Closure , Interior , Neighborhood , Limit Point Of Generalized Maximal Closed Sets In A Topological Space

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Abstract

In this paper we presented closure , interior & neighbourhood of a generalized maximal closed in a topological space. g-m_a -closure ($cl_{g\text{-}ma}(M)$) of M ,where $M \subseteq Y$ in a topological space(Y,η) is the intersection of all g-m_a closed containing M , Simarily g-m_a -interior ($int_{g\text{-}mi}(M)$)of M is the union of all g-m_i open contained in M. Let $N \subseteq Y$ in a topological space (Y,η) is g-m_a- neighborhood of a point $y \in Y$ if $\exists$ a g-m_i open set E $\ni y \in E \subseteq N$. Let $N_{g\text{-}ma}$ be collection of all g-m_a neighborhoods. A point $y \in Y$ is g-m_a limit point of a subset L of (Y,η)if and only if $[E-\{y\}] \cap L \neq \phi$ for each g-m_i open E containing y.

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1. INTRODUCTION

Study of g-closed sets was done by N.Levine and Dunham [1] [2]. Maximal open & Minimal open sets were studied and introduced by F.Nakaoka and N.Oda [5] [4] [3]. S.S. Benchalli ,Suwarnlatha N. Banasode and G. P. Siddapur [6] introduced and characterized generalized minimal closed sets. Further generalized minimal closed was introduced in bitopological spaces by Suwarnlatha N. Banasode & Mandakini Desurkar [7]. The concept of generalized Maximal Closed set[10] was introduced and characterized by Suwarnlatha N. Banasode and Mandakini Desurkar.

2. Preliminaries

In the sequel (Y, η) represents a nonempty topological space on which no separation axioms are assumed unless otherwise explicitly stated.

The compliment, closure and the interior of A ,where A $\subseteq$ Y in a topological space (Y, η) is denoted by A^c , cl (A) and int (A) respectively. Here $int^*(A)$ denotes the interior of generalized open set A and $cl^*(A)$ denotes the closure of generalized closed set A.

Definition:

- Any open (resp closed) E, where is E is a proper subset of (Y, η) is maximal open [5] (resp. maximal closed) set if it contains either E or X.
- Any open (resp. closed) E, where is E is a proper subset of (Y, η) is minimal open [5] if it is contained in E, is either E or ϕ.

Definition:

- A subset E is generalized closed [2](in brief g-closed) if cl (E) $\subseteq$ V when E $\subseteq$ V and V is an open .
- A subset E is generalized open [2](in brief g-open) set if and only if E^c is a generalized closed.
- A subset E is ω-closed [8] if cl (E) $\subseteq$ V when E $\subseteq$ V & V is a semi open .
- An ω-open if & only if E^c is a ω-closed .
- A generalized minimal [6] closed (in brief g-m_i closed) set if cl (E) $\subseteq$ V whenever E $\subseteq$ V and V is a minimal open .
- A α -closed [9] if cl(int (cl (E))) $\subseteq$ E.
- A generalized maximal closed set [10] , if cl (M) $\subseteq$ V whenever M $\subseteq$ V and U is a maximal open.
- For a subset B of (Y, η) , cl^* (B) is the intersection of all the g-closed sets [2] containing B.

3. g-m_a- Closure and g-m_a- Interior

In this section we introduce and characterize generalized maximal (g-m_a) closure and generalized maximal (g-m_a) interior .

Definition III.1: For M $\subseteq$ Y in (Y,η), g-m_a -closure of a subset M of Y is intersection of all g-m_a closed containing M and represented as $cl_{g\text{-}ma}(M)$.

Example III.2: If Y = {a_1, c_1, e_1, g_1} and η = { ϕ,{c_1},{g_1},{c_1, g_1}, {a_1, e_1, g_1},Y}.Let A ={a_1} then $cl_{g\text{-}ma}(A)$ ={a_1}.

