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An S-curve Model on the Maximum Predictive Pricing of Used Cars

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ABSTRACT: A simple linear regression is commonly used as a practical predictive pricing model of used cars. The concept of maximum predictive pricing or efficient frontier model is a popular notion in capital asset pricing model. A non-linear model has been observed to provide a better estimate of price appreciation while describing real-life phenomena. The objective of this paper is to develop a new S-curve model on the maximum predictive pricing of used cars. A dynamic S-shaped Membership Function (SMF) was used as a base function to formulate a new S-curve maximum equation model. A comparison between linear regression, cubic regression, and S-curve model has been made prior to formulating a new S-curve maximum predictive pricing model. Validation of the new S-curve maximum predictive pricing model was done by curve-fitting the new equation into the required data set. This new S-curve model is expected to represent a better and practical model related to the price prediction of used cars and specifically, can offer a more realistic forecast of used car pricing in Malaysia.

Keywords: Maximum predictive pricing, predictive model, S-curve model, used car price.

1. INTRODUCTION

As an owner or a manager of a business organisation, estimating and putting a reasonable price on goods or products is one of the most difficult tasks to undertake. In Microeconomics, a pricing model involves a company's setting the estimated price within a market for a given product. If pricing is done inaccurately, the company may face certain losses.

The used car industry is becoming more popular in the market segment these days. Due to the current lifestyle, it has become a necessity for families to own more than one car per household. In addition, brand-new cars are becoming more expensive for purchase. Many people tend to consider buying used cars, which are supposedly affordable. Thus, in the context of Malaysia, predicting or estimating the price of used cars is highly important before deciding to proceed with purchase.

In the car industry, regardless of new or used cars, economic situations and financial circumstances dictate the selling price, in addition to the cost of motor insurance, road tax, fuel, and maintenance. For many people, the purchase of a new car is considered a luxury. Thus, many people prefer the cheaper alternative of buying a decent preowned or used car.

Notably, predictive and forecasting models have become essential tools in the business world for many years. The prediction of used car pricing poses a remarkably interesting problem in real-life situations. Historically, various techniques or models have been used by researchers for predictive analyses. Pudaruth [1] predicted the pricing of used cars by applying machine learning techniques. Aimin and Shunxi [2] on the other hand, looked into the global prediction of the developing trend and demand for electric cars, based on a logistic model. Sharma and Sinha [3] did a sales forecast of the car industry by applying a multiple linear regression model.

Finding the most suitable tool to provide the best estimate of used car prices can be quite challenging. Predictive modelling can be done using statistical regressions or machine learning algorithms. Traditionally, linear regression has been used frequently in price modelling. Furthermore, linear regression is the oldest type of regression methods used specifically for prediction. However, due to the nature or the assumptions of linear regression, its residual errors tend to grow larger as the data are scattered away from the central mean. Thus, its accuracy in describing a linear relationship in real-life scenarios can be imprecise.

Non-linear models, on the other hand, have been found to outperform linear models in many research studies. One of the non-linear models, the S-curve, has been considered as the suitable shape that describes many real-life phenomena [4]. Aydin [5] also found that a spline S-curve model has given the best predictive performance in forecasting natural gas production compared to six other regression models, namely, the linear, logarithmic, power, exponential, inverse, and growth models. The idea now is to move from linear regression to the S-curve predictive pricing model. Therefore, this paper focuses on the maximum predictive pricing of used cars in Malaysia using a new S-curve model.

2. LITERATURE REVIEW

2.1 Linear Regression

A simple linear regression is the most basic statistical technique and has been commonly used in predictive analyses. This method typically models the linear relationship between two variables, namely X (independent variable or predictor) and Y (dependent variable or response). Fitting these linear models has remained as one of the most frequent activities engaged by many researchers till today.

Linear regression has also been used to determine the pricing model. Years ago, pricing model, or financial modelling, was developed to generate greater incomes and revenues. Estimation is one of the ways to determine the appropriate price of an item. Several researchers have used linear regression in pricing models and found a positive linear relationship between a variable and its predictor [6], [7].

