tilde{P}_{1,0}(\theta) - P_{1,0}(0) = -\lambda\tilde{P}_{1,0}(\theta) + \left[P_{11}(0) + \sum_{l=1}^{M} Q_{l1}(0) + G_1(0)(1-p)\right]\tilde{S}(\theta)$ (11)

$\theta\tilde{P}_{1,j}(\theta) - P_{1,j}(0) = -\lambda\tilde{P}_{1,j}(\theta) + [(1-p)G_{1+j}(0) + P_{1,j+1}(0) + \pi\sum_{l=1}^{M} Q_{l,j+1}(0)]\tilde{S}(\theta)$

$-\sum_{k=1}^{j} P_{1,j-1}(\theta)\lambda g_k, \quad j \geq 1$ (12)

$\theta\tilde{Q}_{1,0}(\theta) - Q_{1,0}(0) = \lambda\tilde{Q}_{1,0}(\theta) - [(1-\pi)P_{m,0}(0) + pG_0(0)]\,\tilde{V}(\theta),$ (13)

$\theta\tilde{Q}_{1,n}(\theta) - Q_{1,n}(0) = \lambda\tilde{Q}_{1,n}(\theta) - [(1-\pi)P_{1,n}(0) + pG_n(0)]\tilde{V}(\theta) + \sum_{k=1}^{n}\tilde{Q}_{1,n-k}(\theta)\lambda g_k,$
$n \geq 1$ (14)

$\theta\tilde{Q}_{j,0}(\theta) - Q_{j,0}(0) = \lambda\tilde{Q}_{j,0}(\theta) - Q_{j-1,0}(0)\tilde{V}(\theta) \quad 2 \leq j \leq M$ (15)

$\theta\tilde{Q}_{j,n}(\theta) - Q_{j,n}(0) = \lambda\tilde{Q}_{j,n}(\theta) - Q_{j-1,n}(0)\tilde{V}(\theta) - \sum_{k=1}^{n}\tilde{Q}_{j,n-k}(\theta)\lambda g_k, 2 \leq j \leq M$ (16)

$\theta\tilde{G}_0(\theta) - G_0(0) = \lambda G_0(\theta) - \pi P_{1,0}(0)\tilde{G}(\theta)$ (17)

$\theta G_n(\theta) - G_n(0) = \lambda G_n(\theta) - \pi P_{1,0}(0)\tilde{G}(\theta) - \sum_{k=1}^{n}\tilde{G}_{n-k}(\theta)\lambda g_k, n \geq 1$ (18)

$\lambda T_0(x) = \sum_{l=1}^{M} Q_{l0}(0)$ (19)

QUEUE SIZE DISTRIBUTIONS

To obtain the system size distribution let us define PGF's as follows:

$$\tilde{P}_1(z,\theta) = \sum_{j=0}^{\infty}\tilde{P}_{1,j}(\theta)\,z^j \quad and \quad P_1(z,0) = \sum_{j=0}^{\infty} P_{1,j}(0)z^j\,;$$

$$\tilde{Q}_i(z,\theta) = \sum_{j=0}^{M}\tilde{Q}_{i,j}(\theta)\,z^j \quad and \quad Q_j(z,0) = \sum_{j=0}^{M} Q_{1,j}(0)z^j\,; j \geq 1,$$

$\tilde{G}(z,\theta) = \sum_{j=0}^{\infty}\tilde{G}_n(\theta)\,z^j \quad and \quad G(z,0) = \sum_{j=0}^{\infty} G_n(0)z^j.$ (20)

Applying the above distribution from the equations (11) to (18) we get

$$(\theta - \lambda + \lambda X(z))\tilde{Q}_1(z,\theta) = Q_1(z,0) - \tilde{V}(\theta)\,[(1-\pi)\,P_{m,n}(0) + C_n(0)]z^n, n \geq 1 \quad (21)$$

$$(\theta - \lambda + \lambda X(z))\tilde{Q}_j(z,\theta) = Q_j(z,0) - \tilde{V}(\theta)Q_{j-1,n}(0)z^n, \qquad 2 \leq j \leq M \quad (22)$$

$$\left(\theta - \lambda + \lambda X(z)\right)\tilde{G}(z,\theta) = G(z,0) - \tilde{G}(\theta)P_{m,n}(0)z^n, \qquad n \geq 1 \quad (23)$$

$$\left(\theta - \lambda + \lambda X(z)\right)\tilde{P}_1(z,\theta) = P_1(z,0)\frac{\tilde{S}(\theta)}{z}$$
$$[P_m(z,0) - \ P_{m,j}(0)z^j + \sum_{l=1}^{M}[Q_l(z,0) - \ \sum_{j=0}^{b-1} Q_{l,j}(0)z^j] + \ G(z,0) - (1-P)G_n(0)z^n \quad (24)$$