Theorem III.3: For any y ϵ Y where (Y,η) is a topological space, yϵ $cl_{g\text{-}ma}(M)$ if & only if M∩ E ≠ϕ for each g-m_i open E containing y.

Proof : let yϵ $cl_{g\text{-}ma}(M)$. Take E as a g-m_i open containing y ∍ M ∩ E = ϕ ⇒ M $\subseteq$ Y -E . Therefore
$cl_{g\text{-}ma}(M) \subseteq$ Y-E ⇒ y $\notin$ $cl_{g\text{-}ma}(M)$ which is a contradiction .Thus M∩E ≠ ϕ ∀ g-m_i open set E containing y.

Conversely, suppose y $\notin$ $cl_{g\text{-}ma}(M)$ then ∃ a g-m_a closed set V containing M ∍ y $\notin$ V. Then y ∈ Y-V and Y-V is g-m_i open set. Thus M∩(Y-V) = ϕ ,which is a contradiction .Thus y ∈ $cl_{g\text{-}ma}(M)$.

Remark III.4: If M $\subseteq$ Y then M $\subseteq$ $cl_{g\text{-}ma}(M) \subseteq$ cl(M).

Example III.5: Let Y= $\{b_1, f_1, h_1, m_1\}$ and $\eta = \{\phi,\{f_1\},\{m_1\},\{f_1,m_1\},\{b_1,h_1,m_1\},Y\}$. If M = $\{h_1,m_1\}$ then cl(M) = $\{b_1,h_1,m_1\}$ and $cl_{g\text{-}ma}(M) = \{h_1,m_1\}$.Thus $M\subseteq cl_{g\text{-}ma}(M)\subseteq cl(M)$.

Theorem III.6: If Q is g-m_a closed , cl(g-m_a(Q))= Q.

Proof: Let M be g-m_a closed . Since $M\subseteq M$ & M is g-m_a closed set, it belongs to all g-m_a closed containing M. This implies M is equal to intersection of all g-m_a closed containing M. Thus M = $cl_{g\text{-}ma}(M) \subseteq M$. Therefore $cl_{g\text{-}ma}(M) = M$.

Remark III.7: For any subsets M of Y , $cl_{g\text{-}ma}(M) \neq cl(M)$.

Example III.8: Let $Y=\{k_1,m_1,p_1\}$ and $\eta =\{ \phi,\{k_1\},\{m_1,p_1\},Y\}$. Let M =$\{m_1\}$ be g-m_a closed set. Clearly $cl_{g\text{-}ma}(M) = \{m_1\}$ and $cl(M) = \{m_1,p_1\}$. Thus $cl_{g\text{-}ma}(M) \neq cl(M)$.

Remark III.9: For any subsets M & N of Y, if $M \subseteq N$ then $cl_{g\text{-}ma}(M) \neq cl_{g\text{-}ma}(N)$

Example III.10: Let$Y=\{k_1,m_1,p_1,l_1\}$and

$\eta=\{\phi,\{k_1\},\{m_1\},\{p_1\},\{k_1,m_1\}, \{m_1,p_1\},\{k_1,p_1\},\{m_1,l_1\},\{k_1,m_1,p_1\},\{k_1,m_1,l_1\},\{m_1,p_1,l_1\},Y\}$.
Let M =$\{m_1\}$ and N =$\{m_1,p_1,l_1\}$.Clearly $M\subseteq N$ but $cl_{g\text{-}ma}(M) = \{m_1,l_1\}$ and $cl_{g\text{-}ma}(N) = \{k_1,m_1,l_1\}$.
Therefore $cl_{g\text{-}ma}(M) \neq cl_{g\text{-}ma}(N)$.

Remark III.11: For any g-m_a closed sets M and N if $M\subseteq N$ then $cl_{g\text{-}ma}(M) \subseteq cl_{g\text{-}ma}(N)$.

