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# A Three-Tropic Level Food Chain Model Considering Holling Type II And IV Functional Responses For Impulsive Pest Control Strategy

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***Abstract: Pest management is a matter of great concern nowadays. To achieve the same, a three tropic level food chain model is proposed considering plant, pest and natural enemies. Two different type of functional responses are taken for mid level and top level predator. Threshold value of the impulsive period is calculated for extinction of mid level predator using Floquet theory of impulsive differential equations, Lyapnuov functions and stroboscopic map. Mid level predator plays the role of pest. Permanence of system is also established. Some complex dynamics is also observed at higher value of impulsive period greater than threshold value. Further, validation of theoretical findings is done using MATLAB. Food chain Impulsive control strategy pest management Permanence mixed functional response***

## 1 Introduction

Preservation of non-renewable resources and protection of environment for coming generations while satisfying human requirements for fodder is the main aim of sustainable agriculture. It’s biggest component is pest management. In order to prevent major economic and production loss, it is the need of hour to control pest population. Pesticides are widely being used to eradicate pests [1,2]. But there are some big issues with use of pesticides. Firstly, these are responsible for environmental pollution up to great extent and identified as a health hazard to mankind. Secondly, aquatic bodies suffer due to water pollution caused by pesticides. Pesticides are harmful to beneficial insects such as pollinators. Further, due to high cost, small scale farmers are finding it hard to use chemical pesticides [3]. Moreover, after long term use, pests even became resistant to pesticides.

Therefore, chemical pesticides must be combined with some other pest control techniques to get maximum benefit and minimum loss. This is called Integrated Pest Management. Biological control is proved to be boon for te same. It includes identifying specific natural enemies of the targeted pest population. These enemies can be predators, parasites or some microbial control agents [4]. All these help to suppress growth of pest population. Natural enemies either kill the pests or hinder their biological process resulting in death of pests. Biological control is used for both open crop field crops and greenhouses. In Netherlands and United Kingdom, the parastoid *Encarsia Formosa* is used on wider scale to control tomato pest *Trialeurodes Vaporariorum* [5].

In this paper, pesticides are applied along impulsive release of natural enemies to manage the pest population. It is observed that many of these insect pests do not cause much damage in their native habitat. But, the problem becomes serious when they migrate into the region where there are no natural enemies. Hence, specified natural enemies can be reared or stocked under favorable environmental conditions and then released periodically in targeted regions to kill pests [6]. Therefore, in our work threshold value of impulsive period is calculated in order to check pest population. Since pesticides and natural enemies are released periodically, so this can be well analyzed using impulsive differential equations. There are plethora of applications of impulsive differential equations in Ecology and other applied sciences [9]. Also pest management can be studied effectively with the help of perturbed prey-predator interactions. Great achievements have been made by eminent researchers by considering prey as pest and natural enemies as predators.

Further, functional response of prey population to predator has an important role in predation. This response can be prey dependent (Holling type) or both prey and predator dependent (Beddington-DeAngelis type). Liu and Chen [10] analyzed Lotka-Volterra predator-prey system with impulsive perturbations using Holloing Type II functional response and studied the chaotic behavior of system. Zhang [11] established two pest-one natural enemy model, and found threshold value of impulsive period for pest free equilibrium. Similarly, valuable results have been obtained in [12, 13, 14, 15] considering food chain and food web models for impulsive pest control strategy. Zhang [16] studied the bifurcation analysis of prey-predator impulsive pest control model with Holling type IV functional response. He found that bifurcation depends on the impulsive release amount of natural enemies. Differnt threshold values of impulsive period have been obtained in [10, 18, 19] for permanence of the system.

Furthermore, Furthermore, good biological understanding of different life stages (immature larva, mature adult) of pests and natural enemies must be there for effectiveness of biological pest control. Hence, Jatav and Dhar [20-22] considered a stage structured (in natural enemies) plant-pest-natural enemy (food chain) model to find the conditions for permanence of the system. Again, Bhanu *et.al.* [23] extended the above work by analyzing stage- structure in pests also.

Motivated by above, a three tropic level plant-pest-natural enemy food chain model is developed using Holling type II and IV functional responses for impulsive pest control strategy. Pesticides and natural enemies are released periodically and simultaneously with impulsive period $\tau$ to manage pest population.

## 2 Mathematical model

The following predator-prey food chain model is proposed in this paper. Here, prey act as plant crop, mid level predator plays the role of pest and top predator is the specified natural enemy.

$$\left.\begin{cases} \frac{dx_c}{dt} = \alpha x_c\left(1-\frac{x_c}{\beta}\right) - \frac{\alpha_c x_c y_p}{1+\gamma_1 x_c}, \\ \frac{dy_p}{dt} = \frac{\alpha_c x_c y_p}{1+\gamma_1 x_c} - \frac{\alpha_p z_{ne} y_p}{1+\gamma_2 y_p^2} - \delta_1 y_p \,, \\ \frac{dz_{ne}}{dt} = \frac{\alpha_p z_{ne} y_p}{1+\gamma_2 y_p^2} - \delta_2 z_{ne} \,, \end{cases}\right\} t \neq n\tau, \tag{1}$$

$$\left.\begin{cases} \Delta y_p(t) = (1-\theta_1), \\ \Delta z_{ne}(t) = \theta_2, \end{cases}\right\} t = n\tau \,, n \in Z_+.$$

The above model is formulated under some assumptions as follows:
$(A_1)$The Prey (plant) grows logistically in the absence of predator.
$(A_2)$ Prey response to mid level predator is Holling type II and mid level prey response to top predator is Holling type IV.
$(A_3)$ Pesticides do not cause any harm to natural predators.
$(A_4)$ For the integrated pest control, pesticides and natural enemies are released periodically at time $t = n\tau$ with intensities $\theta_1, \theta_2$ respectively where $\tau$ is the impulsive period.

