Current Mathematical Models and Numerical Simulation of SIR Model for Coronavirus Disease - 2019 (COVID-19)

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Abstract: This paper deals with seven mathematical models within the current COVID-19 pandemic situations. We developed a number of mathematical models which are compartment solutions of nonlinear differential equations. These models are useful for research scholars, faculty members and academicians in the area of mathematical biology. Also, we study these models and parameter estimation from real-world problem (data of COVID-19 in World Health Organization (WHO)). The researchers discussed to analyze the possible solutions of each model in the general discussion section. The researchers recommended that at the end of the year 2020, there will be a reduction on the spread and an increased recovery rate of COVID-19. These situations are fully changed and return to the normal life very soon. In this paper to the researchers discuss the simple SIR model compared to real-life data for COVID-19 pandemic in Tamilnadu by district wise. This model helps to predict the future calculations of Susceptible, Infections, and Removed people from the total population and reproduction number R_0. The researchers conclude that the infection rate is increased for the next two months (September and October, 2020), and death rate percentage is also possible to decrease from the number of total populations. Also the researchers recommend the continuous lockdown for these two months and for all people to follow the instruction given by the Tamilnadu Government.

Keywords: COVID-19, Mathematical Models, Nonlinear Differential Equations, SIR Model.

1. INTRODUCTION

Globally, as of 7th August 2020, there have been 18,902,735 cases (278,291) confirmed cases of COVID-19, which includes 709,511 deaths (6,815) reported to the World Health Organization (WHO) [1]–[5]. It is possible to develop lot of new models from these seven models. All data are available in WHO's data base and gather data bank [6]-[10]. A number of researchers under the current pandemic COVID-19 situation found many mathematical, with initial and boundary, conditions [11]-[20]. The researchers obtain to predict future

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solutions of the existing COVID-19 pandemic and to check it our system, if it is valid or not [21]. Each mathematical model has numerical and approximation solution with calculate the current situation report from real life data of COVID-19 [22]-[30]. Also, it is easy to predict the future report from the current COVID-19 data [11]. All real-life data of COVID-19 are useful to develop new differential equation with parameter estimations such as SIR, SEIR, etc. The new model validations and development are used such as the system of nonlinear differential equations [31]-[33]. The researchers obtain good analytical results for these types of system with environments fluctuations. In Figure 1, it shows the different types of COVID-19 structures which are provided by the World Health Organization in their official website. We have used the SIR model for real-life theoretical outcome.