Example III.12: In Example III.10, the set M =$\{p_1,l_1\}$ and N=$\{m_1,p_1,l_1\}$ be g-m_a closed sets then $M\subseteq N$. Now $cl_{g\text{-}ma}(M) =\{p_1,l_1\}$and $cl_{g\text{-}ma}(N) =\{m_1,p_1,l_1\}$. Hence $cl_{g\text{-}ma}(M) \subseteq cl_{g\text{-}ma}(N)$.

Remark III.13: For M , N $\subseteq$ Y , $cl_{g\text{-}ma}(M) = cl_{g\text{-}ma}(N)$ does not imply M = N.
Example III.14: In Example III.10 , the set M =$\{m_1\}$ and N =$\{m_1,l_1\}$ then $cl_{g\text{-}ma}(M) =\{m_1,l_1\}$and $cl_{g\text{-}ma}(N) =\{m_1,l_1\}$. Clearly $cl_{g\text{-}ma}(M) = cl_{g\text{-}ma}(N)$ but $M \neq N$.

Theorem III.15: If Q , T $\subseteq$ Y in (Y,η),then

- $cl_{g\text{-}ma}(\phi) = \phi$
- $cl_{g\text{-}ma}(Q)$ is g-m_a closed set in Y.
- If $Q\subseteq T$ then $cl_{g\text{-}ma}(Q) \subseteq cl_{g\text{-}ma}(T)$.
- $cl_{g\text{-}ma}(Q\cap T) = cl_{g\text{-}ma}(Q)\cap cl_{g\text{-}ma}(T)$

Proof: Results (i) (ii) and (iii) are obvious from the definition III.1. (iv) We know that $Q\cap T\subseteq Q$ and $Q\cap T\subseteq T$ from (iii) we have $cl_{g\text{-}ma} (Q\cap T)\subseteq cl_{g\text{-}ma} (Q)$ and $cl_{g\text{-}ma} (Q\cap T)\subseteq cl_{g\text{-}ma} (T)$.
Thus $cl_{g\text{-}ma}(T\cap Q) \subseteq cl_{g\text{-}ma}(Q)\cap cl_{g\text{-}ma}(T)$ −(i).
Let $y\in cl_{g\text{-}ma}(Q) \cap cl_{g\text{-}ma}(T)$ this implies $y\in cl_{g\text{-}ma}(Q)$ and $y\in cl_{g\text{-}ma}(T)$. By definition III.1 ∃g-m_a closed L & O ∍ $Q\subseteq L$ & $T\subseteq O$, $y\in L\cap O$. Thus $Q\cap T\subseteq L\cap O$ & $L\cap O$ is g-m_a closed set by theorem 2.13[10]. Thus
$y\in cl_{g\text{-}ma}(Q) \cap cl_{g\text{-}ma}(T) \Rightarrow y\in cl_{g\text{-}ma}(Q\cap T)$. Hence $cl_{g\text{-}ma}(Q) \cap cl_{g\text{-}ma}(T) \subseteq cl_{g\text{-}ma}(Q\cap T)$ −(ii).
From (i) and (ii) , we have $cl_{g\text{-}ma}(Q\cap T) = cl_{g\text{-}ma}(Q)\cap cl_{g\text{-}ma}(T)$.

Remark III.16: For M , $N\subseteq Y$ in (Y,η) then $cl_{g\text{-}ma} (M \cup N) \neq cl_{g\text{-}ma} (M) \cup cl_{g\text{-}ma}(N)$.

Example III.17: Consider $Y=\{m_1,a_1,k_1,l_1\}$and $\eta=\{\phi,\{a_1\},\{l_1\},\{a_1,l_1\},\{m_1,k_1,l_1\},Y\}$.

