Operations On Triangular Fuzzy Number To Determine The Replacement Time For Fuzzy Replacement Problem

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ABSTRACT:
The purpose of this paper is to find the optimal solution of the fuzzy replacement problem with the triangular fuzzy number in a different way. The proposed method is easy to apply and understand. We compared results with the existing one and a new algorithm is also developed.

Keywords:Fuzzy Replacement Problem, fuzzy set, α- cut , Triangular fuzzy number, Fuzzy Ranking.

1. INTRODUCTION

The efficiency of all industrial and military equipments deteriorates with time. Sometimes the equipment fails completely and affects the whole system. “The maintenance costs (running costs) of equipment also go on increasing with time. Thus it becomes more economical to replace the old equipment with a new one. For machine the maintenance cost always increase with time and stage comes when the maintenance cost becomes so large that it is economical to replace the machine with a new one. Thus the problem of fuzzy replacement in this case is to find the best optimal time at which the old machine should be replaced by the new one.” P.Biswas and S.Pramanik [1] suggested that using yager’s ranking method we can covert the fuzzy numbers into a crisp number this ranking method is also one of the way to find optimal time in replacement problem. Bellman [2] first developed the replacement problem as a dynamic programming to find out the optimal age to replace the equipment. Mahdavi, M., and Mahdavi, M [11] suggested reliability based heuristic model for the replacement problem.Dreyfus [15] analyses replacement problem by considering operating cost and replacement costs as exponentially bounded.

2. PRELIMINARIES

Definition 2.1

A fuzzy set $\tilde{X}$ defined on the set of real numbers R is said to be a fuzzy number if its membership function $\mu_{\tilde{X}} : R \rightarrow [0,1]$ should satisfy the following condition:

(1) $\tilde{X}$ is convex, $\forall x_1, x_2 \in R$ and $\lambda \in [0,1]$

$\mu_{\tilde{X}}(\lambda x_1 + (1-\lambda)x_2) \geq \min\{\mu_{\tilde{X}}(x_1), \mu_{\tilde{X}}(x_2)\}$

(2) $\tilde{X}$ is normal, which means that there exists an $x \in R$ such that $\mu_{\tilde{X}}(x) = \tilde{1}$

(3) $\tilde{X}$ is piecewise continuous .

Definition 2.2

A fuzzy number $\tilde{X}$ on R is said to be a triangular fuzzy number if its membership function $\mu_{\tilde{X}} : R \rightarrow [0,1]$ has the following conditions:

$$\mu_{\tilde{X}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, \text{for } a_1 \leq x \leq a_2 \\ = 1, \text{ for } x = a_2 \\ \dfrac{a_3 - x}{a_3 - a_2}, \text{for } a_2 \leq x \leq a_3 \\ = 0, \text{otherwise} \end{cases}$$

2.3 Arithmetic Operations on Triangular Fuzzy Numbers

For any triangular fuzzy numbers $\tilde{X} \approx (m_1, \alpha_1, \beta_1)$, $\tilde{Y} \approx (m_2, \alpha_2, \beta_2)$,the arithmetic operations are defined by

(i)Addition

$\tilde{X} + \tilde{Y} \approx (a_1, a_2, a_3) + (b_1, b_2, b_3) \approx (m_1, \alpha_1, \beta_1) + (m_2, \alpha_2, \beta_2)$

$= (m_1 + m_2, \max\{\alpha_1, \alpha_2\}, \max\{\beta_1, \beta_2\})$

(ii)Subtraction

$\tilde{X} - \tilde{Y} \approx (a_1, a_2, a_3) - (b_1, b_2, b_3) \approx (m_1, \alpha_1, \beta_1) - (m_2, \alpha_2, \beta_2)$

$= (m_1 - m_2, \max\{\alpha_1, \alpha_2\}, \max\{\beta_1, \beta_2\})$

(iii)Multiplication

$\tilde{X}\tilde{Y} \approx (a_1, a_2, a_3)(b_1, b_2, b_3) \approx (m_1, \alpha_1, \beta_1)(m_2, \alpha_2, \beta_2)$

$= (m_1 m_2, \max\{\alpha_1, \alpha_2\}, \max\{\beta_1, \beta_2\})$

(iv)Division

$$\frac{\tilde{X}}{\tilde{Y}} \approx \frac{(a_1, a_2, a_3)}{(b_1, b_2, b_3)} \approx (\frac{m_1}{m_2}, \max\{\alpha_1, \alpha_2\}, \max\{\beta_1, \beta_2\})$$