By substituting $\theta = \lambda - p\lambda X(z)$ from (21) to (24) we get

$$Q_1(z,0) = \tilde{V}(\lambda - \lambda X(z))\,[(1-\pi)\,P_{1,n}(0) + G_n(0)]z^n \quad (25)$$

$$Q_j(z,0) = \tilde{V}(\lambda - \lambda X(z))Q_{j-1,n}(0)z^n, \qquad 2 \leq j \leq M \quad (26)$$

$$G(z,0) = \tilde{G}(\lambda - \lambda X(z))\pi P_{1,n}(0)z^n, \qquad n \geq 1 \quad (27)$$

$$P_1(z,0) = \frac{\tilde{S}(\lambda - \lambda X(z))}{z}\,[P_1(z,0) - P_{1,j}(0)z^j + \sum_{l=1}^{M}[Q_l(z,0) - Q_{l,j}(0)z_j + G(z,0) - \ (1-P)G_n(0)z^n]$$

$$zP_1(z,0) = \tilde{S}(\lambda - \lambda X(z))\{\ P_1(z,0) - P_{1,j}(0)z^j + \sum_{l=1}^{M}\left(Q_l(z,0) - Q_{l,j}(0)z^j\right)\} + (G(z,0) - G_n(0)z^n) \quad (28)$$

$$\left(z - \tilde{S}\left(\lambda - \lambda X(z)\right)\right)P_1(z,0) = \tilde{S}\left(\lambda - \lambda X(z)\right)f(z) \quad (29)$$

Where $f(z) = \tilde{S}\left(\lambda - \lambda X(z)\right)\,[P_1(z,0) + \sum_{l=1}^{M}[Q_{l,i}(z,0) + (1-P)G_n(0)]\,Z^n - P_{1,j}(0)z^j$

$$+ \tilde{V}(\lambda - \lambda X(z))\,[(1-\pi)\,P_{1,n}(0) + G_n(0)]z^n - \sum_{l=1}^{M} Q_{ln}(0)z^n + \tilde{G}(\lambda - \lambda X(z))\pi P_{1,n}(0)z^n - \ G_n(0)z^n \quad (30)$$

By Substituting (25) to (28) in (21) to (24) and after doing some algebra, we get

$$\tilde{Q}_1(z,\theta) = \frac{\tilde{V}\left(\lambda - \lambda X(z) - \tilde{V}(\theta)\right)\,[(1-\pi)\,P_{1,n}(0) + PG_n(0)]z^n}{\left(\theta - \lambda + \lambda X(z)\right)} \quad (31)$$

$$\tilde{Q}_j(z,\theta) = \frac{\widetilde{(V}\left(\lambda - \lambda X(z) - \tilde{V}(\theta)\right)Q_{j-1,n}(0)z^n}{\left(\theta - \lambda + \lambda X(z)\right)} \qquad 2 \leq j \leq M \quad (32)$$

$$\tilde{G}(z,\theta) = \frac{\tilde{G}\left(\lambda - \lambda X(z) - \tilde{G}(\theta)\right)\pi P_{1,n}(0)z^n}{\left(\theta - \lambda + \lambda X(z)\right)} \qquad n \geq 1 \quad (33)$$

$$\tilde{P}_1(z,\theta) = \frac{\widetilde{(S}\left(\lambda - \lambda X(z) - \tilde{S}(\theta)\right)f(z)}{\left(\theta - \lambda + \lambda X(z)\right)\left(z - \tilde{S}\left(\lambda - \lambda X(z)\right)\right)} \quad (34)$$

Where $f(z)$ is given in (30).