Let $M = \{a_1\}$ and $N = \{m_1,k_1\}$then $M \cup N = \{m_1,a_1,k_1\}$. Clearly $cl_{g\text{-}ma}(M)=\{a_1\}$, $cl_{g\text{-}ma}(N) =\{m_1,k_1\}$ but $cl_{g\text{-}ma}(M \cup N)= \phi$. Hence $cl_{g\text{-}ma}(M \cup N) \neq cl_{g\text{-}ma}(M) \cup cl_{g\text{-}ma}(N)$.

Theorem III.18: For any subset A of Y, $cl_{g\text{-}ma}(cl_{g\text{-}ma}(A))= cl_{g\text{-}ma}(A)$.

Proof: Consider U to be g-m_a closed containing A . By definition III.1 $cl_{g\text{-}ma}(A) \subseteq U$. Since U is a g-$m_a$ closed contained in each g-m_a closed containing A, we have $cl_{g\text{-}ma}(cl_{g\text{-}ma}(A)) \subseteq cl_{g\text{-}ma}(A)$. Thus $cl_{g\text{-}ma}(cl_{g\text{-}ma}(A)) = cl_{g\text{-}ma}(A)$. We now introduce g-$m_a$- interior.

Definition III.19: Let $M \subseteq Y$ in (Y,η). g-m_a -interior of M is the union of all g-m_i open contained in M , represented as $int_{g\text{-}mi}(M)$.

Theorem III.20: For any subset $M \subseteq Y$, $Y- int_{g\text{-}mi} (M) = cl_{g\text{-}ma} (Y - M)$.

Proof : Let $y\in Y- int_{g\text{-}mi} (M)$, this implies $y \notin int_{g\text{-}mi}(M)$. Therefore every g-$m_i$ open set E containing y ∍ $E \subseteq M$. Thus for each g-m_i open set E intersects Y - M. Therefore $E\cap(Y - M) \neq \phi$, then by Theorem III.3 $y\in cl_{g\text{-}ma} (Y- M)$. Therefore $Y- int_{g\text{-}mi} (M) \subseteq cl_{g\text{-}ma}(Y- M)$----(i) Conversely , Let $y \in cl_{g\text{-}ma}(Y- M)$,then for each g-m_i open set E containing y intersects Y- M. Therefore $E\cap(Y- M) \neq \phi$, then by definition III.19 $y \notin int_{g\text{-}mi}(M)$, this implies $y\in Y- int_{g\text{-}mi}(M)$. Therefore $cl_{g\text{-}ma}(Y- M) \subseteq Y- int_{g\text{-}mi}(M)$---(ii). From (i) and (ii) we have , $Y- int_{g\text{-}mi}(M) = cl_{g\text{-}ma}(Y- M)$.

Remark III.21: If $Q\subseteq Y$ in (Y,η) then $int\ Q \subseteq int_{g\text{-}mi}(Q) \subseteq Q$.

Example III.22: Consider $Y=\{f_1, b_1, g_1, i_1\}$ and $\tau =\{ \phi ,\{b_1\},\{i_1\},\{b_1,i_1\},\{f_1,g_1,i_1\},Y\}$.
Let $M = \{b_1,g_1,i_1\}$ then $int\ M=\{b_1,i_1\}$ and $int_{g\text{-}mi} (M) = \{b_1,g_1,i_1\}$. Clearly $int\ M \subseteq int_{g\text{-}mi}(M) \subseteq M$.

Remark III.23: For a subset M of (Y,η) , $int_{g\text{-}mi}(M) \neq int\ M$.

Example III.24: It is clearly seen in Example III.22.thus $int_{g\text{-}mi} (M) \neq int\ M$.

Remark III.25: Let $Q ,T \subseteq Y$ in (Y,η) ,then $int_{g\text{-}mi}(Q) = int_{g\text{-}mi}(T)$ does not imply Q = T.