Unfortunately, real data barely produce a linear relationship. A valuable item is not linearly valued or depreciated with regard to time. Prediction errors become wider as data move farther from the central mean in the linear regression model. Mamipour and Jezeie [8] in their study found that a non-linear relationship exists between independent and dependent variables in economics and finance. Ferrari, Marchese, and Tei [9] evaluated different effects of business elements on shipbuilding activities in relation to different economic-cycle phases and found that using a non-linear econometric approach outperforms the linear approach. However, Seifbarghy, Amiri, and Heydari [10] developed two-estimated cost functions and assessed their accuracy through a simulation. They discovered that non-linear regression performs better than linear regression techniques in estimating the total cost function of a two-echelon inventory system with one central warehouse and several non-identical retailers. Thus, linear regression techniques are not considered the best techniques to approach price prediction, forecasting, or estimation.

2.2 Non-Linear Models

Non-linear models bring a larger variety of behaviour descriptions [11]. In fact, fitting a non-linear model would be the best solution if a linear model failed. Real data hardly produce a straight-line relationship. Previous studies have observed that non-linear econometric models outperform linear predictive models [12], [13]. Therefore, a non-linear model is expected to solve prediction problems in item pricing. At the same time, a non-linear model is also expected to manage any prediction error with better accuracy or better estimation for model pricing in interpreting real-life phenomena.

2.3 S-curve Models

An S-curve is a well-established tool, which has been identified throughout modern computational sciences. It has many applications, especially in the areas of neural network, fuzzy logic, artificial intelligence, etc. One example of an S-curve function is the sigmoid function, which is widely used in artificial neural network to introduce a nonlinearity model [14]–[16] and back-propagation learning [17]. The sigmoid function has also been used to predict how much money could be saved based on the available amount in a current account balance [18].

An S-curve often refers to the special case of a continuous logistic function, which has a mathematical equation as follows [19]:

$$f(x) = \frac{1}{1+e^{-x}} \qquad (1)$$

A logistic function is frequently used to model a population growth of some sets. It is also commonly used in statistics to describe a cumulative distribution function. A logistic function is a continuous and increasing function for all values of *x*. Its gradient is always non-negative for all real values of *x*, whereas its second derivative changes from concave upward to concave downward. The graph of Equation (1) is shown in Figure 1.

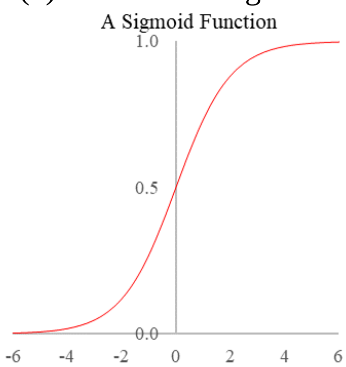

Figure 1: The Logistic S-curve

The S-curve can be divided into three phases. The first phase is where the graph starts slowly, before accelerating in the second phase, and subsequently slows down again to reach its saturation level in the final phase. Such curves can be used to capture a diffusion process [20]. The logistic S-curve of natural growth is a basic model of the Volterra-Lotka equations, which are reliable for describing and forecasting different forms of competition and technology substitution [21].

One of the mathematical membership functions that has an ‘S’ shape is called an S-shaped Membership Function (SMF). It is a spline-based curve, which maps the input vector ***x*** to a membership value (or degree of membership) between 0 and 1. A typical example of SMF is as follows:

$$f(x)=\begin{cases} 0, & x \le a \\ 2\left(\dfrac{x-a}{b-a}\right)^2, & a < x \le \dfrac{a+b}{2} \\ 1-2\left(\dfrac{x-b}{b-a}\right)^2, & \dfrac{a+b}{2} < x < b \\ 1, & b \le x \end{cases} \quad (2)$$

An S-curve has also been found in a technical analysis of comparative index by the second author [22]. Figure 2 shows the graph of Equation (2), which follows an “S” shape with extreme parameters a = -3 and b = 3.

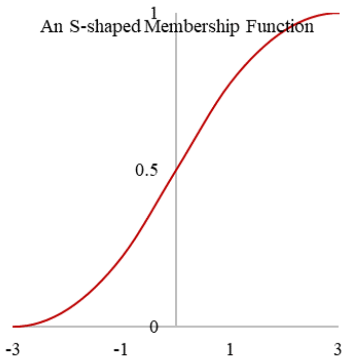

Figure 2: An S-shaped Membership Function with parameters a = -3 and b = 3

The Constructal Law was proposed by Adrian Bejan as a summary of all design generation, evolution phenomena, bio and non-bio nature. Accordingly, the S-curve is one of nature’s most common phenomena, e.g. the spreading of populations, tumours, contaminants, innovations, economic activities, etc. [23]. The S-curve model was also used in graphical project management instruments for planning, tracking, controlling, analysing, and forecasting the status, advancement, and achievement of some management and financial activities [24]. In addition, the S-curve has been widely used in engineering, such as in the fields of civil engineering, mechanical engineering, etc., especially when dealing with project management [25], [26].

