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Group Acceptance Sampling Plans For Life Tests Based On Exponentiated Inverted Weibull Distribution

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Abstract.
When the lifetime of an item follows an Exponentiated inverted Weibull Distribution, a group acceptance sampling plan based on reduced lifetimes is developed in this article. The minimal number of groups and acceptance number required for a particular group size are determined for a specified consumer's risk and the test termination time. The minimum ratios of the true average life to the prescribed life at a certain producer's risk are determined using the values of the operational characteristic function for various quality levels. Examples are used to demonstrate the conclusions.

Key words and phrases. Exponentiated inverted Weibull distribution, group acceptance sampling plans, consumer's risk, operating characteristic (OC) function, producer's risk, truncated life test.

1. Introduction

In the quality control analysis, a reliability study is very important. An experimenter can save time and money by using the results of this study to determine whether to accept or reject the submitted lot. An acceptance sampling plan is a method for determining the minimal sample size for testing. This is especially essential if a product's quality is determined by its lifetime. When constructing a sampling plan, it is frequently believed that only one item will be placed in a tester. In practise, however, testers who can handle a large number of items at once are used since testing time and money can be saved by evaluating objects at the same time. A group of objects in a tester can be considered, and the number of items in a group is referred to as the group size. Determining the sample size is comparable to determining the number of groups in this type of test. In the case of a sudden death testing, this type of tester is frequently accepted. A group acceptance sampling plan is an acceptance sampling plan based on such groups of items (GASP). When the GASP is combined with truncated life tests, it is referred to as a GASP based on truncated life tests, which assumes that the product's lifetime follows a given probability distribution.Studies regarding GASP can be found in Aslam (2007) [1], Aslam et al(2009) [2], Aslam and Jun(2009) [3], Aslam and Jun(2009) [4],Srinivasa Rao (2009) [9], Srinivasa Rao (2009) [10], Aslam et al(2011) [5], Aslam et al(2011a) [6], Ramaswamy and Anburajan (2012) [8].

In Section 2, we present the distributional features of truncated life tests and their group acceptance sampling plan (GASP) when the lifetime of a product follows an exponentiated inverted Weibull distribution. Section 3 describes the operating characteristic (OC). Section 4 details the producer's risk. In Section 5, there are examples for an illustration. Section 6 concludes the paper with a summary and conclusions.

2. The Group Acceptance Sampling Plans (GASP)

The Exponentiated Inverted Weibull Distribution (EIWD)'s probability density function (pdf) is given by

$$f(x) = \theta\beta x^{-(\beta+1)}\left(e^{-x^{-\beta}}\right)^{\theta} \qquad x > 0, \beta > 0, \theta > 0 \qquad (2.1)$$

It has a cdf (cumulative distribution function).

$$F(x) = \left(e^{-x^{-\beta}}\right)^{\theta} \qquad x > 0, \beta > 0, \theta > 0 \qquad (2.2)$$

The hazard function is

$$h(x) = \frac{\theta\beta x^{-(\beta+1)}\left(e^{-x^{-\beta}}\right)^{\theta}}{1-\left(e^{-x^{-\beta}}\right)^{\theta}} \qquad (2.3)$$

The Exponentiated Inverted Weibull Distribution is a skewed, unimodal distribution on the positive real line. The K^{th} moment and median of EIWD are given by

$$E(x^k) = \theta^{\frac{k}{\beta}}\,\Gamma\left(1-\frac{1}{\beta}\right). \qquad (2.4)$$

$$\text{Median} = \left(\frac{\theta}{\ln 2}\right)^{\frac{1}{2}} \qquad (2.5)$$

The largest order statistics X(n) have a pdf that looks like this:

$$\alpha_{(n)} = n\,\theta\beta x^{-(\beta+1)}\left(e^{-x^{-\beta}}\right)^{\theta}\left[\left(e^{-x^{-\beta}}\right)^{\theta}\right]^{n-1} \qquad (2.6)$$

The smallest order statistics X(1) have a pdf that is given by

$$\alpha_{(1)} = n\,\theta\beta x^{-(\beta+1)}\left(e^{-x^{-\beta}}\right)^{\theta}\left[1-\left(e^{-x^{-\beta}}\right)^{\theta}\right]^{n-1} \qquad (2.7)$$

The other distributional properties are thoroughly discussed by Flaih et al(2012) [?].

