

# Anti-Vague Deductive Systems Of Subtraction Algebras

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**Abstract:** *We introduced the notion of anti-vague deductive systems in subtraction algebra in this paper and study some findings on anti-vague deductive subtraction algebra systems and we for all intents and purposes come out with sort of possible theorems and evidence in a sort of major way. The relation between the anti-vague deductive system and the really crisp deductive system for all intents and purposes is also discussed, demonstrating how the relation between the anti-vague deductive system and the particularly crisp deductive system actually is also discussed*

**Keywords:** *Deductive system, Vague deductive system, anti-vague deductive system, crisp subset*

## 1. INTRODUCTION:

The notion of Vague theory recently introduced by Gau and Buehrer is of interests to us. For each such generalization one or more extra edge is added with the fuzzy theory with specialized type of aim and objective. Let 'A' be a vague set of all "good-in-maths students" of the universe U, and B be a fuzzy set of all "good-in-maths students" of U. Suppose that an intellectual Manager M<sub>1</sub>, proposes the membership value  $\mu_B(x)$  for the element x in the fuzzy set B by his best best intellectual Manager M<sub>2</sub>. Proposes independently two membership values  $t_A(x)$  and  $f_A(x)$  for the vague same elements in the vague set A by his best intellectual capacity. The amount  $t_A(x)$  is the true membership value of x and  $f_A(x)$  is the false membership value of x in the vague set A. Both M<sub>1</sub> and M<sub>2</sub> begin human agents have their limitation of perception, Judgement, processing capacity with real life complex situations

## 2. BASIC RESULTS ON SUBTRACTION ALGEBRAS

By subtraction algebra, we mean an algebra (X,-) with a single binary operation "-" that satisfies the following identities : for any x, y, z  $\in$  X.

(s1)  $x-(y-x) = x$

(s2)  $x-(x-y) = y-(y-x)$

$$(s3) (x-y)-z = (x-z)-y$$

The last identity permits us to omit parenthesis in expression of the form  $(x-y)-z$ . The subtraction determines an order relation on  $X$ .  $a \leq b \Leftrightarrow a-b = 0$  Where  $0 = a-a$  is an element that doesn't depend on the choice  $a \in X$ . The ordered set  $(X, \leq)$  is a semi-Boolean algebra in the sense of [1] that is. It meet a semi lattice with zero in every interval  $[0, a]$  is a Boolean Algebra with respect to the induced order. Hence  $a \wedge b = a-(a-b)$ . The complement of a element  $b \in [0, a]$  is  $a-b$  and if  $b, c \in [0, a]$  then

$$\begin{aligned} b \vee c &= (b \wedge c) = a - ((a-b) \wedge (a-c)) \\ &= a - ((a-b) - ((a-b) - (a-c))) \end{aligned}$$

In a subtraction Algebra the following are true

- (a1)  $(x-y)-y = x-y$
- (a2)  $x-0 = x$  and  $0-x = 0$
- (a3)  $(x-y)-x = 0$
- (a4)  $x-(x-y) \leq y$
- (a5)  $(x-y)-(y-x) = x-y$
- (a6)  $x-(x-(x-y)) = x-y$
- (a7)  $(x-y)-(z-y) \leq x-z$
- (a8)  $x \leq y$  if and only if  $x = y-w$  for some  $w \in X$ .
- (a9)  $x \leq y$  implies  $x-z \leq y-z$  and  $z-y \leq z-x$  for all  $z \in X$
- (a10)  $x, y \leq z$  implies  $x-y = x \wedge (z-y)$
- (a11)  $(x \wedge y) - (x \wedge z) \leq x \wedge (y-z)$

**Proposition:** Let 'X' be a subtraction algebra and Let  $x, y \in X$ . If  $w \in X$  is a upper bound for  $x$  and  $y$ , then the element  $x \vee y = w - ((w-y)-x)$  is a least upper bound for  $x$  and  $y$

### 3. BASIC RESULTS ON VAGUE SETS

**Definition 3.1** A Vague set  $A$  in the universe of discourse of  $U$  is characterized by two membership function given by  $A$  truth membership function  $t_A : U \rightarrow [0,1]$  and