Example III.26: Let $Y = \{p_1,w_1,z_1,h_1\}$ and $\eta = \{ \phi ,\{p_1\},\{p_1,w_1\},\{z_1,h_1\},\{p_1,z_1,h_1\},Y\}$. Let $E = \{z_1,h_1\}$ and $F = \{w_1,z_1,h_1\}$, then $int_{g\text{-}mi}(E) = \{z_1,h_1\}= int_{g\text{-}mi}(F)$ but $E \neq F$.

Remark III.27: For any subsets F and Q of (Y,η), $int_{g\text{-}mi}(F) \cup int_{g\text{-}mi}(Q) \neq int_{g\text{-}mi}(F \cup Q)$.
ExampleIII.28: Let$Y=\{p_1,w_1,z_1,h_1\}$ and $\eta=\{\phi,\{p_1\},\{w_1\},\{z_1\},\{p_1,w_1\},\{p_1,z_1\},\{w_1,z_1\},\{w_1,h_1\},\{p_1,w_1,z_1\},\{p_1,w_1,h_1\}, \{w_1,z_1,h_1\},Y\}$. Let $R =\{p_1,w_1\}$ and $K =\{p_1,h_1\}$ then $R \cup K = \{p_1,w_1,h_1\}$. Clearly $int_{g\text{-}mi}(R) = \{p_1,w_1\}$, $int_{g\text{-}mi}(K) = \{p_1\}$ and $int_{g\text{-}mi} (R \cup K) = \{p_1,w_1,h_1\}$. Thus $int_{g\text{-}mi}(R) \cup int_{g\text{-}mi}(K) \neq int_{g\text{-}mi}(R \cup K)$

4.g-ma- Neighborhoods and g-ma-Limit Points

Here we introduce g-m_a- neighbourhood(briefly g-m_a- nbhd) , g-m_a-limit and g-m_a-derived set.
Definition IV.1: A set $N\subseteq Y$ in (Y,η) is g-m_a- neighborhood of a point $y\in Y$ if ∃ a g-m_i open set E ∍ $y\in E\subseteq N$.

Let $N_{g\text{-}ma}$ be collection of all g-m_a neighborhoods.
Definition IV.2 : A set $N\subseteq Y$ in (Y,η) is g-m_a- nbhd of D where $D\subseteq Y$, if ∃ a g-m_i open set E ∍ $D\in E\subseteq N$.

Theorem IV.3: A set $E \subseteq Y$ in (Y,η) is g-m_i open if & only if E is g-m_a nbhd of everyone of its points.

Proof : Consider E to be g-m_i open , then for any $y \in Y$, $y \in E \subseteq E$. Therefore E is g-m_a nbhd of everyone of its points.

Conversely, let E contains a g-m_a neighborhood of each of its points. For each $y \in E$, $\exists$ a neighborhood N_y of y $\ni$ $y \in N_y \subseteq E$. By definition IV.1 $\exists$ a g-m_i open set V_y $\ni$ $y \in V_y \subseteq N_y \subseteq E$. We now intend to prove that $E = \bigcup\{V_y: y \in E\}$. Let $y \in E$, by definition $\exists$ g-m_i open set $V_y \ni y \in V_y$. Therefore $y \in \bigcup \{V_y: y \in E\}$ which implies $E \subseteq \bigcup \{V_y: y \in E\}$. Let $z \in \{V_y: y \in E\}$ so z belongs to some V_y for some $y \in E$. Hence $z \in E$. Therefore $\bigcup\{V_y: y \in E\} \subseteq E$. Hence $E = \bigcup\{V_y: y \in E\}$ for each V_y , where V_y is g-m_i open set. Hence E is g-m_i open set.

Theorem IV.4: If L be a g-m_a closed subset of (Y,η) and $y \in cl_{g\text{-}ma}(L)$ if and only if for each g-m_a nbhd N of y , $N \cap L \neq \phi$.