Assume that a product's lifespan is determined by an Exponentiated inverted Weibull distribution with σ as the scale parameter. F(.) is its cumulative distribution function

$$F(t) = \left(e^{-\left(\frac{t}{\sigma}\right)^{-\beta}}\right)^{\theta} \qquad (2.8)$$

Given 0 < q < 1, the 100th percentile is calculated as follows:

$$t_q = \sigma\left(-\frac{1}{\theta}\log(q)\right)^{-\frac{1}{\beta}} \qquad (2.9)$$

In the scaled form, we get by substituting σ in the equation 2.8

$$F(t) = \left[e^{-\delta^{-\beta\left(\frac{1}{\theta}\log(q)\right)^{-1}}}\right]^{\theta} \qquad (2.10)$$

Where $\delta = \frac{t}{t_q}$. In life testing, it is usual practise to terminate the test at a predetermined period t, need a probability of rejecting a bad lot of at least p*, and have the maximum permitted number of defective items to accept the lot equal to c. Under a truncated life test, the acceptance sampling plan for percentiles is to establish the minimum sample size n for a particular acceptance number c so that the consumer's risk, or the chance of taking a bad lot, does not exceed 1-p*. The true 100th percentile, t_q, is below a given percentile, t_q^0, in a bad lot. As a result, the probability p* is a confidence level in the sense that the chance of rejecting a bad lot with $t_q < t_q^0$ is at least p*. As a result, the proposed acceptance sampling plan can be described by the triplet for given p*. $(n, c, \delta) = (n, c, \frac{t}{t_q^0})$ where $\delta = \frac{t}{t_q^0}$.

When a distribution is symmetric, the mean and median are clearly the same. When the distribution is skewed, that is, one side of the tail is longer than the other, the mean is expected to tend towards that side of the distribution. We can make the mean considerably greater and bigger by increasing the amount of skewness, in which case the proportion of the population below the mean can be made excessively enormous. This is what it means when someone says that the mean would not represent the distribution's centre because more than 80% of the population could be below it. However, if the median is used, there is always 50% of the population less than the median. We use q=0.50 because the median is a better approximation of the population mean for making decisions regarding quality of life in our current skewed population. As a result, we may conclude that EIWD sampling plans based on population median are more expense in terms of sample size than those based on population mean.

Let μ be the true value of the median of a product's life time distribution, and μ_0 be the prescribed median, assuming that an item's life time follows the Exponentiated inverted Weibull distribution. We wish to test the hypothesis based on the failure data.H_0:$\mu \geq \mu_0$ against H_1:$\mu < \mu_0$. A lot is considered as good if $\mu \geq \mu_0$ and bad if $\mu < \mu_0$. The following group acceptance sampling technique is used to test this hypothesis:

1. To establish the sample size for a lot n=g.r, choose the number of groups g and assign predetermined r items to each group.
2. Choose a group's acceptance number c and the experiment timet_0.
3. Carry out the experiment for all g groups at the same time and keep track of how many times each group fails.
4. Accept the lot if each group has no more than c failures.
5. If any group has more than c failures, the experiment should be stopped and the lot should be rejected.

In the case of the EIWD and various values of acceptance number c, we are interested in estimating the number of groups g necessary, whereas the group size r and the termination time t_0 are considered to be known. We shall consider $t_0 = \delta\mu_0$ for a given constant δ (termination ratio) because it is simpler to fix the termination time as a multiple of the provided value μ_0 of the median. The producer's risk is the probability (α) of rejecting a good lot, whereas the consumer's risk is the probability (β) of accepting a bad lot. The recommended sample plan's parameter value g is determined to ensure the consumer's risk β. The consumer's risk β is frequently indicated by the consumer's level of confidence. If the confidence level is p^*, then the consumer's risk will be $\beta = 1 - p^*$. We'll figure out how many groups g to include in the recommended sampling plan such that the consumer's risk β doesn't exceed a certain threshold. We can use the binomial distribution to create the GASP if the lot size is large enough. According to the GASP, a lot of products is only acceptable if each of the g groups has at most c failures. As a result, the probability of a lot being accepted is given by

$$\left(\sum_{i=0}^{c}\binom{r}{i}\mathrm{p}_0^{\mathrm{i}}(1-\mathrm{p}_0)^{r-i}\right)^{\theta} \leq \beta \qquad (2.11)$$

Where $p_0 = F_t(\delta_0)$ is the probability of a failure during the time$t = \delta t_q^0$. To save space, only the result of small sample size for $\beta = 0.25,0.10,0.05,0.01 r = 2(1)7; c = 0(1)5; \delta = 0.7,0.8,1.0,1.2,1.5,2.0$
are displayed in table 1.