Many research studies have been carried out using S-curves in forecasting and prediction analyses. One of the leading forecasting journals, the International Journal of Technological Forecasting and Social Change, has published numerous articles mentioning the S-curve.

Further reading on the types of forecasting methods can be referred to [27], covering all types of forecasting methods from various experts, with some mentioning the S-curve in their chapters.

Although numerous researchers have used the S-curve as a form of prediction, none have produced an S-curve model for an improved estimation of price. Thus, the objective of this research is to develop a new S-curve model to estimate the maximum predictive price of used cars.

3. A CONCEPTUAL S-CURVE MODEL ON USED CAR PRICES

Making a swift decision with minimal knowledge or information is a challenging and difficult task. Theoretically, a statistical model is typically ideal, but can also be realistic. In this present research, the real-life scenario in a regression model is to be emulated. The basic predictive model is linear regression. However, a valuable item does not depreciate linearly over time; thus, linear regression is unsuitable to be the best practice for predictive analysis, as it has sensitivity to outliers, is subject to over-fitting in extremely rare cases when ordinary least square assumptions perfectly exist [28].

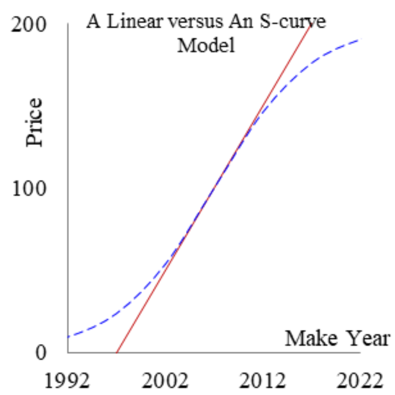

Figure 3: A linear versus an S-curve model

Figure 3 shows the relationship between the prices of used car and the make year of the car. The straight line is the linear regression line, while the dotted line represents an S-curve line. The range of the car make year is from 1992 to 2022. A car older than 12 years is considered not roadworthy. The life span of a car is no more than twelve years in most countries. It clearly shows that if the best fit line is a straight line, then the price of a used car in 1997 is 0. This is not possible, since the car must have some value even if it is already too old to be roadworthy. The value of the car made in 1997 should be set as a scrap value. In 2017, the price of a preowned car logically cannot be higher compared to that of 2012 because of depreciation in value. Therefore, the pricing of used cars cannot follow the linear model; instead, it is more accurate and practical to follow the S-curve.

In general, a regression model will start from the centre mean point $(\bar{x}, \bar{y})$, which is estimated by the sample mean point. At this centre point, the regression model is expected to have the lowest error. The error grows larger as the estimate moves away from the centre point. A practical objective of an S-curve is to better fit real-world situations. In this case, the S-curve model is meant to provide a better price prediction for used cars in Malaysia, instead of using linear regression which projects the price reduction at the same pace per year throughout a life span of a car.

Based on Figure 4, to minimise prediction error during curve fitting, the strategy is to remove the points on the top right corner above the linear regression line, and the points on the bottom left corner below the linear line. Under the current linear regression model, the variance is wider the farther the year is from the centre median year. However, the values of an accurate predictive model should be higher on both sides. Thus, producing a better regression model which will give a more accurate prediction over the whole range is exceptionally challenging.

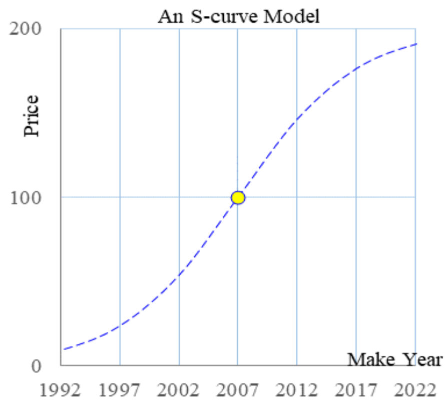

Figure 4: An S-curve model on used car prices

4. METHODOLOGY

Secondary data on used car prices are available from several websites. For this preliminary purpose, data were obtained from Carlist website. Real data were collected on the prices and the car make years for two existing car models in the market, which are BMW 325*i* 3 Series and a VOLVO S60 Sedan. Other features which may affect the prices of used cars, such as mileage, colour, and engine conditions were disregarded. The data's range of interest is from the year 2000 to the year 2016. However, the available data on the website for BMW 325*i* are only from 2004 to 2012, while for VOLVO S60, from 2002 to 2014. The samples of the collected data are shown in Table 1 and Table 2.

