

Non-Minimally Coupled Tachyonic Scalar Field

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Abstract : *In past few decades it has been proved that tachyon scalar field is a very promising candidate of dark energy and it provides explanation of accelerated universe as well as coupling between different components (dark energy, matter, radiation) of scalar fields. In this article we analyze non-minimally coupled tachyonic scalar fields to find the variation of tachyonic potential $V(t)$ with time (t) with the change of equation of state (ω) .*

1. Introduction

The cosmological observations for type Ia Supernova [1, 2] indicates strongly the accelerated expansion of the universe. To explain the observed accelerated expansion concept of dark energy introduced as modified matter. The remarkable feature of this dark energy is its negative pressure which provide it equation of state $\omega_{\text{dark-energy}} < -1/3$ and this negative pressure dominate over its energy density gives the accelerated expansion of the universe. To follow the mechanism of accelerated expansion of the universe there are several candidate of dark energy proposed. Cosmological constant is one of the simplest candidate of dark energy bear the equation of state $\omega_\lambda = -1$. The dominated cosmological constant in the universe provide the exponential expansion of the universe and corresponding scenario known as inflation. Cosmological constant as a candidate of dark energy suffer by two serious problem one is cosmological constant problem and other is coincidence problem. To alleviate these two issue of Λ CDM model the interacting dark energy model proposed by a number of authors [4]-[12]. In this article we have considered non-minimally coupled tachyonic scalar field which can be varied by a coupling parameter Γ . Non-minimal couplings are generated by quantum corrections to the scalar field theory and they are essential for the renormalizability of the scalar field theory in curved space [13]. We have mainly discussed the form of tachyonic potential $V(\phi)$ for $\phi = \phi_0 t$. Among all the available forms of nonminimal coupling [14] we have chosen a non-minimal derivative in the form $\Gamma G_{\mu\nu} \partial^\mu \phi \partial^\nu \phi$ where G is the newton's gravitational constant. So, The action and Lagrangian of non-minimally coupled tachyonic scalar field written as,

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{pl}^2}{2} R - V(\phi) \sqrt{1 - \partial^\mu \phi \partial_\mu \phi} + \Gamma G_{\mu\nu} \partial^\mu \phi \partial^\nu \phi \right] \quad (1)$$

where M_{pl} is the plank mass and the corresponding Lagrangian,

$$\mathcal{L} = -V(\phi) \sqrt{1 - \partial^\mu \phi \partial_\mu \phi} + \Gamma G_{\mu\nu} \partial^\mu \phi \partial^\nu \phi \quad (2)$$

for a spatially flat FLWR metric $ds^2 = -dt^2 + a^2(t)dr^2$ and for homogeneous ϕ the equation of motion can be found by varying the action as [15],

$$\frac{\ddot{\phi}}{1-\dot{\phi}^2} + 3H\dot{\phi} + \frac{V'(\phi)}{V(\phi)} + \frac{\sqrt{1-\dot{\phi}^2}}{V(\phi)} [6\Gamma H^2 \ddot{\phi} + 18\Gamma H^3 \dot{\phi} + 12\Gamma H \dot{H} \dot{\phi}] = 0 \quad (3)$$

Where $\dot{\phi} = \frac{\partial\phi}{\partial t}$ and, $V'(\phi) = \frac{\partial V(\phi)}{\partial \phi}$. The stress energy tensor for \mathcal{L} ,

$$\mathcal{T}^{\mu\nu} = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \partial^\nu \phi - g^{\mu\nu} \mathcal{L} \quad (4)$$

The 00 component of stress energy tensor Eq.(4) gives us energy density of the tachyonic scalar field as,

$$\rho_\phi = \frac{V(\phi)}{\sqrt{1-\dot{\phi}^2}} + 9\Gamma H^2 \dot{\phi}^2 \quad (5)$$

the other components i.e. ii components gives us the pressure of the tachyonic scalar field as,

$$p = -V(\phi)\sqrt{1-\dot{\phi}^2} - \Gamma(3H^2 + 2\dot{H}^2)\dot{\phi}^2 - 4\Gamma H \dot{\phi} \ddot{\phi} \quad (6)$$

The individual equation of continuity of scalar field and matter is given by,

$$\dot{\rho}_\phi + 3H(1 + \omega)\rho_\phi = -Q \quad (7)$$

$$\dot{\rho}_m + 3H(1 + \omega_m)\rho_m = Q \quad (8)$$

where $Q = 0$ and $\omega =$ equation of state which for tachyonic scalar field vart from $-1 \leq \omega \leq 0$ [16][17] mimics the results of Λ CDM in which the scalar field components are independent of each other.