TABLE1.Minimum number of groups(g) for the proposed plan in the case of EIWD

	r	c						
			0.7	0.8	1.0	1.2	1.5	2.0
0.25	2	0	2	1	1	1	1	1
0.25	3	1	3	2	2	2	2	2
0.25	4	2	5	4	3	3	3	3
0.25	5	3	10	7	4	4	4	4
0.25	6	4	20	12	6	5	5	5
0.25	7	5	38	20	9	6	6	6
0.10	2	0	2	2	2	2	1	1
0.10	3	1	5	4	3	3	2	2
0.10	4	2	9	6	4	4	3	3
0.10	5	3	17	11	6	5	4	4
0.10	6	4	32	19	10	6	5	5

0.10	7	5	63	34	14	9	6	6
0.05	2	0	3	3	2	2	2	1
0.05	3	1	6	5	3	3	3	2
0.05	4	2	11	8	5	4	4	3
0.05	5	3	22	14	8	6	5	4
0.05	6	4	42	25	12	8	6	5
0.05	7	5	82	43	19	11	7	6
0.01	2	0	4	4	3	3	2	2
0.01	3	1	9	7	5	4	3	3
0.01	4	2	17	12	8	6	4	4
0.01	5	3	33	22	12	8	6	5
0.01	6	4	64	38	19	12	8	6
0.01	7	5	126	67	28	17	10	7

3. Operating characteristic of the sampling plan

The probability of acceptance can be regarded as a function of the deviation of the specified value μ_0 of the median from its true valueμ. This function is called operating characteristic (OC) function of the sampling plan. After obtaining the least number of groups g, one may be interested in determining the probability of a lot's acceptance when the quality is evaluated good if $\mu \geq \mu_0$ or $\frac{\mu}{\mu_0}$. The OC is given by

$$L(p) = \left(\sum_{i=0}^{c} \binom{r}{i} \mathrm{p}_0^{\mathrm{i}} (1-\mathrm{p})^{r-i}\right)^{g} \qquad (3.1)$$

For any sampling plan, the OC values can be calculated using Equation 3.1. We offer the OC values for sampling plans using to save space. $\frac{\mu}{\mu_0}$=2,4,6,8,10,12; $\beta = 0.25,0.10,0.05,0.01;$
$\delta = 0.7,0.8,1.0,1.2,1.5,2.0;$
r=4,6,7,9; are given in table 2

4. Producer's Risk

The producer may be interested in improving the product's quality so that the acceptance probability exceeds a predetermined level. The smallest ratio can be determined by meeting the following inequality for a given value of the producer's risk, say δ.

$$\left(\sum_{i=0}^{c} \binom{r}{i} \mathrm{p}_0^{\mathrm{i}} (1-\mathrm{p})^{r-i}\right)^{g} \geq 1-\gamma \qquad (4.1)$$

Table 3 shows the period minimum values of the ratio $\frac{\mu}{\mu_0}$ = 2 in the case of an Exponentiated inverted weibull distribution based on the values in table 1 for the acceptance of a lot at a producer's risk of γ = 0.05.