The different parameters used in (1) are defined as follows

1. $x_c(t), y_p(t), z_{ne}(t)$be the densities of prey, mid level predator and top predator at time t.
2. $\alpha > 0$ is the intrinsic reproduction rate of prey and$\beta > 0$ is the carrying capacity.
3. $\alpha_c > 0, \alpha_p > 0$ be the discovery rates by Holling and $\gamma_1 > 0, \gamma_2 > 0$ be the half saturation constants.
4. $\delta_1, \delta_2$ be the death rates of mid level and top predator.

## 3 Preliminaries

Let$R_+ = [0, \infty), R_+^3 = \{x \in R^3 : x \geq 0\}, \Omega = intR_+^3$. The map defined by the right hand of the system (1) is given as $g = (g_1, g_2, g_3)^T$ . Let $S_0 = \left\{V: R_+ \times R_+^3 \to R \; is \; continuous \; on \; (n\tau, (n+1)\tau] \times R_+^3 \; and \lim_{(t,y)\to(n\tau,x) t>n\tau} S(t,x) = S(n\tau^+, x) exists\right\}$.

### 3.1 Definition

If $S \in S_0$, then for $(t,x) \in (n\tau, (n+1)\tau] \times R_+^3$, the upper right derivative of $S(t,x)$ with respect to the impulsive differential system (1) is defined as

$$D^+(t,x) = \lim_{h\to 0^+} sup \frac{1}{h}\left[S\big(t+h, x+h(f,x)\big) - S(t,x)\right]. \tag{2}$$

### 3.2 Definition

Consider that$P(t) = (x_c(t), y_p(t), z_{ne}(t))^T$be the solution of (1). It is piece-wise continuous function from$R^+$ to$R_+^3$, because solution changes its behavior only at moments of impulse. Therefore, $P(t)$ is continuous in the interval $(n\tau, (n+1)\tau], n \in Z_+$ and $\lim_{t\to n\tau^+}\big(P(t)\big) = P(n\tau^+)$ exists also $\lim_{t\to n\tau^-}\big(P(t)\big) = P(n\tau)$ is true in case of IDE.

The required system (1) is said to be permanent if $\exists\, Q \geq q > 0$ such that $q < x_c(t), y_p(t), z_{ne}(t) < Q$ for sufficiently large$t$ and$P(0^+) > 0$.

Our main aim here is to suppress the pests in a targeted region beneath a tolerable limit so that it does not cause major production loss. To achieve the same, we need the following lemma.

**Lemma 1** Consider the following impulsive system:

$$\begin{cases} \frac{d\psi(t)}{dt} = -c\psi(t), \; t \neq n\tau, \\ \psi(t^+) = \psi(t) + d, \; t = n\tau, \; n \in Z^+. \end{cases} \tag{3}$$

It has periodic solution$\bar{\psi}(t)$ (globally stable) and for any solution$\psi(t)$of (3)

$$|\psi(t) - \bar{\psi}(t)| \to 0 \; as \; t \to \infty \; where \; \bar{\psi}(t) = \frac{d\exp(-c(t-n\tau))}{1-\exp(-c\tau)}.$$

## 4 Boundedness and Global Stability

### 4.1 Upper bound of all the variables

Here, in this section, firstly, upper bound for all the variables of system(1) are obtained in the coming lemma.

**Lemma 2**For sufficiently large$t$,there exists a constant$L > 0$ such that$x_c \leq L, y_p \leq L, z_{ne} \leq L$.. That is there is an upper bound for every solution of (1).

*Proof* .Suppose$(x_c(t), y_p(t), z_{ne}(t))$ be any solution of (1).
Let$Q(t) = x_c(t) + y_p(t) + z_{ne}(t)$then for$t \neq n$

$$D^+Q(t) + pQ(t) = \alpha x_c - \frac{\alpha x_c^2}{\beta} - \frac{\alpha_c x_c y_p}{1+\gamma_1 x_c} + \frac{\alpha_c x_c y_p}{1+\gamma_1 x_c} - \frac{\alpha_p z_{ne} y_p}{1+\gamma_2 y_p^2} + \frac{\alpha_p z_{ne} y_p}{1+\gamma_2 y_p^2}$$
$$-\delta_1 y_p - \delta_2 z_{ne} + p(x_c + y_c + z_{ne})$$
$$= (\alpha + p)x_c - \frac{\alpha x_c^2}{\beta} - (\delta_1 - p)y_p - (\delta_2 - p)z_{ne}$$

This implies $D^+Q(t) + pQ(t) \leq (\alpha + p)x_c - \frac{\alpha x_c^2}{\beta} \leq \frac{\beta}{4\alpha}(\alpha + p)^2 = L_0$.
$Q(n\tau^+) = Q(n\tau) + \theta_2$ for $t = n\tau$.

Therefore by Theorem 1.4.1 of [7],

$$Q(t) \leq Q(0)exp\left(\int_0^t (-p)ds\right) + \theta_2 \sum_{0<n\tau<t} exp\left(\int_{n\tau}^t (-p)ds\right) + \int_0^t \left(L_0 exp \int_s^t (-pd\sigma)\right) ds$$
$$\leq Q(0)\exp(-pt) + \theta_2 \sum_{0<n\tau<t} \exp\big(-p(t - n\tau)\big) + \frac{L_0}{\theta}(1 - \exp(-pt))$$
$$\leq Q(0)\exp(-pt) + \frac{L_0}{p}(1 - \exp(-pt)) + \frac{\theta_2 \exp(-p(t - n\tau))}{1 - \exp(-p\tau)} + \frac{\theta_2 \exp(pt)}{\exp(p\tau) - 1}$$
$$\to \frac{L_0}{p} + \frac{\theta_2 \exp(pt)}{\exp(p\tau) - 1} \quad as \ t \to \infty$$

This implies $Q(t) \leq L$ where $L = \frac{L_0}{p} + \frac{\theta_2 \exp(pt)}{\exp(p\tau) - 1}$.
Therefore, $Q(t)$ is uniformly bounded. Hence, ∃ the constant L such that $x_c \leq L, y_p \leq L, z_{ne} \leq L$.