Table 1: A sample data collection of 65 used car price for BMW 325i from 2004 to 2012

Year	**Price (RM)**	**Year**	**Price (RM)**
2004	36800	2010	91989
2004	37800	2010	108000
2004	35800	2010	110000
2005	62800	2010	96000
2005	45800	2010	81800
2005	37800	2010	89900
2005	53800	2010	102800

2005	38800	2010	115000
2005	58888	2010	98800
2005	28800	2010	106800
2008	63800	2010	103888
2008	79888	2010	105000
2008	56800	2010	93800
2008	70000	2010	78888
2008	59800	2010	115000
2008	73800	2010	103800
2009	53800	2010	97800
2009	53988	2011	115000
2009	106988	2011	112000
2009	88000	2011	118000
2009	98800	2011	93800
2009	97000	2011	101800
2009	84800	2011	99888
2009	78800	2011	108500
2009	88800	2011	103800
2009	95800	2012	89800
2009	79800	2012	87800
2009	83800	2012	93999
2009	83888	2012	125000
2009	109900	2012	109880
2009	63900	2012	111888
2009	78000	2012	108888
2009	146888		

Table 2: A sample data collection of 45 used car price for VOLVO S60 from 2002 to 2014

Year	**Price (RM)**	**Year**	**Price (RM)**
2002	19000	2011	85000
2002	22800	2011	86800
2002	21800	2011	159000
2002	18300	2011	115000
2002	22500	2011	95000
2002	24799	2011	107888
2003	19800	2011	97500
2003	17800	2011	105888
2003	23800	2011	88800
2004	28500	2011	89800
2004	28700	2012	105800
2004	29800	2012	91800
2004	29900	2012	99800
2005	32900	2012	92800
2005	23222	2012	98,800
2005	28800	2012	102800

2005	26800	2012	88500
2005	29799	2012	112000
2006	32800	2012	88858
2006	47500	2013	93800
2007	31800	2013	105800
2008	29800	2014	129800
2008	35800		

5. RESULT AND DISCUSSION

Based on data collected, the graphs of linear regression, cubic regression, and S-curve model were produced. Using simply Microsoft Excel, Figure 5 and Figure 6 show the graphs of linear regression and cubic regression of the BMW 325*i* car model, respectively.

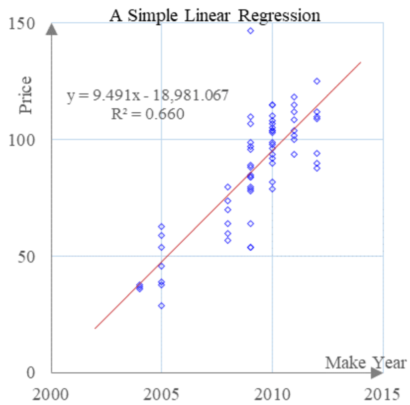

Figure 5: A linear regression for BMW 325i used car prices

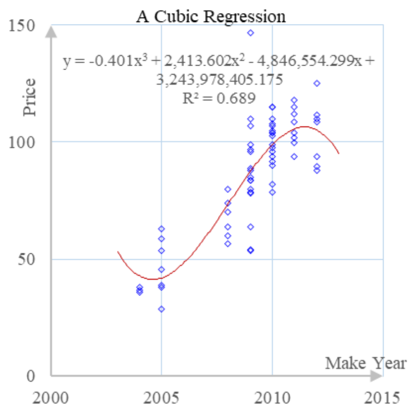

Figure 6: A cubic regression for BMW 325i used car prices

For this preliminary study, to graph an S-curve model using the same data set, the author used an S-shaped Membership Function (SMF) as stated in Equation (2). To produce an S-curve model, the minimum and maximum values of the data needed to be predetermined for the equation to produce correct results. Based on the data collected, the minimum price for the S-curve model is set at RM40,000 while the maximum price is set at RM110,000. The equation of the S-curve model for BMW 325*i* would be as follows:

$$f(x)=\begin{cases}0, & x\le 2003\\ 2\left(\dfrac{x-2003}{10}\right)^2, & 2003<x\le 2008\\ 1-2\left(\dfrac{x-2013}{10}\right)^2, & 2008<x<2013\\ 1, & x\ge 2013\end{cases} \quad (3)$$

Thus, the predicted price is given as $\hat{y}=40000+70000f(x)$. The S-curve model graph for BMW 325*i* is shown in Figure 7.

