

The Ising Model And Social Dynamics

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Abstract: *Ising model is essentially used to see the ferromagnetic to paramagnetic phase transition. However, this is not the only limit of the model. It is in fact widely used to study the social behavior of human. Despite the fact that the human collective behavior is a result of many physiological and psychological processes, yet the mathematical modelling of a society has yielded many information which go in line with the overall dynamism of the society. This article discusses the possible ways to deal with social dynamics using the Ising model.*

I. Introduction:

The primary theme of studying social dynamics is to understand the transition from an initial disordered state to the ultimate ordered state of the social behavior. There are however various aspects of the society, such as variation in opinion, voting style, language and cultural dynamics, which can be dealt with various physical models. Let us suppose that, there is a consensus vote going on in a society of small population. The individual views can be put in the form of either 'yes' or 'no'. In physical modelling this is given either a spin 'up' or 'down' status. Consider the diagram as shown in figure-1.

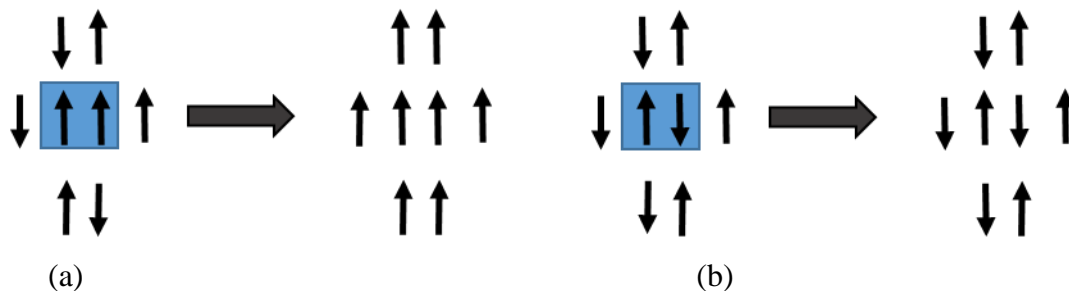


Figure 1: figure showing a pair neighboring agents with the same opinion convince all their neighbors while there is no influence if they disagree.

If, two near neighbors are paired up and votes for an 'up' or 'yes' (figure (a)), then the entire population gradually goes with a 'yes'. Contrary to this, if they disagree (figure 1(b)), then the entire population remains the same, with no common consensus building among them. This is in accordance with Sznajd model- one of the models of opinion dynamics [1], [2].

Ising model was initially restricted to the study of phase transitions especially to visualize the paramagnetic to the ferromagnetic phase transitions. In ferromagnetic substances such as Iron, Nickel etc., spontaneous magnetization is caused by the alignment of the magnetic spins in the same direction. This however happens below a certain temperature which is known as the Curie temperature. It has been observed that if the spins are aligned in the same direction, the total energy is decreased than in the case of being antiparallel [3]. Therefore, the substance always tends to be in the minimal energy state and the spins have a tendency to be

aligned. At a temperature more than the Curie temperature, average thermal energy of the spins is increased and the substance has approximately half of the spins aligned opposite to each other. This creates a net zero magnetization and the substance behaves as a paramagnetic material. Then, the material is said to have changed its phase from being ferromagnetic to paramagnetic.

The concepts of laws of nature are of statistical origin and this is a firmly believed fact. The success of statistical physics is far reaching and has been gaining success even in modeling the society. Statistical physics is now widely applied to understand the dynamical behavior of a common society. The only point of concern is that in social phenomena the basic constituents are not the particles, but the humans. The human societies are highly interacting in nature and hence the opinion of each of its constituent matters [4-10]. People not only interact but also influence others. Now, although the extent of interaction of an individual is limited as compared to the total number of people in the social system, yet there is a stunning global regularity. There are transitions from disorder to order, spontaneous formation of culture, language and emergence of agreement over a specific concern.

II. The Ising Model

Let us consider a system of N spins; the spins can assume only two values either +1 or -1. At the initial time, it may also be considered that the spins are randomly oriented. The resulting Hamiltonian of such a system may be written as [11]

$$H = -J \sum_{\langle i,j \rangle} S_i S_j$$

Here, the sum $\langle i, j \rangle$ is over the nearest neighbor. J is the strength of interaction. A brief algorithm to study the phase transition is presented below:

- a. Arrange the atoms with random spins (or any specified spins such as +1 or -1) over a 2D lattice of definite size.
- b. For any chosen random spin, calculate the energy of the lattice. Call it initial energy.
- c. Give the chosen spin a trial flip and again calculate the energy of the lattice. Call it the final energy.
- d. The difference in energy is $dE = E_{final} - E_{initial}$.
- e. If $dE \leq 0$, accept the trial flip, as the system goes into a minimum energy configuration. Else, compute $u = \exp(-dE/kT)$ and compare this with a computer generated random number (r).
- f. If $r < u$ accept the move, otherwise reject it.
- g. Do it for all temperatures and find out the averaged thermodynamic quantities.

III. The Ising Model and the Society

In social dynamic model, especially in Potts model, each spin can assume one out of the q values and equal nearest neighbor values are energetically favored. For Ising model $q=2$. In the case of social modelling, the dynamical aspect requires the monitoring of the suitable quantity which is able to properly identify the buildup of order. Magnetization is not that suitable quantity in social modelling. Rather, the appropriate quantity is the correlation function between pairs of spins at a distance r from each other. This can be thought to as the monitoring agent of the ordering process [12].

$$C(r, t) = \langle S_i(t) S_{i+r}(t) \rangle - \langle S_i(t) \rangle^2$$

IV. Conclusion

Physical models such as the Ising model is not only restricted to the study of the material, but, it is well applicable to the study of the dynamical behavior of the society as well. There could be limitation due to finite-size effects and other psycho-physical evolutionary nature of the society, yet the physical models have predicated the society correctly to a great extent.

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