**Lemma 3**If $V(t)$be any solution of system (1) with $V(0^+) \geq 0$, then $V(t) \geq 0$ for all $t \geq 0$. Also,$V(t) > 0$ for all$t \geq 0$ if $V(0^+) > 0$.

After using Chemical pesticides and natural enemies, when pest population becomes extinct, then$y_p = 0$, the impulsive system (1)reduces to

$$\left\{\begin{aligned} \frac{dx_c}{dt} &= \alpha x_c\left(1 - \frac{x_c}{\beta}\right) \\ \frac{dz_{ne}}{dt} &= -\delta_2 z_{ne}, \end{aligned}\right\} t \neq n\tau,$$

$\{\Delta z_{ne}(t) = \theta_2,\} t = n\tau, n \in Z_+$.
(4)

Now, first equation of (4) is simply logistic model. It has two equilibrium points 0 and$\beta$. $x_c = 0$ is unstable while$x_c = \beta$ is stable. Also, applying Lemma 1 on second and third equation of (4), we get globally asymptotically stable periodic solution$\bar{z}_{ne}$as

$$\bar{z}_{ne} = \frac{\theta_2 \exp((-\delta_2(t-n\tau))}{1-\exp(-\delta_2\tau)} \quad ; \quad \bar{z}_{ne}(0^+) = \frac{\theta_2}{1-\exp(-\delta_2\tau)} \tag{5}$$

Now, system (1) has two pest extinction equilibrium points $\left(0,0,\bar{z}_{ne}(t)\right)$ and $(\beta, 0, \bar{z}_{ne}(t)$

**Theorem 1**Let $(x_c(t), y_p(t), z_{ne}(t))$ be any solution of system (1), then

1.$\bar{X}_1 = (0,0,\bar{z}_{ne}(t))$is unstable.

2. There exists a threshold value $\tau_{max}$of the impulsive period such that if$\tau \le \tau_{max}$ , then the pest eradication solution$\bar{X}_2 = (\beta, 0, \bar{z}_{ne}(t))$is locally asymptotically stable and if$\tau > \tau_{max}$ , it is unstable where,

$$\tau_{max} = \left(\frac{\theta_2\alpha_p}{\delta_2} - \ln(1-\theta_1\right)\left(\frac{1+\gamma_1\beta}{\alpha_c\beta - \delta_1 - \delta_1\gamma_1\beta}\right).$$

*Proof* 1. Here, we use small perturbation method to prove the local stability of the required solution. Let$\zeta_1(t), \zeta_2(t), \zeta_3(t)$be the small perturbations in$0,0,\bar{z}_{ne}(t)$respectively. Then

$$x_c(t) = \zeta_1(t), y_p(t) = \zeta_2(t), z_{ne}(t) = \bar{z}_{ne}(t) + \zeta_3(t).$$

Putting these values in system (1) and after linearisation, it reduces to

$$\left\{\begin{array}{c} \frac{d\zeta_1(t)}{dt} = \alpha\zeta_1(t), \\ \frac{d\zeta_2(t)}{dt} = -(\alpha_p\bar{z}_{ne}(t) + \delta_1)\zeta_2(t)\,, \\ \frac{d\zeta_3(t)}{dt} = \alpha_p\zeta_2(t)\bar{z}_{ne}(t) - \delta_2\zeta_3(t)\,, \end{array}\right\} t \ne n\tau,$$

$$\left\{\begin{array}{c} \zeta_1(t^+) = \zeta_1(t), \\ \zeta_2(t^+) = (1-\theta_1)\zeta_2(t), \\ \zeta_3(t^+) = \zeta_3(t), \end{array}\right\} t \ne n\tau,\ n \in Z_+. \tag{6}$$

Then (6)represents system of linear differential equations, which can be written in matrix form. Hence for$t = n\tau,$ the coefficient matrix is given as

$$B = \begin{bmatrix} \alpha & 0 & 0 \\ 0 & -(\alpha_p\bar{z}_{ne}(t) + \delta_1) & 0 \\ 0 & \alpha_p\bar{z}_{ne}(t) & -\delta_2 \end{bmatrix}$$

and for$t \ne n\tau$

$$\begin{bmatrix} \zeta_1(n\tau^+) \\ \zeta_2(n\tau^+) \\ 3(n\tau^+) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1-\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \zeta_1(n\tau) \\ \zeta_2(n\tau) \\ \zeta_3(n\tau) \end{bmatrix}$$

Let$\phi(t)$be the fundamental solution of (6), then,

$$\frac{d\phi(t)}{dt} = B\phi(t)$$

$$\phi(\tau) = \phi(o)exp\left(\int_0^\tau Bdt\right) \tag{7}$$

With$\phi(0) = I,$ the identity matrix. On solving, we have ,

$$\phi(\tau)=\begin{bmatrix} exp\left(\int_0^\tau \alpha dt\right) & 0 & 0 \\ 0 & exp\left(\int_0^\tau -(\alpha_p \bar{z}_{ne}(t)+\delta_1\right)dt & 0 \\ 0 & exp\left(\int_0^\tau (\alpha_p \bar{z}_{ne}(t)\right)dt & exp\left(\int_0^\tau -\delta_2 dt\right)dt \end{bmatrix}$$

Now according to Floquet Theory of impulsive differential equations(Theorem 3.1and 3.5 of [8]), if absolute values of all the eigen values of Monodromy matrix$M$ are less than one, then the required solution is globally stable where,

$$M=\begin{bmatrix}1 & 0 & 0\\ 0 & 1-\theta_1 & 0\\ 0 & 0 & 1\end{bmatrix}\phi(\tau)$$

The eigen values of$M$are

$$\lambda_1 = exp\left(\int_0^\tau \alpha dt\right),$$

$$\lambda_2=(1-\theta_1)exp\left(\int_0^\tau -(\alpha_p\bar{z}_{ne}(t)+\delta_1\right)dt,$$

$$\lambda_3 = exp\left(\int_0^\tau -\delta_2 dt\right)dt$$

(8)

Now, it is obvious from (8), that$|\lambda_1|>1$ $(since\ \alpha>0)$. Hence the equilibrium$(0,0,\bar{z}_{ne}(t))$is unstable.