Proof : Consider N to be g-m_a neighborhood of a point y in (Y,η) $\ni$ $N \cap L = \phi$, then by definition IV.1 $\exists$ an g-m_i open set E $\ni$ $y \in E \subseteq N$. Therefore $E \cap L = \phi$, so $y \in Y$- E. Thus $cl_{g\text{-}ma}(L) \in Y$- E. Therefore $y \notin cl_{g\text{-}ma}(L)$ which is a contradiction. Thus $N \cap L \neq \phi$.

Conversely , Let $y \notin cl_{g\text{-}ma}(L)$ thus by Definition III.1 $\exists$ a g-m_a closed E $\ni$ $L \subseteq E$ and $y \notin E$. Thus $y \in Y$- E & Y- E is g-m_i open. Thus Y-E is g-m_a neighbourhood of y but $L \cap (Y\text{-}E) = \phi$, which is a contradiction. Thus $y \in cl_{g\text{-}ma}(L)$.

Remark IV.5: If Q & G are g-m_a neighborhood , $Q \cap G$ need not be g-m_a neighborhood.

Example IV.6: Let $Y =\{d_1,h_1,l_1\}$ and $\eta = \{\phi,\{d_1\},\{l_1\},\{d_1 ,l_1\},\{d_1, h_1\},Y\}$
Clearly $Q =\{d_1, h_1\}$ and $G= \{d_1,l_1\}$ are any two members of $N_{g\text{-}ma}(y)$ but $\{d_1,h_1\} \cap \{d_1,l_1\} = \{d_1\}$ is not a member of $N_{g\text{-}ma}(y)$.

Theorem IV.7: Let $y \in Y$ in (Y,η). If $N_{g\text{-}ma}(y)$ is the collection of all g-m_a neighborhood of y then $N_{g\text{-}ma}(y) \neq \phi$ and y belongs to each member of $N_{g\text{-}ma}(y)$.

Proof: Let Y be g-m_i open set containing m thus by Theorem IV.III , it is a g-m_a neighborhood of each of its points. Hence $\exists$ atleast one g-m_a neighborhood namely y $\ni$ $N_{g\text{-}ma}(m) \neq \phi$. Let $M \in N_{g\text{-}ma}(m)$, M is a g-m_a neighborhood of m , by definitionIV.1 $\exists$ a g-m_i open set V $\ni$ $m \in V \subseteq M$ this implies $m \in M$. Therefore m belongs to every member M of $N_{g\text{-}ma}(m)$.

Remark IV.8: If K be a g-m_a neighbourhood of y , for any $y \in Y$ and $M \subseteq K$, then M need not be g-m_a neighbourhood of y.

Example IV.9: Let $Y= \{d_1, f_1, l_1\}$ and $\eta = \{\phi,\{d_1\},\{f_1\},\{d_1, f_1\},\{d_1, l_1\},Y\}$.
Let $N = \{d_1, l_1\}$ be g-m_a neighbourhood of Y and let $M = \{d_1\}$ .Clearly $M \subseteq N$ but M is not g-m_a neighbourhood of

Definition IV.10: Consider L to be a subset of (Y,η) and $y \in Y$. Then a point $y \in Y$ is g-m_a limit point of L if and only if for each g-m_i open E containing y contains atleast one point of L other than y i.e $[E\text{-}\{y\}] \cap L \neq \phi$.

A set of all g-m_a limit points of subset L of Y is g-m_a derived set of L & is symbolized as $d_{g\text{-}ma}(A)$.

Theorem IV.11: For M, $N \subseteq Y$ in (Y,η), then

- $d_{g\text{-}ma}(\phi) = \phi$
- (ii) If $M \subseteq N$ then $d_{g\text{-}ma}(M) \subseteq d_{g\text{-}ma}(N)$.