Let us define

$$p_i = P_{m,i}(0), \quad q_i = \sum_{l=1}^{M} Q_{l,i}(0), \qquad G_i = G_i(0) \quad and\; c_i = p_i + q_i + G_i$$

Let P (z) be the PGF of the system size at an arbitrary time epoch

$$P(z) = \tilde{P}_1(z,0) + \sum_{l=1}^{M} \tilde{Q}_l(z,0) + \tilde{G}(z,0) + T_0(z,0) \qquad (35)$$

Using the equations (32) to (34)with θ=0 in (35), we get the PGF of a queue size $P(z)$ at an arbitrary time epoch as

$$P(z) = \frac{\left[\tilde{S}(\lambda-\lambda X(z))-1\right]\left[(z-1)C_0+(\tilde{V}(\lambda-\lambda X(z))-(1-\pi)C_0(p)+\tilde{G}(\lambda-\lambda X(z))-(\tilde{V}(\lambda-\lambda X(z))(z-1)\pi p_0\right.}{(-\lambda+\lambda X(z))\left(z-\tilde{S}(\lambda-\lambda X(z))\right)} \qquad (36)$$

The probability q_0 can be expressed in terms of p_0 as,

$\sum_{l=1}^{m} q_l z^l = (p_0+q_0)\sum_{i=0}^{M} \alpha_i z^i$, $\quad p_0 = (p_0+q_0)\alpha_0$, and $q_0 = (1- \alpha_0)\, p_0$.

Here p_0 and q_0 are the probabilities of no customers in the queue at service and vacation completion epoch respectively. The PGF of the queue size has some unknowns say, $p_0, q_0 and\; c_0$. By Rouche's theorem of complex variables, it can be proved that $z - \tilde{S}(\lambda - \lambda X(z))$ has one zero on the unit circle |z| = 1. Since $P(z)$ is analytic on the unit circle, the numerator must vanish at these points, which gives the unknown constants. These equations can be solved by any suitable techniques.

Particular case:

When π=0 and p=0 then the PGF obtained in (35) reduces to the following form

$$P(z) = \frac{(\tilde{S}(\lambda-\lambda X(z))(1-\rho)(z-1)}{(z-\tilde{S}(\lambda-\lambda X(z)))} \qquad (37)$$

This is the Pollaczek-Khinchin formula for $M^X/G/1$.

PERFORMANCE MEASURES

Some important performance measures using the PGF of the queue size of $P(z)$of (36) are derived.

Expected Busy Period

Theorem

Let B be the random variable for 'busy period'. Then the expected busy period is

$$E(B) = \frac{E(S)}{(1-\pi)p_n+G_n} + \pi E(G).$$

Proof:

Let T be the residence time that the server is rendering service or under second optional re-service. Therefore, T=S with probability π and T=S+G with probability 1-π.

Define a random variable J as:

J= 0, if the server finds no customer in the queue after a residence time,
1, if the server finds at least one customer in the queue after a residence service,

Now, the expected length of busy period is given by

E (B) = E(B/J=0) P (J=0) +E (B/J=1) P (J=1)

$$= \{\pi[E(S)+E(G)]+(1-\pi)E(S)\}P(J=0)+[E(S)+E(B)]P(J=1)$$
$$= \{\pi[E(S)+E(G)]+(1-\pi)E(S)\}P(J=0)+[E(S)+E(B)][1-P(J=0)].$$

On solving for E (B), we get

$$E(B)=\frac{E(S)}{(1-\pi)p_n+G_n}+\pi E(G) \tag{38}$$

EXPECTED IDLE PERIOD

Theorem

Let I be the idle period random variable, then the expected idle period is

$$E(I)=\frac{E(V)}{1-\sum_{i=0}^{n}\alpha_i[(1-\pi)p_{n-i}+G_{n-i}]} \tag{39}$$

Hereα_i is the probability that there are 'n' customers arriving during a vacation.

Proof

We define the random variable U as,

U=0, if the server finds one customer after the first vacation.
=1, if the server finds no customer after the first vacation.

Now, the expected length of idle period due to multiple vacations E(I) is given by

E(I)= E(I/U=0) P(U=0)+E(I/U=1) P(U=1)
= E(V)P(U=0)+[E(V)+E(I)]P(U=1).