A false membership function  $f_A : U \rightarrow [0,1]$

Where  $t_A(u)$  is a lower bound of the grade of membership of 'U' derived from the "evidence for U" and  $f_A(u)$  is a lower bound negation of 'U' derived from "evidence for U" and

$$t_A(u) + f_A(u) \leq 1$$

Thus the grade of membership of 'U' in the Vague set  $A$  is bounded by a subinterval  $[t_A(u), 1-f_A(u)]$  The Vague set is written as  $A = \{(u, [t_A(u), 1-f_A(u)]) \mid u \in U\}$  Where the interval  $[t_A(u), 1-f_A(u)]$  is called the vague set of  $U$  in 'A' and is denoted by  $V_A(u)$

**Definition 3.2** A vague set  $A$  of set  $U$  is called

- (1) The Vague set of  $U$  if  $t_A(u) = 0$  and  $f_A(u) = 1$  for all  $u \in U$
- (2) The unit Vague set of  $U$  if  $t_A(u) = 1$  and  $f_A(u) = 0$  for all  $u \in U$
- (3) The  $\alpha$ -Vague set of  $U$  if  $t_A(u) = \alpha$  and  $f_A(u) = 1 - \alpha$  for all  $u \in U$ , where  $\alpha \in (0,1)$

For  $\alpha, \beta \in [0,1]$  we now define  $\alpha, \beta$  – cut and  $\alpha$ -cut of a Vague set

**Definition 3.3** Let ‘A’ be a Vague set of a Universe X with the true membership function  $t_A$  and false membership  $f_A$ . The  $\alpha, \beta$  – cult of the Vague set A is a crisp subset  $A_{(\alpha,\beta)}$  of the set given by  $A_{(\alpha,\beta)} = \{x \in X \mid V_A(x) \leq [(\alpha,\beta)]\}$  clearly  $A_{(0,0)} = X$ , The  $(\alpha,\beta)$  – cuts are also called Vague sets of the Vague set A.

**Definition 3.4** The  $\alpha$  – cut of the vague set A is a crisp subset  $A_\alpha$  of the set X given by  $A_\alpha = A_{(\alpha, \alpha)}$

Note that  $A_0 = X$  and if  $\alpha \geq \beta$  then  $A_\beta \subseteq A_\alpha$  and  $A_{(\alpha,\beta)} = A_\alpha$ . Equivalently, we can define the  $\alpha$ -cut as  $A_\alpha = \{x \in X \mid t_A(x) \leq \alpha\}$ .

**Notation 3.5** Let  $I[0,1]$  denote the family of all closed subintervals of  $[0,1]$ . If  $I_1 = [a_1, b_1]$  and  $I_2 = [a_2, b_2]$  be two elements of  $I[0,1]$ , we call  $I_1 \geq I_2$  if  $a_1 \geq a_2$  and  $b_1 \geq b_2$

Similarly we can understand the relations  $I_1 \leq I_2$  and  $I_1 = I_2$  clearly the relation  $I_1 \geq I_2$  does not necessarily imply that  $I_1 \geq I_2$  and conversely. We define the term “imax” to mean the maximum of two intervals as

$$\text{imax}(I_1, I_2) = [\max(a_1, a_2), \max(b_1, b_2)]$$

Similarly we define “imin”. The concept of “imax” and “imin” could be extended to define “isup” and “iin” of infinite number of elements  $I[0,1]$

it is obvious that  $L = \{I[0,1], \text{isup}, \text{iin}, \leq\}$  is a lattice with universal bounds  $[0,0]$  and  $[1,1]$

#### 4. ANTI-VAGUE DEDUCTIVE SYSTEMS

In what follows let x be a subtraction algebra unless otherwise specified

**Definition 4.1** A non-empty set D of X is called a deductive system of X. If it satisfies

- (1)  $0 \in D$
- (2)  $(\forall x \in X)(\forall y \in D) (x-y \in D \Rightarrow x \in D)$

**Definition 4.2** A Vague set of X is called a anti-vague deductive system of X if the following conditions are true

- (C1)  $(\forall x \in X)(V_A(0) \leq V_A(x))$
- (C2)  $(\forall x, y \in X)(V_A(x) \leq \max\{V_A(x-y), V_A(y)\})$

That is  $t_A(0) \leq t_A(x)$ ,  $1-f_A(0) \leq 1-f_A(x)$ ,

$$t_A(x) \leq \max\{t_A(x-y), t_A(y)\}$$

$$1-f_A(x) \leq \max\{1-f_A(x-y), 1-f_A(y)\} \forall x, y \in X$$

**Example 4.3** Consider a subtraction algebra  $X = \{0, x, y\}$  with the following table

-	0	x	y
0	0	0	0
x	x	0	x
y	y	y	0

Let 'A' be a Vague set in X define as follows  $A = \{(0, [0.2, 0.6]), (x, [0.3, 0.6]), (y, [0.5, 0.3])\}$  is a routine to verify that 'A' is a anti-vague deductive system of X.