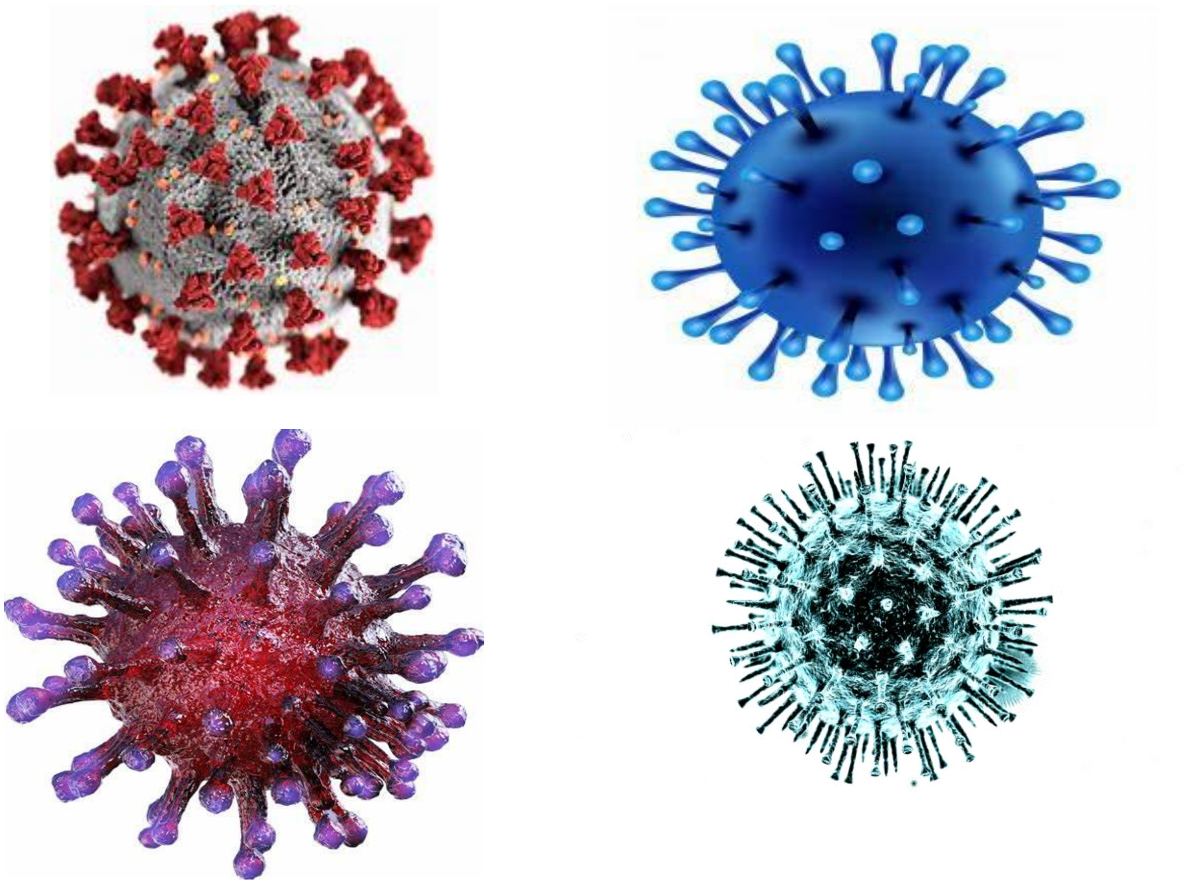

Figure 1 Different Shapes of COVID-19

2. THEORETICAL FRAMEWORK

These models are useful for studies which involve COVID-19 and [22]. Let us consider the following procedure for handling the nonlinear ODE:

- collect COVID-19 current data from WHO and MoHFW (Ministry of Health and Family Welfare, Government of India);
- find the parameter estimation through any suitable model;

- create a new mathematical model from already published suitable model, and use the current situation;
- analyze stability, sensitivity, and bifurcation etc;
- discuss the optimal control problem for nonlinear systems;
- choose any of the analytical method of the developed model in the study; and
- choose any of the current numerical methods for simulation.

The researchers utilized a number of useful links for data collection and a review of related literature for the present situation. This type of equation is solved analytically through different methods such as “Homotopy Perturbation Method, Homotopy Analysis Method, Homotopy Perturbation Transform Method, Differential Transform Method, Variation Iteration Method, Modified Variation Iteration Method, Modified F-Expansion Method, Generalized Separable Variable Method, and Variation Homotopy Perturbation Method”. Current Numerical Methods are used through Finite Difference Methods, Finite Element Methods, B-Spline Finite Element Method, Wavelet Galerkin Finite Element Method, Keller Box Method, Differential Quadrature Method, etc., in order to estimate numerical simulation. To estimate the relevant parameters from the data and to validate the proposed model, numerical analysis and computer simulations will be undertaken. Generally, the researchers analyze the COVID-19 model such as positivity and boundedness, biological feasible region, basic reproduction number, “disease free equilibrium point” (locally& global asymptotically stable), unique “Endemic equilibrium point”, and stability analysis (local and global stability, direction of bifurcation (forward) at R_0 and sensitivity). Also, the researchers minimized the control optimal solution for the cost function subject to the model equations with initial conditions. Though “Pontryagin's Maximum Principle” and optimality condition, the researchers obtained the required optimal control functions and use Forward-Backward Sweep Method [31].

3. METHODOLOGY

In **Mathematical Model 1**, it is possible to write three types of distinct models such as host model, people model, and reservoir model. Hence, we have to develop the three different solutions from this first model.

$$\frac{dS_X}{dt} = \varphi_X - d_X S_X - \beta_X S_X I_X$$

$$\frac{dE_X}{dt} = \beta_X S_X I_X - \mu_X E_X - d_X E_X$$

$$\frac{dI_X}{dt} = \mu_X E_X - \left(\alpha_X + d_X\right) I_X$$

$$\frac{dR_X}{dt} = \alpha_X I_X - d_X R_X$$

$$\frac{dS_Y}{dt} = \varphi_Y - d_Y S_Y - \beta_{XY} S_Y I_X - \beta_Y S_Y I_Y$$

$$\frac{dE_Y}{dt} = \beta_{XY} S_Y I_X + \beta_Y S_Y I_Y - \mu_Y E_Y - d_Y E_Y$$

$$\frac{dI_Y}{dt} = \mu_Y E_Y - \left(\alpha_Y + d_Y\right) I_Y$$

$$\frac{dR_Y}{dt} = \alpha_Y I_Y - d_Y R_Y$$
$$\frac{dS_P}{dt} = \varphi_P - d_P S_P - \beta_P S_P \left(I_P + rF_P \right) - \beta_C S_P C$$
$$\frac{dE_P}{dt} = \beta_P S_P \left(I_P + rF_P \right) + \beta_C S_P C - \left(1 - \gamma_P \right) \mu_P E_P - \delta_P \mu'_P E_P - d_P E_P$$
$$\frac{dI_P}{dt} = \left(1 - \gamma_P \right) \mu_P E_P - \left(\alpha_P + d_P \right) I_P$$
$$\frac{dF_P}{dt} = \gamma_P \mu'_P E_P - \left(\alpha'_P + d_P \right) F_P$$
$$\frac{dR_P}{dt} = \alpha_P I_P + \alpha'_P F_P - d_P R_P$$
$$\frac{dC}{dt} = eC \frac{I_Y}{N_Y} + \rho_P I_P + \rho'_P F_P - \delta C$$