Solving for E (I), we get

$E(I) = \frac{E(V)}{P(U=0)}$From (32), we get

$$Q_i(z,0) = \tilde{V}(\lambda - \lambda X(z)) \sum_{n=1}^{M} \left((1-\pi)p_n + G_n\right) z^n$$

Using the fact that $Q_{1,n}$ (0) is the probability that 'n' customers being in the queue after first vacation.

$$\sum_{n=0}^{M} Q_{1,n}(0) z^n = \tilde{V}(\lambda - \lambda X(z)) \sum_{n=0}^{M} \left((1-\pi)p_n + G_n\right) z^n$$
$$= \left(\sum_{n=0}^{M} \alpha_n z^n\right)\left[\sum_{n=0}^{M} \left((1-\pi)p_n + G_n\right) z^n\right].$$

Equating the coefficients of z^n (n=0, 1, 2, 3… a-1) on both sides, we get,

$$Q_{1,n}(0) = \sum_{i=0}^{n} \alpha_i \left((1-\pi)p_{n-i} + G_{n-i}\right).$$

Also, $P(U=0) = 1 - \sum_{n=0}^{a-1} Q_{1,n}(0) = 1 - \sum_{n=l}^{M} \sum_{i=0}^{n} \alpha_i \left((1-\pi)p_{n-i} + G_{n-i}\right).$

Expected Queue Size:

The expected queue size E (Q) at an arbitrary time epoch is obtained by P' (z) at z =1 and is given by

$$E(Q) = \frac{\left\{\begin{matrix} f_1(X,S)\sum_{i=a}^{b-1}\left(b(b-1)-i(i-1)\right)c_i + f_2(X,S)\sum_{i=a}^{a-1}(b-i)c_i + f_3(X,S,V)\sum_{i=0}^{a-1} c_i \\ + f_4(X,S,V)\sum_{i=0}^{a-1} i c_i + f_5(X,S,V,G)\pi\sum_{i=0}^{a-1} p_i + f_6(X,S,V,G)\sum_{i=0}^{a-1} i.p_i \end{matrix}\right\}}{2(\lambda(E(X)(b-S1)))^2} \quad (40)$$

Where

$$f_1(X,S) = T.S1, f_2(X,S) = T.S2 - T2S1,$$

$f_3(X,S,V) = T.(b.V2 + b(b-1)V1) - T2.b.V1,$ f_4(X, S, V) =T.2b.V1,

f_5(X,S,V,G)=b.X1.T.(G2-V2)+b(b-1)(G1-V1)-b.T2(G1-V1)λ.B1,

f_6(X, S, V, G) =2.b.X1.T. (G1-V1),

S1=λE(X) E(S), G1=λE(X) E (G), V1=λE(X) E (V), X1=E(X), X2=X" (1),

S2=λX2E(S) +$\lambda^2 E(X)^2 E(S^2)$, G2=λX2E(G)+$\lambda^2 E(X)^2 E(G^2)$,V2=λX2E (V) +$\lambda^2 E(X)^2$ E (V^2), and

T=X1(b (b-S_1), T2=X1(b (b-1)-S2) +X2(b-S1).

Cost Model:

The importance of finding an optimal policy for a queueing system was discussed in detail in the recent survey by Choudhry [3,4] and [5]. By means of developing total expected cost function per unit time and the optimal values of the queuing system parameters can be obtained. Following the same cost structures by Lee et al. [8], Tadj and Ke [12,13], we let C_s be the start-up cost, C_h be the holding cost per customer per unit time, C_0, be the operating cost per unit time, C_r be the reward per unit time due to vacation, C_u be the re-service cost per unit time. The length of cycle is the sum of the idle period and busy period. Now, the expected length of cycle, E (T_c) is obtained as

$$E(T_c) = E(B) + E(I).$$

The Total average cost= start-up cost per cycle + holding cost of number of customer in the queue per unit time + operating cost per unit time* ρ − reward due to vacation per unit time.

$$\text{Total average cost} = \left[C_s - C_r \frac{E(V)}{P(U=0)} + \pi C_u E(G)\right] \frac{1}{E(T_c)} + c_h E(Q) + C_0 \rho.$$

We present some numerical example to illustrate the above solution in the next section.

Numerical Illustrations:

Computational analysis of this model for various combinations of service time distributions is exhibited. The unknown probabilities of the PGF of the queue size at an arbitrary time epoch and other computations are carried out using MATLAB. Numerical results are shown in the tables that are self-explanatory. In this model the cost values are assumed as $C_s=4, C_v=1, C_g=2, C_0=5$ and $C_h=0.25$. The values assumed for μ_1, the rate of an essential service, μ_2, the rate of optional re-service and α, the vacation rate are given in the bottom of the respective tables. Also, the probability for requesting the re-service is assumed as $\pi=0.1$.

Table 1: Performance measures for $M^{[x]}/G/1$ (exponential re-service and exponential vacation time)($\mu_1=4, \alpha=7, \mu_2=6$).