**Example 4.4** Consider a subtraction algebra  $X = \{0, a, b, c, d\}$  with the following table

-	0	a	b	c	d
0	0	0	0	0	0
a	a	0	a	0	a
b	b	b	0	0	b
c	c	b	a	0	c
d	d	d	d	d	0

Let 'A' be a Vague set in X define as follows  $A = \{(0, [0.6, 0.2]), (a, [0.7, 0.2]), (b, [0.7, 0.2]), (c, [0.7, 0.2]), (d, [0.7, 0.2])\}$  is a routine to verify that 'A' is a anti-vague deductive system of X.

**Proposition 4.5** Every Anti-Vague deductive system 'A' of X satisfies  
 $(\forall x, y \in X)(x \leq y \Rightarrow V_A(x) \leq V_A(y))$

**Proof** Let  $x, y \in X$  be such that  $x \leq y$ . Then  $x - y = 0$  and so  $t_A(x) \leq \max\{t_A(x-y), t_A(y)\} = \max\{t_A(0), t_A(y)\} = t_A(y)$   
 so  $t_A(x) \leq t_A(y)$   
 $1 - f_A(x) \leq \max\{1 - f_A(x-y), 1 - f_A(y)\}$   
 $\quad = \max\{1 - f_A(0), 1 - f_A(y)\}$   
 $\quad = 1 - f_A(y).$

This shows that  $V_A(x) \leq V_A(y)$

**Proposition 4.6** Every anti-vague deductive system A of X satisfies  
 $(\forall x, y, z \in X) (V_A(x-z) \leq \max\{V_A((x-y)-z), V_A(y)\})$

**Proof** Using (C2) and (S3) we have  
 $V_A(x-z) \leq \max\{V_A((x-z)-y), V_A(y)\}$   
 $\quad = \max\{V_A((x-y)-z), V_A(y)\}$   
 for all  $x, y, z \in X$

**Theorem 4.7** If A is a Vague set in X satisfying (C1) and (4). Then A is a anti-vague Deductive system of X.

**Proof** Taking  $z = 0$  in (4) and using (a2) we have  
 $V_A(x) = V_A(x-0)$   
 $\quad \leq \max\{V_A((x-y)-0), V_A(y)\}$

$$= \max\{V_A(x-y), V_A(y)\} \text{ for all } x, y \in X$$

Hence A is a anti-vague deductive system of X.

**Corollary 4.8** Let A be a Anti-vague set in X. Then A is a anti vague deductive system of X. If and only if it satisfies the following conditions (C1) and (C4)

**Theorem 4.9** Let A be a Vague - set in X. Then A be a anti-vague deductive system of X if and only if it satisfies the following conditions :

$$(\forall x, y \in X)(V_A(x-y) \leq V_A(x))$$

**Proof :** Assume that 'A' be anti-vague deductive system of X. Using (a3) , (c1) and (c2) we get

$$\begin{aligned} V_A(x-y) &\leq \max\{V_A((x-y)-x) , V_A(x)\} \\ &= \max\{V_A(0), V_A(x)\} = V_A(x) \quad \forall x,y \in X \end{aligned}$$

$$\text{Since } (x-((x-a)-b))-a = (x-a) - ((x-a)-b) \leq b$$

it follows from (3) that

$$\begin{aligned} V_A(x-((x-a)-b)) &\leq \max\{V_A((x-a)-b)-a , V_A(a) \} \\ &= \max\{V_A(a) , V_A(b)\} \end{aligned}$$

$$(\forall x, a, b \in X)(V_A(x) - x-((x-a)-b)) \leq \max\{V_A(a), V_B(b)\}$$

Conversely let A be a anti-vague set in X satisfying conditions (5) and (6) if we take  $y = x$  in (5) then  $V_A(0) = V_A(x-x) \leq V_A(x)$  for all  $x \in X$  using (6) we obtained

$$\begin{aligned} V_A(x) &= V_A(x-0) \\ &= V_A(x-((x-y)-(x-y))) \\ &= V_A(x-((x-(x-y))-y)) \\ &\leq \max\{V_A(x-y), V_A(y)\} \end{aligned}$$

for all  $x, y \in X$ . Hence a is a Anti-vague deductive system of X.