2. Similarly, we can discuss the local stability of second pest extinction equilibrium point$\left(\beta,0,\bar{z}_{ne}(t)\right)$. Here

$$x_c(t)=\beta+\zeta_1(t), y_p(t)=\zeta_2(t), z_{ne}(t)=\bar{z}_{ne}(t)+\zeta_3(t).$$

Proceeding similarly as above, the Monodromy matrix$M$in this case is

$$M=\begin{bmatrix}1 & 0 & 0\\ 0 & 1-\theta_1 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}-\alpha & -\frac{\alpha_{c\beta}}{1+\gamma_1\beta} & 0\\ 0 & \frac{\alpha_{c\beta}}{1+\gamma_1\beta}-(\alpha_p\bar{z}_{ne}(t)+\delta_1) & 0\\ 0 & \alpha_p\bar{z}_{ne}(t) & -\delta_2\end{bmatrix}$$

The eigen values of $M$ are

$$\lambda_1=-\alpha\tau<1,$$

$$\lambda_2=(1-\theta_1)exp\left(\int_0^\tau \frac{\alpha_{c\beta}}{1+\gamma_1\beta}-(\alpha_p\bar{z}_{ne}(t)+\delta_1\right)dt,$$

$$\lambda_3=-\delta_2\tau<1$$

(9) Now, it is obvious from (9), that $|\lambda_1|\le 1, |\lambda_3|\le 1$ and$|\lambda_2|\le 1$ if $\tau\le\tau_{max}$. Hence the required result.

### 4.2 Global Stability

**Theorem 2** There is a threshold value $(\check{\tau})$ of the impulsive period such that if $\tau < \check{\tau}$ then the pest eradication solution $(\beta, 0, \bar{z}_{ne}(t))$ is globally asymptotically stable where,

$$\check{\tau} = \left(\frac{\theta_2 \alpha_p}{\delta_2} - \ln(1-\theta_1)\right)\left(\frac{1}{\alpha_c \beta - \delta_1}\right).$$

*Proof.* Let $(x_c(t), y_p(t), z_{ne}(t))$ be arbitrary solution of (1). Given that $\tau < \check{\tau}$, so, it is possible to find sufficiently small $\tilde{\varepsilon}_1 > 0$ uch that

$$\int_0^{\tau} (\alpha_c (\beta + \tilde{\varepsilon}_1) - \alpha_p(\bar{z}_{ne}(t) + \tilde{\varepsilon}_1) - \delta_1) dt = \rho_1 < 0 \quad (10)$$

From (1),

$$\frac{dx_c}{dt} \le \alpha x_c \left(1 - \frac{x_c}{\beta}\right) \quad (11)$$

Consider its comparison system

$$\frac{du_c}{dt} = \alpha u_c \left(1 - \frac{u_c}{\beta}\right) \quad (12)$$

Using comparison theorem of ordinary differential equations, $x_c \le u_c \to \beta$ as $t \to \infty$. Therefore, $x_c \le \beta + \tilde{\varepsilon}_1$ for $t \ge \kappa_1 \tau$. Again from system (1)

$$\begin{cases} \dfrac{dz_{ne}}{dt} \ge -\delta_2 z_{ne}, & t \ne n\tau, \\ \Delta z_{ne}(t) = \theta_2, & t = n\tau, \ n \in Z_+ . \end{cases} \quad (13)$$

Using comparison analysis technique of impulsive differential equations and applying lemma 1, solution of (13) satisfies $z_{ne}(t) \ge \bar{z}_{ne}(t) - \tilde{\varepsilon}_1 \ \forall \ t \ge \kappa_2 \tau$. Again from (1)

$$\begin{cases} \dfrac{dy_p}{dt} \ge (\alpha_c(\beta + \tilde{\varepsilon}_1) - \delta_1 - \alpha_p(\tilde{z}_{ne} - \tilde{\varepsilon}_1)), & t \ne n\tau, \\ \Delta y_p(t) = -\theta_1 y_p(t), & t = n\tau, \ n \in Z_+ . \end{cases} \quad (14)$$

Integration of first equation of (14) on $(\kappa_2 \tau, (\kappa_2 + 1)\tau]$ gives

$$y_p(\kappa_2 + 1)\tau \le y_p(\kappa_2 \tau)\exp(\rho_1) \ \text{ where } \rho_1 \text{ is given by (10).} \quad (15)$$

After using impulsive factor from (14), we obtain the stroboscopic map

$$y_p(\kappa_2 + 1)\tau \le (1-\theta_1) y_p(\kappa_2 \tau)\exp(\rho_1). \ \text{ This implies}$$

$$y_p(\kappa_2 + q)\tau \le (1-\theta_1)^q y_p(\kappa_2 \tau)\exp(q\rho_1) \to 0 \ as \ t \to \infty (\rho_1 < 0 \ from \ (10)). \quad (16)$$

This implies, there exists a positive integer $\kappa_3 > \kappa_2$ and sufficiently small $\tilde{\varepsilon}_2 > 0$ such that $y_p(t) < \tilde{\varepsilon}_2$ for $t \ge \kappa_3 \tau$ and $\tilde{\varepsilon}_2 < \frac{\delta_2}{\alpha_p}$. Using maximum value of $y_s(t)$ in the first equation of system (1), we get

$$\frac{dx_c}{dt} \ge \alpha x_c \left(1 - \frac{x_c}{\beta} - \alpha_c \tilde{\varepsilon}_2\right).$$