TABLE 2.Operating characteristic values of the group sampling plan for EIWD

β	g	δ	$\frac{\mu}{\mu_0}$					
			2	4	6	8	10	12

0.25	5	0.7	0.9295	0.9999	1.0000	1.0000	1.0000	1.0000
0.25	4	0.8	0.8895	0.9996	1.0000	1.0000	1.0000	1.0000
0.25	3	1.0	0.7631	0.9969	0.9999	1.0000	1.0000	1.0000
0.25	3	1.2	0.5914	0.9852	0.9997	0.9999	1.0000	1.0000
0.25	3	1.5	0.3679	0.9355	0.9969	0.9998	0.9999	1.0000
0.25	3	2.0	0.1519	0.7631	0.9686	0.9969	0.9997	0.9999
0.10	9	0.7	0.8768	0.9998	1.0000	1.0000	1.0000	1.0000
0.10	6	0.8	0.8262	0.9994	1.0000	1.0000	1.0000	1.0000
0.10	4	1.0	0.6973	0.9959	0.9999	1.0000	1.0000	1.0000
0.10	4	1.2	0.4964	0.9803	0.9997	0.9999	1.0000	1.0000
0.10	3	1.5	0.3674	0.9355	0.9969	0.9998	0.9999	1.0000
0.10	3	2.0	0.1519	0.7361	0.9686	0.9969	0.9997	0.9999
0.05	11	0.7	0.8515	0.9998	1.0000	1.0000	1.0000	1.0000
0.05	8	0.8	0.7753	0.9993	1.0000	1.0000	1.0000	1.0000
0.05	5	1.0	0.6372	0.9949	0.9999	1.0000	1.0000	1.0000
0.05	4	1.2	0.4964	0.9803	0.9996	0.9999	1.0000	1.0000
0.05	4	1.5	0.2636	0.9150	0.9959	0.9998	0.9999	1.0000
0.05	3	2.0	0.1519	0.7631	0.9686	0.9969	0.9997	0.9999
0.01	17	0.7	0.7801	0.9997	1.0000	1.0000	1.0000	1.0000
0.01	12	0.8	0.6827	0.9989	0.9999	1.0000	1.0000	1.0000
0.01	8	1.0	0.4862	0.9918	0.9999	1.0000	1.0000	1.0000
0.01	6	1.2	0.3498	0.9706	0.9994	0.9999	1.0000	1.0000
0.01	4	1.5	0.2636	0.9150	0.9954	0.9998	0.9999	1.0000
0.01	4	2.0	0.0810	0.6973	0.9584	0.9959	0.9996	0.9999

Table 3.Minimum ratio of the values of true median and the specified median for producer's risk of $\gamma = 0.05$ in the case of EIWD

β	r	c	δ					
			0.7	0.8	1.0	1.2	1.5	2.0
0.25	2	0	4.1243	4.1283	5.1082	6.1298	7.6622	10.2163
0.25	3	1	2.6462	2.8193	3.5241	4.2289	5.2862	7.0482
0.25	4	2	2.1107	2.3325	2.7859	3.3430	4.1788	5.5717
0.25	5	3	1.8627	2.0300	2.3400	2.8080	3.5100	4.6800
0.25	6	4	1.7046	1.8323	2.0894	2.4427	3.0534	4.0712
0.25	7	5	1.5857	1.6888	1.1944	2.1745	2.7181	3.6242
0.10	2	0	4.1243	4.7135	5.8918	7.0702	7.6622	10.2163
0.10	3	1	2.8681	3.1676	3.7803	4.5363	5.2862	7.0482
0.10	4	2	2.2915	2.4768	2.9157	3.4988	4.1788	5.5717
0.10	5	3	1.9893	2.1550	2.4836	2.9032	3.5100	4.6800
0.10	6	4	1.7963	1.9366	2.2381	2.5073	3.0534	4.0712

0.10	7	5	1.6691	1.7910	2.0239	2.2972	2.7181	3.6242
0.05	2	0	4.4396	5.0739	5.8918	7.0702	8.8377	10.2163
0.05	3	1	2.9463	2.9463	3.7803	4.5363	5.6704	7.0482
0.05	4	2	2.3523	2.5778	3.0153	3.4988	4.3735	5.5717
0.05	5	3	2.0500	2.2209	2.5839	2.9803	3.6290	4.6800
0.05	6	4	1.8488	1.9981	2.2904	2.6082	3.1342	4.0712
0.05	7	5	1.7122	1.8356	2.0986	2.3572	2.7768	3.6242
0.01	2	0	4.6610	5.3269	6.3423	7.6108	8.8277	11.7836
0.01	3	1	3.1185	4.4423	4.0973	4.7515	5.6704	7.5606
0.01	4	2	2.4828	2.7183	3.2222	3.7152	4.3735	5.8314
0.01	5	3	2.1444	2.3428	2.7235	3.1007	3.7253	4.8386
0.01	6	4	1.9292	2.0909	2.4207	2.7485	2.2603	4.1789
0.01	7	5	1.7817	1.9191	2.1923	2.4858	2.9110	3.7023