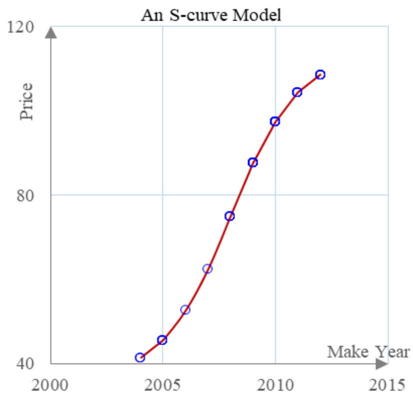

Figure 7: An S-curve model for BMW 325i used car prices using SMF equation

For VOLVO S60 data set, a similar process was done. Using simply Microsoft Excel, Figure 8 and Figure 9 show the graphs of linear regression and cubic regression for VOLVO S60 car model, respectively.

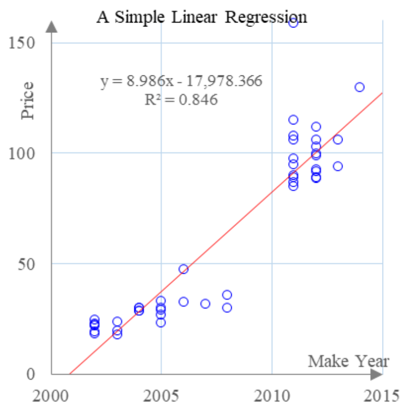

Figure 8: A linear regression for VOLVO S60 used car prices

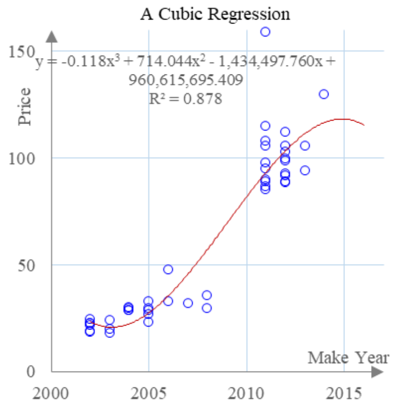

Figure 9: A cubic regression for VOLVO S60 used car prices

To produce an S-curve model, the minimum price for set at RM20,000 while the maximum price is set at RM110,000. The equation of the S-curve model for VOLVO S60 would be as follows:

$$f(x) = \begin{cases} 0, & x \leq 2001 \\ 2\left(\dfrac{x-2001}{7}\right)^2, & 2001 < x \leq 2008 \\ 1-2\left(\dfrac{x-2015}{7}\right)^2, & 2008 < x < 2015 \\ 1, & x \geq 2015 \end{cases} \qquad (4)$$

Thus, the predicted price is given as $\hat{y} = 20000 + 90000 f(x)$. The S-curve model graph for VOLVO S60 is shown in Figure 10.

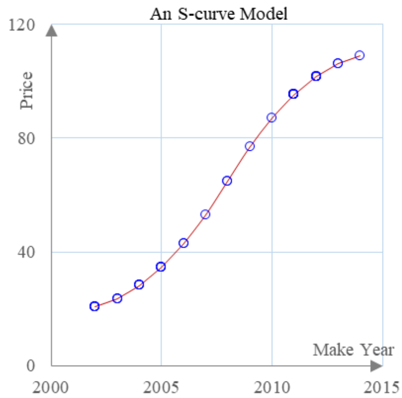

Figure 10: An S-curve model for VOLVO S60 used car prices using SMF equation

All predicted values, the least square errors, and also the mean squared errors were calculated. The values of the mean squared errors for all the three types of regression for both car models are shown in Table 3 below.

Table 3: Mean Squared Error (MSE) for BMW 325i and VOLVO S60 used car prices

Types of	**Mean Squared Error (MSE)**

Regression	BMW 325i	VOLVO S60
Linear	217.22	237.99
Cubic	198.97	188.93
S	201.99	204.84

From Table 3, the mean squared errors for S-curve is found to be closer to cubic regression, quantitatively. The smaller the mean squared error, the closer it is to finding the line of best fit. Depending on the data, it may be inconceivable to achieve a very small value for the mean squared error due to the quantitative nature of the data.