V(ϕ) when $Q = 0$

When $Q = 0$ the calculations becomes very simple and we can write the solutions of Eq.(7) and Eq.(8) as,

$$\frac{\rho_\phi}{\rho_\phi^0} = \frac{a}{a_0} x^{-3(1+\omega)} \quad (9)$$

$$\frac{\rho_m}{\rho_m^0} = \frac{a}{a_0} x^{-3(1+\omega_m)} \quad (10)$$

Putting Eq.(5) on Eq.(7) we get,

$$V(\phi) = \sqrt{1-\dot{\phi}^2} [\rho_\phi^0 x^{-3(1+\omega)} - 9\Gamma H^2 \dot{\phi}^2] \quad (11)$$

by assuming $\phi = \phi_0 t$ and, $a(t) = e^{\alpha t}$ where α is the expansion constant and ϕ_0 tachyonic field we get,

$$V(t) = \sqrt{1-\phi_0^2} [\rho_\phi^0 e^{-3\alpha t(1+\omega)} - 9\Gamma \alpha^2 \phi_0^2] \quad (12)$$

V(ϕ) when $Q \neq 0$

Among all the forms considered by several authors[18]-[22] we will focus on the form $Q =$

$\chi\rho_\phi H$ then the solutions of equations Eq.(7) and Eq.(8) becomes,

$$\left(\frac{\rho_\phi}{\rho_\phi^0}\right) = \left(\frac{a}{a_0}\right)^{-\beta} \quad (13)$$

$$\left(\frac{\rho_m}{\rho_m^0}\right) = \left(\frac{a}{a_0}\right)^{-3(1+\omega_m)} + \frac{\beta\rho_\phi^0}{\rho_m^0[\beta-3(1+\omega_m)]} \left[\left(\frac{a}{a_0}\right)^{-3(1+\omega_m)} - \left(\frac{a}{a_0}\right)^{-\beta} \right] \quad (14)$$

where $\beta = [3(1 + \omega) - \chi]$. Now putting Eq.(13) on Eq.(7) we get,

$$V(t) = \sqrt{1 - \phi_0^2} [\rho_\phi^0 e^{-\alpha\beta t} - 9\Gamma\alpha^2\phi_0^2] \quad (15)$$

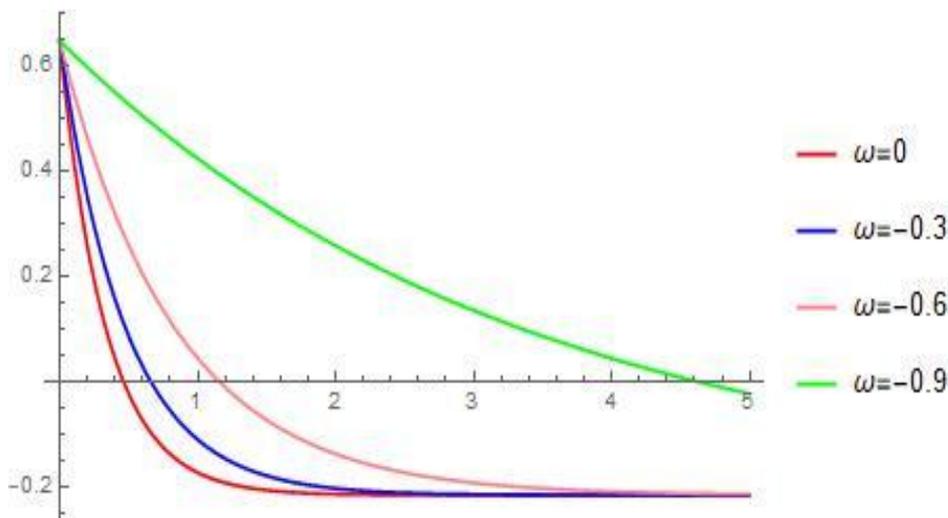


Figure 1: The figure representing the variation of $V(t)$ with t for $\omega = 0, -0.3, -0.6, -0.9$ using fixed values of other parameters as, $\phi_0 = 0.5$, $\Gamma = 1$, $\rho_\phi^0 = 1$ and, $\alpha = 1$.

2. Conclusion

We have analysed non-minimally coupled tachyonic scalar field and obtained the expressions of energy density and pressure denoted by Eq.(5) and Eq.(6). With the help of these equations we have obtained possible forms of *Tachyonic potential* $V(t)$ as a function of time for two cases (i) when there is no coupling between the scalar field components (ii) when there is a finite coupling between the components of scalar fields in the form $Q = \alpha\rho_\phi H$. A schematic variation of *Tachyonic Scalar Potential*(Eq.(12)) with cosmic time t for different values of ω has been shown in Fig.[1] by fixing the values of $\phi_0 = 0.5$, $\Gamma = 1$, $\rho_\phi^0 = 1$ and, $\alpha = 1$. It can be observed from the diagram that as ω decreases from 0 towards -1 the downward slope of the variation curve becomes less steeper and in between $\omega = -0.9$ to -1 the potential shows positive values and tends to remain in positivity.

3. References

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