So, $\lim_{t \to \infty} x_c = \beta$. This implies $x_c \to \beta$ as $t \to \infty$. Again from system (1)

$$\begin{cases} \dfrac{dz_{ne}}{dt} \le (\alpha_p \tilde{\varepsilon}_2 - \delta_2) z_{ne}, & t \ne n\tau, \\ \Delta z_{ne}(t) = \theta_2, & t = n\tau, \ n \in Z_+ . \end{cases} \quad (17)$$

By using comparison analysis technique of impulsive differential equations and applying lemma 1, (17) has periodic solution

$$\bar{w}_{ne} = \frac{\theta_2 \exp(-(\delta_2 - \alpha_p \tilde{\varepsilon}_2)(t - n\tau))}{1 - \exp(-(\delta_2 - -\alpha_p \tilde{\varepsilon}_2)\tau)} \quad ; \quad \bar{w}_{ne}(0^+) = \frac{\theta_2}{1 - \exp\left(-\left(\delta_2 - -\alpha_p \tilde{\varepsilon}_2\right)\tau\right)}$$

.

such that $z_{ne}(t) < \bar{w}_{ne} - \tilde{\varepsilon}_3$ for all $t \ge \kappa_4 \tau$ . As $\tilde{\varepsilon}_1, \tilde{\varepsilon}_2, \tilde{\varepsilon}_3 > 0$ are sufficiently small, therefore$\overline{w}_{ne} \to \bar{z}_{ne}$as$t \to \infty$. Hence it is established that$x_c \to \beta, y_p \to 0$ and$z_{ne} \to \bar{z}_{ne}$as$t \to \infty$.

## 5 Permanence

Firstly, condition for the system to be permanent is established as follows

**Theorem 3** The system (1) is permanent if$\tau > \check{\tau}$.

*Proof.* Upper bound of all the variables $x_c(t), y_p(t), z_{ne}(t)$of the system (1) is already been obtained in lemma 2. Also in the above section, it is proved that

$$z_{ne}(t) \geq \bar{z}_{ne}(t) - \tilde{\varepsilon}_1 = r_1 \forall\ t \geq \kappa_2\tau$$

Also$\frac{dx_c}{dt} \geq \alpha x_c\left(1 - \frac{x_c}{\beta} - \alpha_c L\right)$ . This implies$x_c > (1 - \alpha_c L)\beta = r_2$. (18)

for sufficiently large $t$. Thus, for permanence of the system(3), there must exists a constant$r_3 < \frac{\delta_2}{\alpha_p}$such that$y_p(t) \geq r_3$for sufficiently large$t$. This is done in two steps as follows

**Step I**

To start with, assume that$y_p(t) \geq r_3$is not true$\forall\ t$. Thus$\exists\ \tilde{t}_1$such that $y_p(t) < r_3\ \forall\ t \geq \tilde{t}_1$. Considering this assumption, from (1), we have

$$\begin{cases} \dfrac{dz_{ne}}{dt} \leq (\alpha_p r_3 - \delta_2) z_{ne}\ , & t \neq n\tau, \\ \Delta z_{ne}(t) = \theta_2, & t = n\tau,\ \ n \in Z_+ . \end{cases}$$

Consider the following impulsive system

$$\begin{cases} \frac{du_{ne}}{dt} = (\alpha_p r_3 - \delta_2) z_{ne}\ , & t \neq n\tau, \\ \Delta u_{ne}(t) = \theta_2, & t = n\tau,\ \ n \in Z_+ . \end{cases} \tag{19}$$

Applying lemma 1, (19) has periodic solution

$$\bar{u}_{ne}(t) = \frac{\theta_2 \exp\left(-\left(\delta_2 - \alpha_p r_3\right)(t - n\tau)\right)}{1 - \exp\left(-\left(\delta_2 - -\alpha_p r_3\right)\tau\right)}\ , t \in (n\tau, (n+1)\tau]\ ;$$

where,

$$\bar{u}_{ne}(0^+) = \frac{\theta_2}{1 - \exp\left(-\left(\delta_2 - -\alpha_p r_3\right)\tau\right)}$$

which is globally asymptotically stable. Therefore by Theorem 1.4.1 of [7],$z_{ne}(t) \leq u_{ne} \to \bar{u}_{ne}$ Hence,$\exists$ a positive integer$\kappa_5$such that

$$z_{ne}(t) < \bar{u}_{ne}(t) + \tilde{\varepsilon}_4 = r_1 \forall\ t \geq \kappa_5\tau$$

(20) Therefore,$x_c > r_2$ implies that for$t \geq \kappa_5\tau$, we have the following subsystem of (1)

$$\begin{cases} \frac{dy_p}{dt} \geq (\alpha_c r_2 - \delta_1 - \alpha_p(\bar{u}_{ne} + \tilde{\varepsilon}_4)) y_p\ , & t \neq n\tau, \\ \Delta y_p(t) = -\theta_1 y_p(t), & t = n\tau,\ \ n \in Z_+ . \end{cases} \tag{21}$$

Integration of first equation of (14) on $(\kappa_5\tau, (\kappa_5 + 1)\tau]$ gives the stroboscopic map

$$y_p(\kappa_5 + 1)\tau \geq y_p(\kappa_5\tau)(1 - \theta_1)\exp\left(\int_{\kappa_5}^{(\kappa_5+1)\tau} (\alpha_c r_2 - \delta_1 - \alpha_p(\tilde{u}_{ne} + \tilde{\varepsilon}_4))dt\right)$$

$\geq y_p(\kappa_5\tau)(1 - \theta_1)\exp(\rho_2)$where

$$\rho_2 = (1-\theta_1)\exp\left(\int_{\kappa_5}^{(\kappa_5+1)\tau} (\alpha_c r_2 - \delta_1 - \alpha_p(\tilde{u}_{ne} + \tilde{\varepsilon}_4))dt\right).$$

Because $\tau > \check{\tau}$, so it is possible to find $r_2$ and $\tilde{\varepsilon}_4 > 0$ such that $\rho_2 > 1$. This implies

$$y_p(\kappa_5 + q)\tau \geq y_p(\kappa_2\tau)\exp(q\rho_1) \to \infty \ as \ q \to \infty.$$

This is in contradiction to our assumption that $y_p(t) < r_3 \ \forall \ t \geq \tilde{t}_1$.. HenceThus $\exists \ \tilde{t}_2 > \tilde{t}_1$ such that $y_p(\tilde{t}_2) \geq r_3$.