2.4 Ranking of Triangular fuzzy number

For every triangular fuzzy number $\tilde{X} \approx (a_1, a_2, a_3) \in F(R)$,ranking function $\Re(\tilde{A}) : F(R) \to R$ is defined as the below

$\Re(\tilde{X}) : \dfrac{a_1 + 4a_2 + a_3}{6}$

For any two triangular fuzzy number $\tilde{X} \approx (a_1, a_2, a_3)$ and $\tilde{Y} \approx (b_1, b_2, b_3)$, we have the following comparison

"(i) $\tilde{X} \succ \tilde{Y}$ if and only if $\Re(\tilde{X}) \succ \Re(\tilde{Y})$

(ii) $\tilde{X} \prec \tilde{Y}$ if and only if $\Re(\tilde{X}) \prec \Re(\tilde{Y})$

(iii) $\tilde{X} \approx \tilde{Y}$ if and only if $\Re(\tilde{X}) \approx \Re(\tilde{Y})$

(iv) $\tilde{X} - \tilde{Y} \approx \tilde{0}$ if and only if $\Re(\tilde{X}) - \Re(\tilde{Y}) = 0$"

A triangular fuzzy number "$\tilde{X} \approx (a_1, a_2, a_3) \in F(R)$" is said to be positive if $\Re(\tilde{X}) \succ 0$ .Also if $\Re(\tilde{X}) = 0$,then $\tilde{X}$ is said to be a zero triangular fuzzy number and is denoted by $\tilde{X} \approx \tilde{0}$ and if $\Re(\tilde{X}) \prec 0$, then $\tilde{X} \prec \tilde{0}$ .If $\Re(\tilde{X}) = \Re(\tilde{Y})$,then the triangular numbers "$\tilde{X}$ and $\tilde{Y}$" are said to be equivalent and is denoted by "$\tilde{X} \approx \tilde{Y}$ ".Also if $a_1 = b_1, a_2 = b_2, a_3 = b_3$ then $\tilde{A} = \tilde{B}$.

3. PROPOSED WORK

Fuzzy replacement of an equipment that deteriorate i.e., whose maintenance fuzzy costs increase with the time:

" When time is taken for the continuous variable"

Let $\tilde{C}$: "Capital fuzzy cost of equipment."

$\tilde{S}$: "Scrap fuzzy value of equipment."

n: "Number of years that equipment would be in use."

$\tilde{f}(t)$: "Maintenance fuzzy cost for time t."

$\tilde{A}$ (n): "Average annual fuzzy cost."

Annual fuzzy cost of the equipment at any time t= fuzzy capital cost –scrap fuzzy value +maintenance fuzzy cost at time t

"Total maintenance fuzzy cost in n years" = $\int_0^n \tilde{f}(t)dt$

Total fuzzy cost during n years $\tilde{T}\tilde{C} = \tilde{C} - \tilde{S} + \int_0^n \tilde{f}(t)dt$

Average annual fuzzy cost on the equipment $\tilde{A}\tilde{T}\tilde{C}_n = \dfrac{1}{n}[\tilde{C} - \tilde{S} + \int_0^n \tilde{f}(t)dt]$ ……………………….(3.1)

$$\frac{d}{dn}(\tilde{A}\tilde{T}\tilde{C}_n) = -\frac{1}{n^2}(\tilde{C} - \tilde{S}) - \frac{1}{n^2}\int_0^n \tilde{f}(t)dt + \frac{1}{n}\tilde{f}(n)$$

For $\dfrac{d}{dn}(\tilde{A}\tilde{T}\tilde{C}_n) = 0$,we have

$$\tilde{f}(n) = \frac{1}{n}[\tilde{C} - \tilde{S} + \int_0^n \tilde{f}(t)dt] = \tilde{A}\tilde{T}\tilde{C}_n$$

……………………………………………………………..(3.2)

Sometimes the equipment should replace when the average annual fuzzy cost becomes equal to the current Maintenance fuzzy cost.
"When the time is taken for discrete variable"

Here the total fuzzy cost during n years $\tilde{T}\tilde{C} = \tilde{C} - \tilde{S} + \sum_{t=0}^{n} \tilde{f}(t)$ ……………………………..…....(3.3)

And the average annual fuzzy cost on the equipment

$$\tilde{A}\tilde{T}\tilde{C}_n = \frac{1}{n}[\tilde{C} - \tilde{S} + \sum_{t=0}^{n} \tilde{f}(t)] \quad \text{………………………………………………………………..(3.4)}$$

When $\tilde{A}\tilde{T}\tilde{C}_n$ is "minimum"