**Proposition 4.10** Every anti-vague deductive system of A of X satisfies the following assertion

$$(\forall x, y \in X) (\exists x \vee y \Rightarrow V_A(x \vee y)) \leq \max\{V_A(x), V_A(y)\}$$

**Proof :** Suppose there exist  $x \vee y$  for  $x, y \in X$  . Let w be a upper bound of 'x' and 'y'. Then  $x \vee y = w-((w-y)-x)$  is the least upper bound for x and y and so

$$V_A(x \vee y) = V_A(w-((w-y)-x)) \leq \max\{V_A(x), V_A(y)\} \text{ by (6) this completes the proof}$$

**Proposition 4.11 :** Let A be a vague set in X. Then A is a anti-vague deductive system of X if and only if it satisfies

$$(\forall x, y, z \in X) (\exists x-y \leq z \Rightarrow V_A(x)) \leq \max\{V_A(y), V_A(z)\}$$

**Proof :** Assume that A is a anti-vague deductive system of X. and let  $x, y, z \in X$  be such that  $x-y \leq z$  then  $V_A(y) \leq V_A(z)$  by (3). It follows from (C2) that  $V_A(x) \leq \max\{V_A(x-y), V_A(y)\} \leq \max\{V_A(y), V_A(z)\}$

Conversely suppose that 'A' satisfies (8)

since  $0-y \leq y$  for all  $y \in X$ , we have

$V_A(0) \leq \max\{V_A(y), V_A(x-y)\} = V_A(y)$  by (8)

Thus (C1) is invalid. since  $x-(x-y) \leq y$  for all  $x, y \in X$  by (a4) it follows from (8) that

$$V_A(x) \leq \max\{V_A(x-y), V_A(y)\}$$

Hence 'A' is a anti-vague deductive system of X.

**Theorem 4.12** If a vague set in 'A' in x is a anti-vague deductive system of X, the

$$\prod_{i=1}^n x - \omega_i = (\dots((x - \omega_1) - \omega_2) - \dots) - \omega_n$$

**Proof :** The proof is by induction on n. Let A be a anti-vague deductive system of X. By (3) and (8) we know that the condition (9) is invalid for  $n=1,2$

Suppose that A satisfies the condition (9) for  $n=k$ , that is  $\prod_{i=1}^k x - \omega_i = 0 \Rightarrow V_A(x) \leq \max\{V_A(w_i) \mid i=1,2,\dots,k\}$  for all  $x, w_1, w_2, \dots, w_k \in X$  let  $x, w_1, w_2, \dots, w_k, w_{k+1}$  be such that  $\prod_{i=1}^{k+1} x - \omega_i = 0$  then  $V_A(x-w_1) \leq \max\{V_A(w_j) \mid j = 2,3,\dots,k+1\}$  since A is a anti-vague deductive system of X it follows from (C2) that

$$\begin{aligned} V_A(x) &\leq \max\{V_A(x-w_1), V_A(w_1)\} \\ &\leq \max\{V_A(w_1), \max\{V_A(w_j) \mid j = 2,3,\dots,k+1\}\} \end{aligned}$$

This completes the proof

**Theorem 4.13** Let A be a vague set in X satisfying the condition (9) . Then A is a anti-vague deductive system of X

**Proof :** note that  $(\dots((0-x)-x)-\dots)-x = 0$  for all  $x \in X$  it follows from (9) that  $V_A(0) \leq V_A(x)$  for all  $x \in X$ . Let  $x, y, z \in X$  be such that  $x-y \leq z$

$$\text{then } 0 = (x-y)-z = (\dots(((x-y)-z)-0)-\dots) - 0$$

$$\text{and so } V_A(x) \leq \max\{V_A(y), V_A(z)\}$$

Hence by proposition 4.11, we conclude that 'A' is anti-vague deductive system of X.

**Theorem 4.14** Let A be a anti-vague deductive system of X. then for any  $\alpha, \beta \in [0,1]$  the vague-cut  $A_{(\alpha, \beta)}$  is a crisp deductive system of X.