**Step II**

There is nothing to prove if $y_p(t) \geq r_3 \forall \ t \geq \tilde{t}_2$. But if this is not the situation, let $\tilde{t}_3 = inf\{t,\ y_p(t) < r_3; t > \tilde{t}_2\}$. Thus $y_p(t) \geq r_3 \forall \ t \in [\tilde{t}_2, \tilde{t}_3], \tilde{t}_3 \in (\check{n}_1\tau, (\check{n}_1+1)\tau$ . $y_p(\tilde{t}_3) = r_3$ because of continuity of $y_p(t)$. Let $\tau^* = (\check{n}_2 + \check{n}_3)\tau$ where $\check{n}_2$ and $\check{n}_3$ satisfies the following conditions

$$\check{n}_2\tau > -\left(\frac{1}{\delta_2 - \alpha_p r_3}\right) ln\left(\frac{\tilde{\varepsilon}_4}{L+\theta_2}\right)$$

$$(1-\theta_1)^{(\check{n}_2+\check{n}_3+1)} \exp(\check{n}_3\rho_3 + \mu(\check{n}_2+1)\tau) > 1, \mu = (\alpha_c r_2 - \alpha_p L - \delta_1) < 0 \quad (22)$$

Now, we will prove that $\exists \ t_4 \in ((\check{n}_1+1)\tau, (\check{n}_1+1)\tau + \tau^*]$ such that $y_p(\tilde{t}_4) \geq r_3$.

Suppose this is not true, then $y_p(\tilde{t}_4) < r_3 \ \forall \ t_4 \in ((\check{n}_1+1)\tau, (\check{n}_1+1)\tau+\tau^*]$. If system (19) is considered with $u_{ne}((\check{n}_1+1)\tau^+) = z_{ne}((\check{n}_1+1)\tau^+)$ then using lemma 1 for $t \in ((\check{n}_1+1)\tau, (\check{n}_1+1)\tau+\tau^*]$, we have

$$u_{ne}(t) = \left[u_{ne}((\check{n}_1+1)\tau^+) - \frac{\theta_2}{1-\exp(-(\delta_2 - -\alpha_p r_3)\tau)}\right]\exp(-(\delta_2 - \alpha_p r_3)(t - (\check{n}_1 + 1)\tau)) + \bar{u}_{ne}(t)$$

This implies $|u_{ne}(t) - \bar{u}_{ne}(t)| \leq (L+\theta_2)\exp(-(\delta_2 - \alpha_p r_3)(t - (\check{n}_1+1)\tau)) \leq \tilde{\varepsilon}_4$(by(22)

which depicts that $z_{ne}(t) \leq u_{ne}(t) < \bar{u}_{ne}(t) + \tilde{\varepsilon}_4, (\check{n}_1 + \check{n}_2 + 1)\tau \leq t \leq (\check{n}_1+1)\tau + \tau^*$.

Integrating (21)on$[(\check{n}_1+\check{n}_2+1)\tau, (\check{n}_1+\check{n}_2+\check{n}_3+1)\tau]$ we get

$$y_p(\check{n}_1+\check{n}_2+\check{n}_3+1)\tau \geq y_p(\check{n}_1+\check{n}_2+1)\tau(1-\theta_1)^{\check{n}_3}\exp(\rho_3\check{n}_3) \quad , \quad (23)$$

where $\rho_3 = \int_{\kappa_5}^{(\kappa_5+1)\tau}(\alpha_c r_2 - \delta_1 - \alpha_p(\tilde{u}_{ne} + \tilde{\varepsilon}_4))dt$.

Further, for $t \in [\tilde{t}_3, (\check{n}_1+1)\tau]$ two possibilities are there

**Case(i)**

If $y_p(t) \leq r_3 \ \forall \ t \in [\tilde{t}_3, (\check{n}_1+1)\tau]$ then from above assumption $y_p(t) \leq r_3 \ \forall \ t \in [\tilde{t}_3, (\check{n}_1+1)\tau+\tau^*]$. This implies

$$\begin{cases} \frac{dy_p}{dt} \geq (\alpha_c r_2 - \alpha_p L - \delta_1 - \alpha_p)y_p, & t \neq n\tau, \\ \Delta y_p(t) = -\theta_1 y_p(t), & t = n\tau, \ n \in Z_+. \end{cases} \quad (24)$$

On integrating equation (24)in$[\tilde{t}_3, (\check{n}_1+\check{n}_2+1)\tau]$, we obtain the stroboscopic map

$$y_p(\check{n}_1+\check{n}_2+1)\tau \geq y_p(\tilde{t}_3)(1-\theta_1)^{\check{n}_2+1}\exp(-\mu(\check{n}_2+1)\tau) > r_3 \quad (25)$$

Using (25) in (24) , we have

$$y_p(\check{n}_1+\check{n}_2+\check{n}_3+1)\tau \geq y_p(\tilde{t}_3)(1-\theta_1)^{\check{n}_2+\check{n}_3+1}\exp(\rho_3\check{n}_3)\exp(-\mu(\check{n}_2+1)\tau) > r_3$$

But this contradicts our assumption. Therefore, $y_p(t) \geq r_3$ in $[\tilde{t}_3, (\check{n}_1+\check{n}_2+\check{n}_3+1)\tau]$ for some $t$. Let $\tilde{t}_5 = inf\{t,\ y_p(t) \geq r_3; t > \tilde{t}_4\}$ Due to continuity of $y_p(t), y_p(\tilde{t}_5) = r_3$. Now integration of equation (24) on the interval $[\tilde{t}_3, \tilde{t}_5]$ gives

$$y_p(t) \geq y_p(\tilde{t}_3)(1-\theta_1)\exp((\mu(t-\tilde{t}_3))$$
$$\geq r_3(1-\theta_1)\exp((\mu(t-\tilde{t}_3))$$
$$\geq r_3(1-\theta_1)\exp(\mu(\breve{n}_2+\breve{n}_3+1)\tau) = \overline{r_3} \ .$$

Since$y_p(\tilde{t}_5) \geq \overline{r_3}$ so similar process can be continued for$t > \tilde{t}_5$. Hence$y_p(t) \geq \overline{r_3}\ \forall\ t > \tilde{t}_2$.