λ	Unknown probabilities					E(Q)	E(B)	E(I)	E(W)	ATC
	P(1)	P(3)	P(5)	P(7)	P(9)					
1	0.6845	0.6167	0.3826	0.2317	0.1379	1.7257	0.3945	0.2556	0.2876	6.9924
2	0.5680	0.6916	0.4157	0.2456	0.1433	1.8101	0.3494	0.3261	0.3017	6.6906
3	0.4775	0.1304	0.4728	0.2737	0.1570	2.0333	0.3430	0.3714	0.3389	6.3838
4	0.4049	0.1147	0.5563	0.3173	0.1796	2.3546	0.3533	0.3836	0.3924	6.2912
5	0.3445	0.1029	0.0537	0.3790	0.2125	2.7497	0.3727	0.3722	0.4583	6.3528
6	0.2915	0.0941	0.0516	0.4630	0.2581	3.2021	0.3975	0.3504	0.5345	6.4758
7	0.2421	0.0878	0.0511	0.0231	0.3202	3.7248	0.4251	0.3275	0.6208	6.6052
8	0.1930	0.0836	0.0526	0.0266	0.4045	4.3110	0.4525	0.3078	0.7185	6.7275
9	0.1401	0.0814	0.0562	0.0318	0.0124	4.9858	0.4762	0.2932	0.8310	6.8575
10	0.0784	0.0815	0.0627	0.0395	0.0192	5.7879	0.4911	0.2850	09647	7.0269

It is observed from Table 1 that, various performance measure values are higher if the essential service time follows exponential distribution than when it follows 2-Erlangen distribution for the same re-service time and vacation time distributions.

Table 2: Performance measures for $M^{[x]}/E_2/1$ (exponential vacation and re-service time is 2-Erlangian) (μ_1=4,α=8,μ_2=7).

λ	Unknown probabilities					E(Q)	E(B)	E(I)	E(W)	ATC
	P(1)	P(3)	P(5)	P(7)	P(9)					
1	0.8134	0.7167	0.4826	0.3821	0.2379	3.7257	0.7291	0.0576	2.2576	4.9344
2	0.7993	0.6916	0.5157	0.3502	0.1423	3.9001	0.7494	0.0261	2.3617	4.6456
3	0.7884	0.5144	0.5728	0.2737	0.1872	4.0368	0.7630	0.0764	2.3359	4.3348
4	0.6992	0.5047	0.5963	0.3298	0.1723	4.0983	0.7733	0.0846	2.3904	4.2902
5	0.6832	0.4029	0.3537	0.3790	0.2125	4.7497	0.7787	0.0762	2.4533	4.3008
6	0.6598	0.3241	0.2456	0.4190	0.4682	5.2829	0.7975	0.0534	2.5545	4.4018
7	0.6001	0.3001	0.1511	0.0872	0.4202	5.7259	0.8251	0.0255	2.6608	4.6662
8	0.5910	0.2801	0.1126	0.0341	0.4045	6.1010	0.8525	0.0238	2.7985	4.7015
9	0.5782	0.2013	0.0562	0.0321	0.0564	6.8858	0.8762	0.0932	2.8210	4.8575
10	0.5110	0.1415	0.0627	0.0300	0.0120	7.7002	0.8911	0.0850	2.9643	5.1169

It is observed from Tables 2 that various performance measure values are higher if the essential service time follows exponential distribution than when it follows 2-Erlangian distribution for the same re-service time and vacation time distributions.

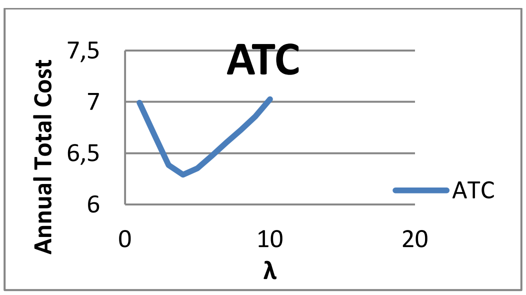

Figure 2: Arrival rate vs. Total average cost

CONCLUSION

In this paper we studied equilibrium and multiple optimizations in the bulk arrival general service queue under Bernoulli service schedule and multiple adaptive vacations assumptions. For the Bernoulli service and binomial controllable policy, we have shown that multiple equilibrium balking strategies exist, and among them some are stable while other is not. We have demonstrated that a unique optimal strategy could be larger than some of the vacation. The impact of cost model is analyzed numerically to decision making process. PGF of queuesize and various characteristics of this model are obtained. Particular case of the model is also provided.

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