The result suggested that the prices of used cars are expected to rise with newer car models. Even though cubic regression produces smaller MSE, the curve at the far ends has already gone into negative derivative. These phenomena negate the principle of rising prices over time. Even though the S-curve shows a slightly higher MSE, it gives a closer price prediction to the practicality in real life.

Based on this result, an observation has been made to further strengthen the novel idea of a new S-curve model, based on SMF equation. The authors have chosen SMF as the base equation due to several observations and the scope of developing a new S-curve. Using several different data sets on the used car prices of several different car models, the authors have proposed an improved S-curve model equation in the following section.

A New S-Curve Model On The Maximum Predictive Pricing Of Used Cars

To obtain the maximum income from the sales of used cars, an S-curve model will be introduced and proposed as the new maximum predictive pricing model. The idea for maximum predictive pricing was inspired by an efficient frontier concept which was first introduced by the Nobel Laureate Harry Markowitz in 1952, to show how a manager attempts to push for a maximum return of investment portfolio at a given risk level [29].

To form a new S-curve maximum predictive pricing model, an algorithm for constructing the model has been proposed as follows:

Let $x_0, x_1, \ldots, x_{n-1}$ be independent identically distributed (i.i.d) maximum random variables and $x_{(0)} < x_{(1)} < \ldots < x_{(n-1)}$.

i. Estimate the central mean point or an inflection point $(\bar{x}_m, \bar{y}_m)$.

ii. Estimate the mean value within the neighbourhood values nearer or closer to x_0 and x_{n-1}. In other words, compute for $(x_0, \bar{y}_0)$ and $(x_{n-1}, \bar{y}_{n-1})$.

iii. The equation of a curve to the left of the central mean point is regressed by:

$$f(x_i) = \left(\frac{x_i - x_0}{\bar{x}_m - x_0} \right)^{\alpha}, x_0 < x_i < \bar{x}_m, 1 \le \alpha \le 4$$

iv. The equation of a curve to the right of the central mean point is regressed by:

$$f(x_i) = 1 - \left(\frac{x_{n-1} - x_i}{x_{n-1} - \bar{x}_m} \right)^{\beta}, \bar{x}_m < x_i < x_{n-1}, 1 \le \beta \le 4$$

v. Thus, the proposed new S-curve maximum predictive pricing model is as follows:

$$\hat{y}_i(x_i) = \begin{cases} \bar{y}_0 + (\bar{y}_m - \bar{y}_0)\left(\dfrac{x_i - x_0}{\bar{x}_m - x_0}\right)^{\alpha} & , x_0 < x_i < \bar{x}_m \ , 1 \le \alpha \le 4 \\ \bar{y}_m + (\bar{y}_{n-1} - \bar{y}_m)\left(1 - \left(\dfrac{x_{n-1} - x_i}{x_{n-1} - \bar{x}_m}\right)\right)^{\beta} & , \bar{x}_m < x_i < x_{n-1} \ , 1 \le \beta \le 4 \end{cases} \quad (5)$$

where $(\bar{x}_m, \bar{y}_m)$ is the central mean point and $\bar{x}_m = \dfrac{x_0 + x_{n-1}}{2}$.

In this new S-curve maximum predictive pricing model, as shown in Figure 11, a piecewise function from Equation (5) provides an avenue for a flexible concavity to the second derivative in the best-fitting option. The S-curve moves dynamically towards the lowest possible error within a certain degree range. At the inflection point, the S-curve should have the steepest or highest slope. This inflection point is extremely crucial since it is the stage where a car owner or seller could push for a higher car price to gain maximum profit.

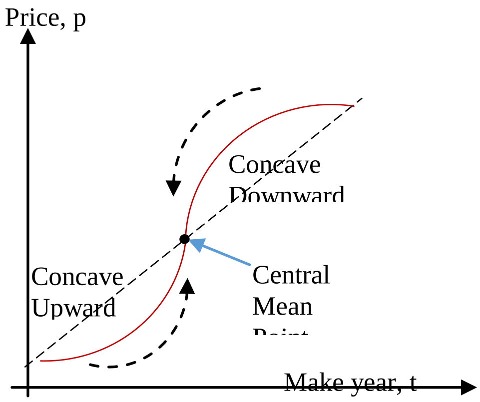

Figure 11: A new S-curve maximum predictive pricing model

To validate the new S-curve maximum equation model, more data set was collected. This time, a robot crawler was used to collect more data. A total of 73,199 used car price data was collected from the same Carlist website. This time, only BMW 3 Series was chosen, but the collected range of car make year was from 1998 to 2018. After selection has been made and the data has undergone a cleaning process to eliminate duplications and outliers, only 726 cleaned data were ready to be used for validation purposes. Figure 12 shows the tabulation of 726 used car price data from 1998 to 2018 make year of BMW 3 Series car model.