We will get

$$\tilde{A}\tilde{T}\tilde{C}_{n-1} > \tilde{A}\tilde{T}\tilde{C}_n < \tilde{A}\tilde{T}\tilde{C}_{n+1}$$

it becomes

$$\tilde{A}\tilde{T}\tilde{C}_{n-1} - \tilde{A}\tilde{T}\tilde{C}_n > 0$$

From (3.4) for period n+1

$$"\tilde{A}\tilde{T}\tilde{C}_{n+1} = \frac{1}{n+1}[\tilde{C} - \tilde{S} + \sum_{t=1}^{n+1} \tilde{f}(t)] = \frac{1}{n+1}[\tilde{C} - \tilde{S} + \sum_{t=1}^{n} \tilde{f}(t) + \tilde{f}(n+1)]"$$

$$= \frac{n}{n+1}[\frac{1}{n}\{\tilde{C} - \tilde{S} + \sum_{t=1}^{n} \tilde{f}(t)\}] + \frac{\tilde{f}(n+1)}{n+1} = \frac{n}{n+1}\tilde{A}\tilde{T}\tilde{C}_n + \frac{\tilde{f}(n+1)}{n+1}$$

$$\tilde{A}\tilde{T}\tilde{C}_{n+1} - \tilde{A}\tilde{T}\tilde{C}_n = \frac{n}{n+1}\tilde{A}\tilde{T}\tilde{C}_n + \frac{\tilde{f}(n+1)}{n+1} - \tilde{A}\tilde{T}\tilde{C}_n$$

$$= \frac{\tilde{f}(n+1)}{n+1} + \tilde{A}\tilde{T}\tilde{C}_n(\frac{n}{n+1} - 1) = \frac{\tilde{f}(n+1)}{n+1} - \frac{\tilde{A}\tilde{T}\tilde{C}_n}{n+1}$$

Since $"\tilde{A}\tilde{T}\tilde{C}_{n+1} - \tilde{A}\tilde{T}\tilde{C}_n > 0"$ it becomes

$$\frac{\tilde{f}(n+1)}{n+1} + \frac{\tilde{A}\tilde{T}\tilde{C}_n}{n+1} > 0 \text{ or } "\tilde{f}(n+1) - \tilde{A}\tilde{T}\tilde{C}_n > 0" \text{ or } "\tilde{f}(n+1) > \tilde{A}\tilde{T}\tilde{C}_n"$$

Similarly $\tilde{f}(n) < \tilde{A}\tilde{T}\tilde{C}_{n-1}$ from $\tilde{A}\tilde{T}\tilde{C}_{n-1} - \tilde{A}\tilde{T}\tilde{C}_n > 0$

$$\tilde{f}(n+1) > \frac{1}{n}[\tilde{C} - \tilde{S} + \sum_{t=0}^{n} \tilde{f}(t)] \quad \text{…………………………………………………………….(3.5)}$$

From final result (3.5) we should replace the equipment at the end of the nth year. We have used the tabular method here.

4. EXAMPLE

4.1 Example.

A company launched a certain type of machine whose fuzzy cost in rupees is (58000, 58500, 59000) and the scrap value is (5000, 5250, 5500). After survey, it has been noticed by company officials that the running cost (maintenance cost (M.C)) in rupees are found from experience (see Table 1).When would the company officials recommend replacing the machine with a new one? (Take 1000 =1 unit).

Table: 1 Maintenance Fuzzy Cost Of Truck For Ten Years

Year	Maintenance fuzzy cost of truck for eight years
1	$\tilde{f}_1$ = [190, 190,190]
2	$\tilde{f}_2$ =[750,750,1000]
3	$\tilde{f}_3$ =[1000,1200,1200]
4	$\tilde{f}_4$ =[1400,1400,1600]
5	$\tilde{f}_5$ =[1800,1800,2000]
6	$\tilde{f}_6$ =[2000,2500,2500]
7	$\tilde{f}_7$ =[3250,3500,3750]
8	$\tilde{f}_8$ =[9000,9250,9500]
9	$\tilde{f}_9$ =[9500,9550,11000]
10	$\tilde{f}_{10}$ =[13000,16800,18100]

Solution:

5. PROPOSED METHOD:

Without converting the fuzzy values into crisp:

Here $\tilde{A} \approx (a_1, a_2, a_3) \approx (m, \alpha, \beta)$

And for $t = 2,3,\ldots 8$ are given below

Table: 2 We convert the fuzzy values into triangular fuzzy number

Year (n)	Yearly Maintenance fuzzy cost $(\tilde{f}_t)$ of truck Where t=1,2,3….8 "$\tilde{A} \approx (a_1, a_2, a_3) \approx (m, \alpha, \beta)$"	(m, α, β) Where t=1,2,3….8
1	$\tilde{f}_1$ = [190, 190,190]	$(\tilde{f}_1) \approx$ [190,0,0]
2	$\tilde{f}_2$ =[750,750,1000]	$(\tilde{f}_2) \approx$ [750,0,150]
3	$\tilde{f}_3$ =[1000,1200,1200]	$(\tilde{f}_3) \approx$ [1200,200,0]
4	$\tilde{f}_4$ =[1400,1400,1600]	$(\tilde{f}_4) \approx$ [1400,0,200]
5	$\tilde{f}_5$ =[1800,1800,2000]	$(\tilde{f}_5) \approx$ [1800,0,200]
6	$\tilde{f}_6$ =[2000,2500,2500]	

		$(\tilde{f}_6) \approx$ [2500,500,0]
7	$\tilde{f}_7$ =[3250,3500,3750]	$(\tilde{f}_7) \approx$ [3500,250,250]
8	$\tilde{f}_8$ =[9000,9250,9500]	$(\tilde{f}_8) \approx$ [9250,250,250]
9	$\tilde{f}_9$ =[9500,9550,11000]	$\tilde{f}_9$ =[9550,50,1450]
10	$\tilde{f}_{10}$ =[13000,16800,18100]	$\tilde{f}_{10}$ =[16800,3800,1300]

Here Fuzzy cost $(\tilde{C}) \approx [58000, 58500, 59000]$ and

Scarp fuzzy value $(\tilde{S}) \approx [5000,\ 5250,\ 5500]$

Triangular fuzzy numbers are $(\tilde{C}) \approx [58500,\ 500,\ 500]$, $(\tilde{S}) \approx [5250, 250, 250]$

Then $(\tilde{C}) - (\tilde{S}) \approx [53250, 500,\ 500]$

Table: 3 To find the replacement time of truck:

Year n	Running fuzzy cost $(\tilde{f}_t)$ Where t=1,2,3….8 (m, α, β)	Cumulative running fuzzy cost $\Sigma(\tilde{f}_t)$ Where t=1,2,3….8 (3)	Depreciation fuzzy cost $(\tilde{C}) - (\tilde{S})$ (4)	Total fuzzy cost (3)+(4)	Average fuzzy cost (5)/n
1	$\tilde{f}_1$ = [190, 190,190]	[190,0,0]	[53250,500,500]	[53440,500,500]	[53440,500,500]
2	$\tilde{f}_2$ =[750,750,1000]	[750,0,150]	[53250,500,500]	[54000,500,500]	[27000,500,500]
3	$\tilde{f}_3$ =[1000,1200,1200]	[1200,200,0]	[53250,500,500]	[54450,500,500]	[18150,500,500]
4	$\tilde{f}_4$ =[1400,1400,1600]	[1400,0,200]	[53250,500,500]	[54650,500,500]	[13662.5,500,500]
5	$\tilde{f}_5$ =[1800,1800,2000]	[1800,0,200]	[53250,500,500]	[55050,500,500]	[11010,500,500]
6	$\tilde{f}_6$	[2500,500,0]	[53250,500,5	[55750,500,50	[9291.67,500,5

	=[2000,2500,2500]		00]	0]	00]
7	$\tilde{f}_7$ =[3250,3500,3750]	[3500,250,250]	[53250,500,500]	[56750,500,500]	[8107.14,500,500]
8	$\tilde{f}_8$ =[9000,9250,9500]	[9250,250,250]	[53250,500,500]	[62500,500,500]	[7812.5,500,500]
9	$\tilde{f}_9$ =[9500,9550,11000]	[9550,50,1450]	[53250,500,500]	[62800,500,1450]	[6977.8,500,1450]
10	$\tilde{f}_{10}$ =[13000,16800,18100]	[16800,3800,1300]	[53250,500,500]	[70050,500,500]	[7005,3800,1300]

By using the above algorithm we got the optimal solution for the existing example. Finally we can conclude that the machine should replaced at the end of the 9^{th} year(from **TABLE: 3**)

6. CONCLUSIONS:

From the above study we can conclude that the different solution approach is used to find optimal solution for the ages of the equipments. A numerical example is solved from the proposed method "without converting the given values into crisp".

Acknowledgment

The authors are grateful to the reviewers for their constructive comments and suggestions, which have helped to significantly improve both the content and exposition of this paper.

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