**Case(ii)**

If $\exists \tilde{t}_6 \in [\tilde{t}_3, (\breve{n}_1+1)\tau]$ such that $y_p(\tilde{t}_6) \geq r_3$ , then let $\tilde{t}_7 = inf\{t\ ,\ y_p(t) \geq r_3; t > \tilde{t}_6\}$ Therefore,$y_p(\tilde{t}_6) = r_3$ Again, integration of equation (24) on the interval$\tilde{t}_3, \tilde{t}_7$gives

$$y_p(t) \geq y_p(\tilde{t}_3)(1-\theta_1)\exp((\mu(t-\tilde{t}_3)) \geq \bar{r}_3$$

Similar argument can be followed for$t > \tilde{t}_7$ Hence, it is concluded that$y_p(t) \geq \bar{r}_3 \forall t > \tilde{t}_2$.

**Step III**

Let$a = min\{r_1, r_2, r_3\}, \Theta = \{R_+^3 : a \leq x_c(t), y_p(t), z_{ne}(t) \leq L\}$ Thus, from above steps and Lemma 2, it is proved that each solution of system (1) will always remain in region $\Theta$. Therefore, by Definition3.2, system(1) is permanent.

## 6 Numerical Analysis and Discussion

A prey-predator food chain model with harvesting of middle prey and stocking of top predator is constituted and investigated in this paper to tackle with outbreak of pest population.Mid level prey is taken as pest and top predator plays the role of natural enemy. Use of pesticides is combined with impulsive release of natural enemies for Integrated pest management. Firstly, global stability of mid level predator (pest) free solution is established and then condition for the permanence of system is derived. For this, threshold value of impulsive period is found that depends on releasing amounts of pesticides and natural enemies population. The initial values of population densities of prey, mid level and top predator are$x_c(0^+) = 0.5, y_p(0^+) = 0.5, z_{ne}(0^+) = 1$. The values of different parameters that are used in system(1) are given in Table 1.

Table 1: Values of different parameters used in system(1)

| Parameter | Representation | Its Value (per week) |
|---|---|---|
| $\alpha$ | Reproduction rate of susceptible pest | 1.1 |
| $\beta$ | Carrying capacity | 1.1 |
| $\alpha_c$ | Predation rate by mid level predator (pest) | 0.9 |
| $\alpha_p$ | Rate of predation by top predator (natural enemy) | 0.9 |
| $\gamma_1$ | half saturation constant for Holling II predation | 0.1 |
| $\gamma_2$ | half saturation constant for Holling IV predation | 0.2 |
| $\delta_1$ | death rate of mid level predator | 0.4 |
| $\delta_2$ | Death rate of top predator (natural enemy) | 0.6 |
| $\theta_1$ | impulsive spraying amount of pesticides | 0.1 |
| $\theta_2$ | impulsive release amount of natural enemies | 3 |

On calculating, we get$\check{\tau} = 7.805$and$\tau_{max} = 9.256653$. Therefore, by Theorem 1, it is obtained that pest eradication solution is locally stable if$\tau \leq 9.25663$ . Also, theorem 2 is verified here that is the pest free solution is globally stable if impulsive period$\tau \leq 7.805$ (see Fig. 1). Hence impulsive perturbations contribute a lot to the dynamics of the system since some complex dynamics is there at higher values of impulsive period greater than threshold value.Thus, combination of chemical and natural control is very effective for pest control.

## 7 Conclusion

The war between pests and humans is going on from several decades and time to time, different pest control techniques are acquired by mankind. Working on the same path, here we investigated a predator-prey three tropic level model for the purpose of integrated pest management. It is found that instead of using pesticides alone, combination of chemical control along with natural enemies is more efficient in pest control. In Theorem 3, threshold value of impulsive period ($\check{\tau}$)is obtained and it is established that pests can coexist with infected pests and natural enemies if$\tau > \check{\tau}$.Also effect of spraying amount of pesticides and natural enemies is discussed and found that greater releasing amount or small impulsive period support pest eradication.