5.Tables and Examples

Table 1 shows the GASP design parameters for various values of the consumer's risk and the test termination time multiplier. It should be noted that n=r x g can be used to get the minimal sample size. Table 1 shows that when the test termination time multiplier δ grows, the number of groups g reduces, implying that if the test termination time multiplier increases at a certain group size, fewer groups are required. Table 1 shows that if β=0.10,r=4, c = 2, and a changes from 0.7 to 0.8, the required values of the GASP design parameters change from g = 9 to g = 6. This trend, however, is not constant because it is influenced by the acceptance rate. Table 2 shows the probability of acceptance for the lot at the median ratio that corresponds to the producer's risk. Finally, for certain parameter values, Table 3 shows the minimum ratios of true median to defined median for the acceptance of a lot with producer's risk γ = 0.05.
Assuming that a product's lifetime follows the exponentiated inverted Weibull distribution, a GASP should be designed to see if the median is more than 1,000 hours, based on a testing time of 700 hours and four testers. The values c = 2 and β= 0.10 are assumed. As a result, the termination multiplier δ is equal to 0.700. Table 1 shows that g = 9 is the minimal number of groups required. As a result, we'll select a random sample of n = 36 items and assign 4 things to each of the 9 groups to test for 700 hours. This means that a total of 36 products are required, with 4 items assigned to each of the 9 testers. We shall accept the lot if no more than 2 failures occur in each of the 9 categories before 700 hours. When the experiment reaches its third failure before the 700th hour, we call it a day. When the true value of the median is μ = 4,000 hours, the probability of acceptance for this proposed sampling plan is p = 0.8768. This the producer's risk is α = 0.1232 if the true value of the median is 4 times the required value μ_0 = 1000 hours. Table 3 can be used to calculate the ratio required to ensure a producer's risk of γ = 0.05.For example, when β = 0.10, r = 4, g =9, c = 2 and δ= 0.700, the required ratio is $\frac{\mu}{\mu_0}$ = 2.2915.

6. Summary and Conclusions

In the case of an exponentiated inverted Weibull distribution, a group acceptance sampling plan based on a truncated life test is proposed in this study. When the consumer's risk (β) and other plan variables are given, the number of groups and acceptance number are determined. As the test termination time multiplier grows, it is seen that the minimal number of groups required lowers. Furthermore, as quality improves, the operating characteristics function grows disproportionately. When a large number of items are being tested at the same time, this GASP can be used. Clearly, such a tester would save time and money during the testing process.

References

1. Aslam.M Double acceptance sampling based on truncated life tests in Rayleigh distribution *European Journal of Scientific Research*, 17:605-611, 2007.
2. Aslam.M.,Jun.C.H., and Ahmad.M., A group sampling plan based on truncated life tests for gamma distribution *Pakistan Journal of Statistics*, 25:333-340, 2009.
3. Aslam.M.,and Jun.C.H., A group sampling plan based on truncated life tests having Weibull distribution *Journal of Applied Statistics*, 36:1021-1027, 2009.
4. Aslam.M.,and Jun.C.H., A group sampling plan based for truncated life tests based on the inverse Rayleigh and log-logistic distribution *Pakistan Journal of Statistics*, 25:107-119, 2009.
5. Aslam.M.,Debasis Kundu., Jun.C.H., and Ahmad.M., Time truncated group acceptance sampling plans for generalized exponential distribution *Journal of Testing and Evaluation*, 39(4): 968-976. 2011.
6. Aslam.M.,Jun.C.H., Ahmad.M., and Rasool.M., Group accepatnce sampling plans for resubmitted lots under Burr-type XII distributions *Chinese Instiof Industrial Engineers*, 28(8): 606-615,2011.
7. Nasiru.S., and Luguterah.A., The new Weibull-Pareto distribution *Pakistan Journal of Statistics and Operations Research*, 11(1):103-114, 2015
8. Ramaswamy.A.R.S., and Anburajan.P., Group acceptance sampling plans using weighted binomial on truncated life tests for inverse Rayleigh and Log-logistic distributions IOSR *Journal of Mathematics*, 2(3): 33-38, 2012.
9. Srinivasa Rao.G. A group acceptance sampling plans for life times following a generalized exponential distribution. *Economic Quality Control* ,24: 75-85,2009.
10. Srinivasa Rao.G. A group acceptance sampling plans for truncated life tests for Marshall-Olkin extended Lomax distribution. *Electronic Journal of Applied Statistical Analysis* ,3(1): 18-28,2009