Proof:

- Let $y \in d_{g\text{-}ma}(\phi)$ that is y is g-m_a limit point of ϕ. Then by definition IV.10 $\forall$ g-m_i open set U containing y, it contains atleast one point of M other than y. Thus we should have $[U\text{-}\{y\}] \cap M \neq \phi$ which is not possible. Hence $d_{g\text{-}ma}(\phi) = \phi$.
- (ii) Let $y \in d_{g\text{-}ma}(M)$, that is y is g-m_a limit point of M . Then by definition IV.10 $\forall$ g-m_i open U containing y contains atleast one point of M other than y. Since $M \subseteq N$ this implies $[U\text{-}\{y\}] \cap M \subseteq [U\text{-}\{y\}] \cap N$. Thus y is g-m_a limit point of N that is $y \in d_{g\text{-}ma}(N)$. Therefore $d_{g\text{-}ma}(M) \subseteq d_{g\text{-}ma}(N)$.

Theorem IV.12: If $Q,T \subseteq Y$ in (Y,η), then $d_{g\text{-}ma}(Q \cap T) \subseteq d_{g\text{-}ma}(Q) \cap d_{g\text{-}ma}(T)$.

Proof : As $Q \cap T \subseteq Q$ & $Q \cap T \subseteq T$. Thus by Theorem IV.11 (ii) we have $d_{g\text{-}ma}(Q \cap T) \subseteq d_{g\text{-}ma}(Q)$ and $d_{g\text{-}ma}(Q \cap T) \subseteq d_{g\text{-}ma}(T)$. Thus $d_{g\text{-}ma}(Q \cap T) \subseteq d_{g\text{-}ma}(Q) \cap d_{g\text{-}ma}(T)$.

Theorem IV.13 : If $Q,T \subseteq Y$ in (Y,η) then $d_{g\text{-}ma}(Q) \cup d_{g\text{-}ma}(T) \subseteq d_{g\text{-}ma}(Q \cup T)$.

Proof : As $Q \subseteq Q \cup T$ and $T \subseteq Q \cup T$. Thus by Theorem IV.11 (ii) we have
$d_{g\text{-}ma}(Q) \subseteq d_{g\text{-}ma}(Q \cup T)$ and $d_{g\text{-}ma}(T) \subseteq d_{g\text{-}ma}(Q \cup T)$. Thus $d_{g\text{-}ma}(Q) \cup d_{g\text{-}ma}(T) \subseteq d_{g\text{-}ma}(Q \cup T)$.

Remark IV.14: The converse of the hypothesis need not be valid, as observed from the accompanying illustration.

Example IV.15: Let $Y = \{a_1,e_1,i_1\}$ and $\eta = \{\phi,\{a_1\},\{i_1\},\{a_1, i_1\},\{e_1, i_1\},Y\}$.
Let $Q = \{a_1,i_1\}$, $T=\{e_1, i_1\}$ and $Q \cup T = \{a_1,e_1,i_1\}= Y$. Clearly $d_{g\text{-}ma}(Q) = \{e_1\}$, $d_{g\text{-}ma}(T) = \{e_1\}$, $d_{g\text{-}ma}(Q) \cup d_{g\text{-}ma}(T) = \{e_1\}$ and $d_{g\text{-}ma}(Q \cup T) = \{a_1,e_1,i_1\}$. Therefore $d_{g\text{-}ma}(Q \cup T) \not\subset d_{g\text{-}ma}(Q) \cup d_{g\text{-}ma}(T)$.

Theorem IV.16: Consider any subset M of (Y,η) and $y \in d_{g\text{-}ma}(M)$ then $y \in d_{g\text{-}ma}(M\text{-}\{y\})$.

Proof: Let $y \in d_{g\text{-}ma}(M)$, that is y is g-m_a limit point of M, then by definition IV.10 $\forall$ g-m_i open U containing y contains at least one point of M other than y of M -{y}. Hence y is g-m_a limit point of M -{y} & $y \in d_{g\text{-}ma}(M\text{-}\{y\})$. Hence $y \in d_{g\text{-}ma}(M) \Rightarrow y \in d_{g\text{-}ma}(M)$.

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