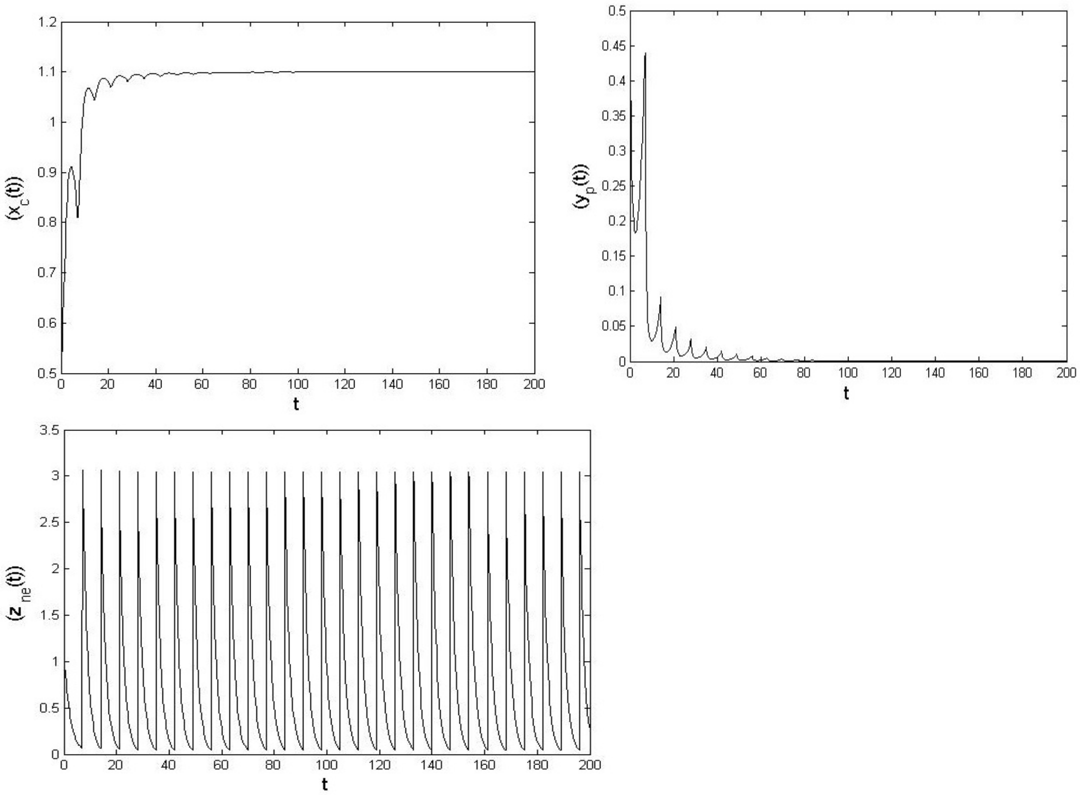
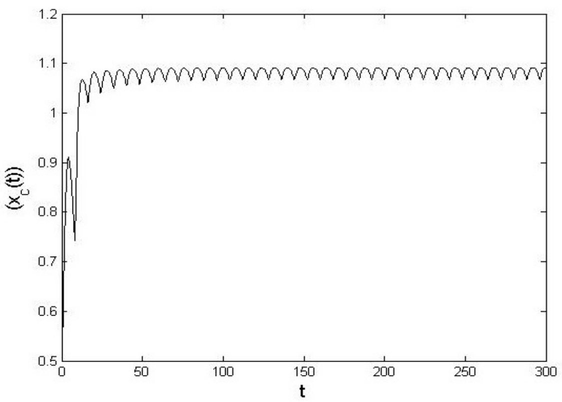
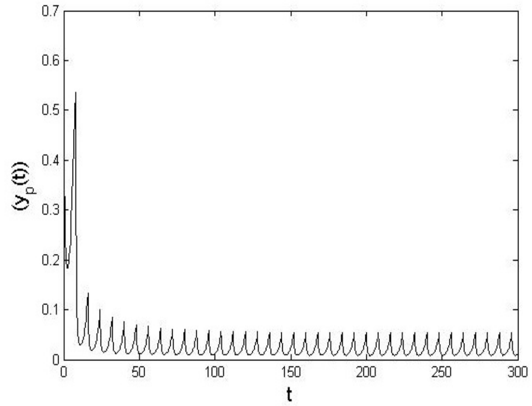
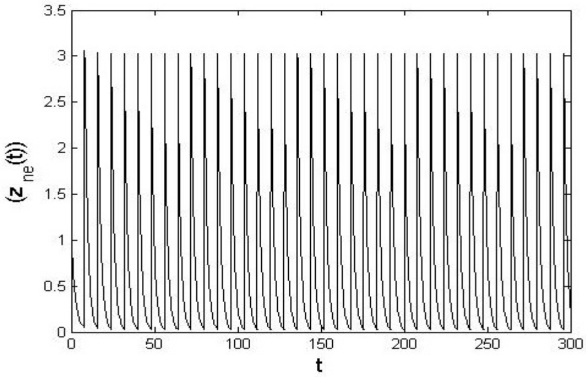
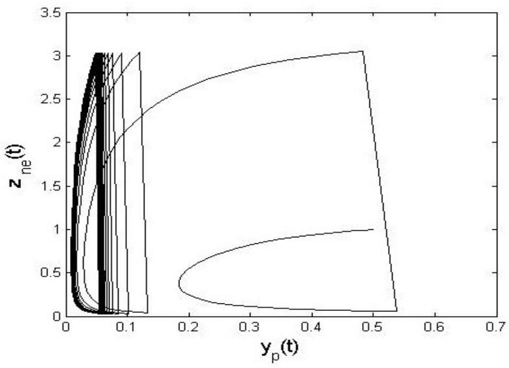



Figure 1: Global stability of pest extinction periodic solution$(0, y_p(t), z_{ne}(t))$ of system (1) at$\tau < \check{\tau}(= 7.805)$.


(x_c(t))
t


(y_p(t))
t


(z_ne(t))
t


z_ne(t)
y_p(t)

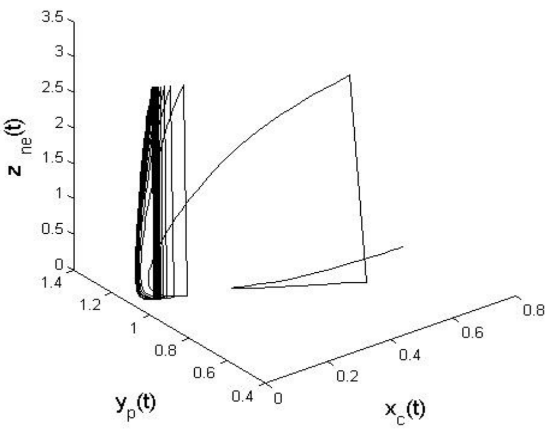



Figure 2: Permanance of the system (1) at $\tau < \check{\tau}(= 7.805)$ with $x_c(0^+) = 0.5, y_p(0^+) = 0.5, z_{ne}(0^+) = 1$, phase portrait of mid level predator and top predator and phase potrait when system (1) is permanent.

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