**Proof :** clearly  $0 \in A_{(\alpha, \beta)}$ . Let  $x, y \in X$  be such that  $y \in A_{(\alpha, \beta)}$  and  $x-y \in A_{(\alpha, \beta)}$  then  $V_A(y) \leq [\alpha, \beta]$  (i.e)  $t_A(y) \leq \alpha$  and  $f_A(y) \leq \beta$  and  $V_A(x-y) \leq [\alpha, \beta]$  (i.e)  $t_A(x-y) \leq \alpha$  and  $1-f_A(x-y) \leq \beta$  it follows from (2) that

$$\begin{aligned} t_A(x) &\leq \max\{t_A(x-y), t_A(y)\} \leq \alpha \\ 1-f_A(x) &\leq \max\{1-f_A(x-y), 1-f_A(y)\} \leq \beta \end{aligned}$$

so that  $V_A(x) \leq [\alpha, \beta]$ . Hence  $x \in A_{(\alpha, \beta)}$  and so  $A_{(\alpha, \beta)}$  is a deductive system of X.

**Proposition 4.15** Let A be a anti-vague deductive system of X. Two vague-cut deductive systems  $A_{(a, b)}$  and  $A_{(c, d)}$  with  $[a, b] < [c, d]$  are equal if and only if there is no  $x \in X$  such that  $[a, b] \geq V_A(x) \geq [c, d]$

**Theorem 4.16** Let  $x$  be a finite and let 'A' be a anti-vague deductive system of  $X$ . consider the set  $V(A)$  given by  $V(A) = \{V_A(x) \mid x \in X\}$  then  $A_i$  are the only vague-cut deductive systems of  $X$ , where  $I \in V(A)$

**Proof :** consider  $[a_1, a_2] \in I[0,1]$  where  $[a_1, a_2] \notin V(A)$  if  $[a, b] < [a_1, a_2] < [c, d]$  where  $[a, b], [c, d] \in V(A)$  then  $A_{(a, b)} = A_{(a_1, a_2)} = A_{(c, d)}$   
if  $[a_1, a_2] > [a_1, a_3]$  where  $[a_1, a_3] = \text{imax}\{x/x \in V(A)\}$  then  $A_{(a_1, a_3)} = X = A_{(a_1, a_2)}$   
Hence for any  $[a_1, a_2] \in I[0,1]$ , the vague-cut deductive system  $A_{(a_1, a_2)}$  is one of  $A_i$  for  $I \in V(A)$  this completes the proof

**Theorem 4.17** Any deductive system of  $D$  of  $X$  is a vague cut deductive system of  $X$

**Proof :** consider the vague set 'A' of 'X' given by

$$V_A(x) = [\alpha, \alpha] \text{ if } x \in D \\ = [0, 0] \text{ if } x \notin D$$

where  $\alpha \in (0,1)$ . since  $0 \in D$  we have

$$V_A(0) = [\alpha, \alpha] \leq V_A(x) \text{ for all } x \in X$$

Let  $x, y \in X$ . If  $x \in D$  then  $V_A(x) = [\alpha, \alpha] \leq \max\{V_A(x-y), V_A(y)\}$

Assume  $x \notin D$  then  $y \notin D$  or  $(x-y) \notin D$  it follows that  $V_A(x) = [0,0] = \max\{V_A(x-y), V_A(y)\}$

Thus 'A' is a anti-vague deductive system of  $X$ . clearly  $D = A_{(\alpha, \alpha)}$

**Theorem 4.18** Let  $A$  be a anti-vague deductive system of  $X$  then the set

$$D = \{x \in X \mid V_A(x) = V_A(0)\} \text{ is a crisp deductive system of } X$$

**Proof :** clearly  $0 \in D$ . Let  $x, y \in X$  be such that  $x-y \in D$  and  $y \in D$ . Then  $V_A(x-y) = V_A(0) = V_A(y)$  and so  $V_A(x) \leq \max\{V_A(x-y), V_A(y)\} = V_A(0)$  by (C2)

Since  $V_A(0) \leq V_A(x)$  for all  $x \in X$ , it follows that  $V_A(x) = V_A(0)$  so that  $x \in D$ . therefore 'D' is a crisp deductive system of  $X$ .

## 5. CONCLUSION

In this paper the concept of Anti-vague deductive system of subtraction algebra is introduced and investigate various elementary properties of this concept is investigated

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