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Reaction Diffusion Model in Understanding the Chemical Nature of Animal Pigmentations

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Abstract

The Animal Coats are unique for each species in the sense that they differ in texture, colour, pattern etc. Major factors such as habitat, lifestyle, camouflaging etc. influence the formation of the animal coat. Melanin is the major pigment in the body of the organism responsible for the pigmentation on surface of the skin and other appendages. Melanin is produced by specialised cells in the basal layer of the epidermis called 'Melanocytes' whose distribution determines the pattern and overall colour concentration of the pigmentation. The distribution of Melanocytes is specific in each organism determined by chemical signals called 'Morphogens.' A number of authors have put forth theories regarding Melanogenesis and its subsequent distribution to form patterns. Turing and Murray have been considered the pioneer in the work of using differentiation to form a reaction diffusion model to explain the movement of melanocytes and the melanin pigment through a concentration gradient by Morphogens. According to them, body surface area and its uniformity throughout the organism plays an important role in dispersion of Melanocytes. Various patterns like strips, spots, colour variations and other criteria can be explained on the basis of such mathematical theorems and models. In this article, we have made an attempt to study a biological concept that is animal coat pattern formation, in a mathematical way. Ordinary differential equations and Partial differential equations are being used for determining the colours and patterns occurring in Animal coats.

Keywords: Animal coats, Diffusion, Differential Equation, Melanin, Patterns

Introduction

Alan Turing

Alan Turing was one of greatest scientists in the 20th century. In 1930's the Turing machine was designed by him. The chemical basis of morphogenesis was taken as an interest of research and written as an article by Alan Turing. Oscillating chemical reactions such as the Belousov–Zhabotinsky reaction was predicted by him which was first observed in the 1960s. Morphogenesis is the development of patterns and shapes in biological organisms. Alan Turing developed an interest towards morphogenesis and suggested that a system of chemicals reacting with each other and diffusing across space, could account for the main phenomena of morphogenesis. A system of Partial Differential Equations was applied to model catalytic chemical reactions.

Murray's Theory

Murray suggests that generation of a common pattern is possible due to the presence of a single mechanism. This mechanism is based on a reaction-diffusion system of the morphogen

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pre patterns, and the subsequent differentiation of the cells to produce melanin simply reflects the spatial patterns of morphogen concentration.

Melanin: Pigment that has an impact on the colour of skin, eye, and hair in humans and other mammals.

Morphogen: The various chemicals in embryonic tissue that influence the movement and organization of cells during morphogenesis by forming a concentration gradient.

The development of color pattern on the skin of mammal occurs towards the end of embryogenesis, but it may reflect an underlying pre-pattern that is laid down much earlier.
To create the color patterns, certain genetically determined cells, called melanoblasts, migrate over the surface of the embryo and become specialized pigment cell, called melanocytes. Hair color comes from the melanocytes generating melanin, within the hair follicle, which then pass into the hair.

Reaction Diffusion Systems

$$U_t = D_U \Delta U + a - U - \rho R(U, V)$$

$$V_t = D_V \Delta V + c\,(b - V) \rho R(U, V)$$

Where, $R(U, V) = \frac{dUV}{e + fU + gU^2}$

- ❑ Domain: Rectangle
- ❑ Boundary conditions:
  i. Head and Tail (No Flux)
  ii. Body Size (Periodic)

Solution of the Reaction Diffusion System

$$\begin{pmatrix} u(t,x,y) \\ v(t,x,y) \end{pmatrix} = \sum_{n,m=0}^{\infty} C_{n,m} e^{\mu_{n,m} t} V_{n,m} \cos\left(\frac{n\pi x}{a}\right) \cos\left(\frac{2m\pi y}{b}\right),$$

where $\mu_{n,m}$ are the eigenvalues of matrix $\lambda J - k_{n,m}^2 D$, J is Jacobian, $D = \begin{pmatrix} 1 & 0 \\ 0 & d \end{pmatrix}$, and $k_{n,m} = \left(\frac{n^2}{a^2} + \frac{4m^2}{b^2}\right)\pi^2$.