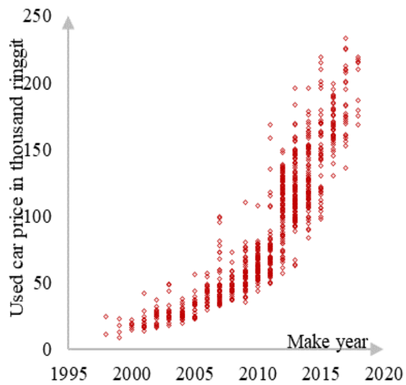

Figure 12: Tabulation of 726 used car price data from 1998 to 2018 make year of BMW 3 Series car model.

Since the idea is to produce an S-curve maximum predictive pricing model, only 2 highest points would be selected for each make year to be considered for a curve-fitting procedure using the new S-curve equation model as stated in Equation (5). Maximum, minimum, and central mean prices were determined prior to defining the degrees of α and β. Using these three details, Equation (5) was fitted into the data which contain 42 maximum prices defined earlier. Thus, the S-curve maximum predictive pricing model for BMW 3 Series, as shown in Figure 13, has the following equation:

$$\hat{y}(x) = \begin{cases} 20{,}450 + 57{,}950\left(\dfrac{x_i - 1999}{10}\right)^2 \\ 78{,}400 + 148{,}900\left(1 - \left(\dfrac{2019 - x_i}{10}\right)\right)^2 \end{cases} \qquad (6)$$

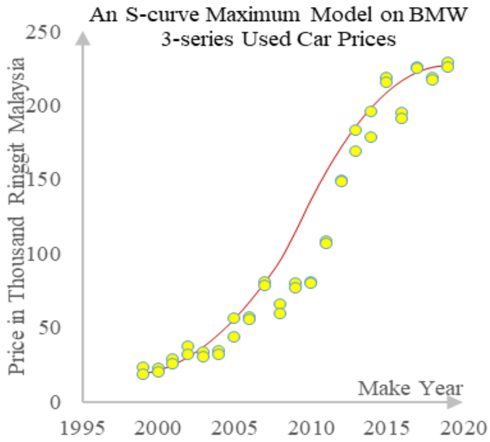

Figure 13: An S-curve maximum predictive pricing model on BMW 3 series used car prices

To build this S-curve model, there is a need to investigate how various correlations among used car pricing affect the depreciation of used car values. Using optimisation techniques, a maximum curve can be computed at each level of make year, based on the combination of different market prices to obtain the best price bargain.

The S-curve maximum predictive pricing model as shown in Figure 13 suggests the best-recommended price for a given used car, based on the market price value. The maximum curve or line suggests the maximum possible price any car dealer would set on the used car. The recommended insured value on any used car would be referred along this line. The car dealer will be capable of pushing the current market value higher than the linear regression line, especially on popular car models.

The new proposed S-curve maximum predictive pricing model has several important implications:

1. Presuming the market is in equilibrium, a new venture will improve the market price of used cars. Improvements are made by injecting an extra niche; therefore, pushing for a higher maximum curve.
2. The S-curve maximum predictive pricing model expects improvement and revision on the

market price of used cars.

3. The S-curve embarks on a non-linear market price and the demand from used car consumers. The S-curve attempts to predict the behaviour of future used car consumers and broadens the understanding of the capital market phenomena of used cars.
4. Direct application of the S-curve measures the performance of car sellers.

6. CONCLUSION

This paper has laid a conceptual study and validation of an actual study on the development of a new S-curve on the maximum predictive pricing model of used cars. The S-curve model does provide a degree of significant proximity to real-life situations. The S-curve regression model has the potential to represent a better model in the world of predicting or forecasting real-life scenarios. The intention of this research was to provide a realistic predictive pricing of used cars. The S-curve model is concluded to be able to offer a more accurate forecast of the prices of used cars in Malaysia.

Acknowledgement

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