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Effect of Scale on Pattern

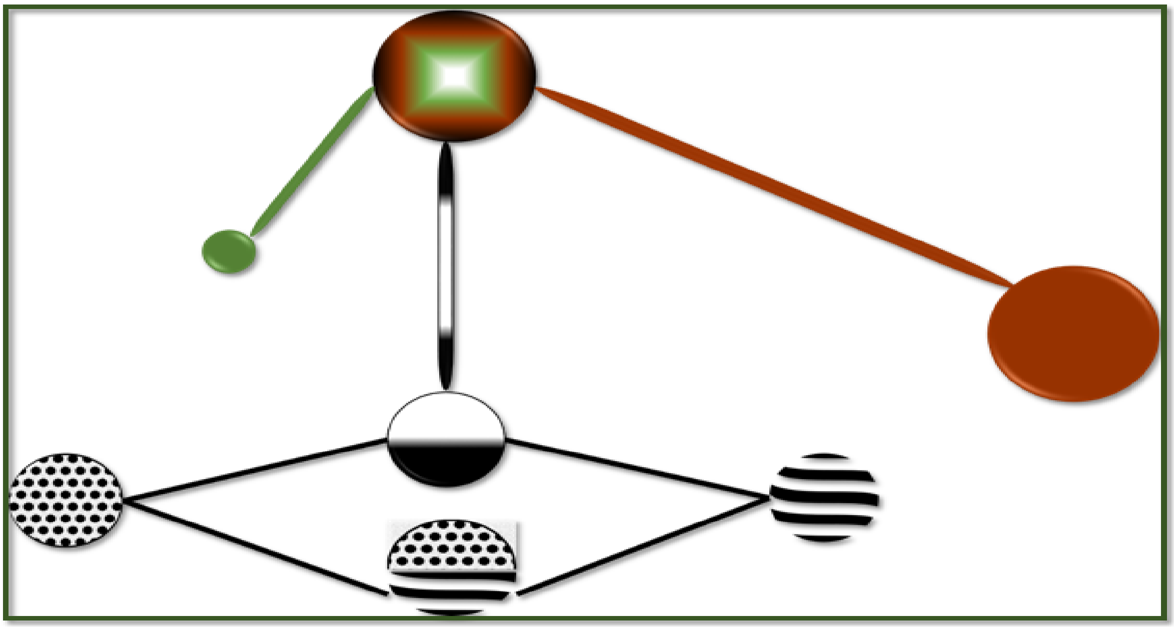
Very small domain: Lambda is small, there is no spatial pattern, and the constant is stable.
Small animals are uniform in colour.
Eg:
Squirrel,
Sheep,
Small dogs'

Medium size domain: Lambda is not too large nor too small, and there are many spatial patterns.
Eg:
Zebra,
Big cats,
Giraffe

Large domain: Lambda is large, and there are patterns but the structure of the pattern is very fine.

Eg.
Elephant,
Bear

Graph Representation of Effect of Scale on Pattern



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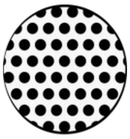
Descriptions

Lambda - λ

Uniform Colour – No Pattern

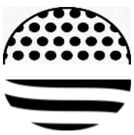
Mono colour - Fine Pattern

Mixed Colour - Spatial Pattern



Spots

Stripes



Combination of Spots & Stripes

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λ is Small

λ is Medium

λ is Large

Natural Patterns of Cos (KX)
Case 1:
Example Taken

Cos (1x): Valais goat

Single color: f(x) =1

Case 2:

i. Cos (2x): Galloway belted Cow
ii. Cos (2x): Giant Panda

Case 3:

i. Cos(3x): Gazelle
ii. Cos(3x): Red Fox

Findings

1. From Turing instability theory, the unstable mode and the spatial pattern have same spatial structure.
2. The unstable mode is determined by the parameter pair (d, λ), where $d = D_V/D_U$, $\lambda = S/D_U$ and S is proportional to the size of the domain.

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3. Spatial patterns can occur only when d is large and when d is large, the pattern occur in order of eigenvalue sequence $k_{n,m} = \left(\frac{n^2}{a^2} + \frac{4m^2}{b^2}\right)\pi^2$
4. When b is smaller, the striped patterns are more likely, which are constant in y direction; and when b is larger, the spots patterns are likely to occur which are not constant in y direction.

Theorems

Theorem 1:

Snakes always have striped (ring) patterns, but not spotted patterns.

Turing-Murray Theory: Snake is the example of b/a is large.

Theorem 2:

There is no animal with striped body and spotted tail, but there is animal with spotted body and striped tail.

Turing-Murray Theory:

The body is always wider than the tail. The same reaction-diffusion mechanism should be responsible for the patterns on both body and tail. Then if the body is striped, and the parameters are similar for tail and body, then the tail must also be striped since the narrower geometry is easier to produce strips.

Examples:

Zebra, Tiger (striped body and tail),

Leopard (spotted body and tail),

Genet, Cheetah (spotted body and striped tail)

Tail Patterns of Big Cats

- Domain: Tapering cylinder, with the width becoming narrower at the end.

- ❑ No-flux boundary condition at the head and tail parts, and periodic boundary condition on the side.
- ❑ Predicted patterns:
    - i. Spots on the wider part and strips on the tail part
    - ii. All spots
    - iii. Or all strips.

Leopard:

The spots almost reach the tip of tail, the pre-natal leopard tail is sharply tapered and relatively short; There are same number of "rings" on the pre-natal and post-natal tails; the sharply tapered shape allow the existence of spots on top part of tail; larger spots are further down the tail, and the spots near the body are relatively small.

Genet:

Uniformly striped pattern; the genet embryo tail has a remarkably uniform diameter which is relatively thin.

Other Related Researches:

These patterns can also be found on aquatic species as well. These patterns occur due to the Chlorite-Iodide-Malonic Acid (CIMA) Reaction. Chemical reactions can be oscillatory (periodic).

Conclusion:

This paper gives a brief idea about formation of patterns in animals which is formulated using differential equations. The various values of λ gives rise to Uniform colour, Spatial Pattern and Fine Pattern on the skin of animals. This is also represented through the formation of a graph.

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