In **Mathematical Model 2**, the researchers developed a number of models for these equations such as SEIR, SEIAR, SEIAHR, SS_qEIR, SEEqIR, SEIHR, etc. Here, this model is very useful for those researchers who are just beginning their studies which are related to of COVID-19.

$$\frac{dS}{dt} = -(\beta c + cq(1-\beta))S(I + \theta A) + \lambda S_q$$
$$\frac{dE}{dt} = \beta c(1-q)S(I + \theta A) - \sigma E$$
$$\frac{dI}{dt} = \sigma \rho E - (\delta_I + \alpha + \gamma_I) I$$
$$\frac{dA}{dt} = \sigma (1-\rho) E - \gamma_A A$$
$$\frac{dS_q}{dt} = (1-\beta) cqS(I + \theta A) - \lambda S_q$$
$$\frac{dE_q}{dt} = \beta cqS(I + \theta A) - \delta_q E_q$$
$$\frac{dH}{dt} = \delta_I I + \delta_q E_q - (\alpha + \gamma_H) H$$
$$\frac{dR}{dt} = \gamma_I I + \gamma_A A + \gamma_H H$$

In **Mathematical Model 3,** we create a number of models for the following equations: SEIR, SEIFR, SEIPFR, SEIPAR, SEIRF, SEIHRF, etc. Here, this model is useful for those who are starting with their studies related to COVID-19.

$$\frac{dS}{dt} = -\beta \frac{I}{N} S - l\beta \frac{H}{N} S - \beta' \frac{P}{N} S$$
$$\frac{dE}{dt} = \beta \frac{I}{N} S + l\beta \frac{H}{N} S + \beta' \frac{P}{N} S - \kappa E$$
$$\frac{dI}{dt} = \kappa \rho_1 E - (\gamma_a + \gamma_i) I - \delta_i I$$

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$$\frac{dP}{dt} = \kappa\rho_2 E - (\gamma_a + \gamma_i)P - \delta_i P$$
$$\frac{dA}{dt} = \kappa(1 - \rho_1 - \rho_2)E$$
$$\frac{dH}{dt} = \gamma_a (I + P) - \gamma_r H - \delta_h H$$
$$\frac{dR}{dt} = \gamma_i (I + P) + \gamma_r H$$
$$\frac{dF}{dt} = \delta_i I + \delta_p P + \delta_h H$$

In **Mathematical Model 4,** similarly, this model also helps to create a number of new good models.

$$\frac{dS}{dt} = d_{qs}Q(t) - f(t) - d_{sq}S(t)$$
$$\frac{dE}{dt} = f(t) - \varepsilon E(t) - d_{eq} - E(t)$$
$$\frac{dD}{dt} = d_{qd}Q(t) + d_{id}I(t) - (\gamma + \delta)D(t)$$
$$\frac{dQ}{dt} = d_{eq}E(t) + d_{sq}S(t) - d_{qs}Q(t) - d_{qd}Q(t)$$
$$\frac{dI}{dt} = \varepsilon E(t) - d_{id}I(t) - \delta I(t)$$
$$\frac{dR}{dt} = \gamma D(t)$$

In **Mathematical Model 5**, similarly, this model helps to create a number of new good models.

$$\frac{ds}{dt} = b - \gamma s - \frac{\delta s(i + \beta a)}{N} - \varepsilon sm$$
$$\frac{de}{dt} = \frac{\delta s(i + \beta a)}{N} + \varepsilon sm - (1 - \nu)\theta e - \nu ae - \gamma e$$
$$\frac{di}{dt} = (1 - \nu)\theta e - (\rho + \gamma)i$$
$$\frac{da}{dt} = \nu ae - (\sigma + \gamma)a$$
$$\frac{dr}{dt} = \rho i + \sigma a - \gamma r$$
$$\frac{dm}{dt} = \tau i + \kappa a - \omega m$$

In **Mathematical Model 6**, similarly, this model helps to create a number of new good models.

$$\frac{dS_h}{dt} = \lambda - \mu S_h - \alpha S_h + \alpha S_a - \beta S_h I_h$$
$$\frac{dI_h}{dt} = \beta S_h I_h - (\delta + \gamma) I_h$$

$$\frac{dS_a}{dt} = \lambda - \mu S_a - \alpha S_a + \alpha S_h - \beta S_a I_a$$
$$\frac{dI_a}{dt} = \beta S_a I_a - (\delta + \gamma) I_a$$
$$\frac{dQ_a}{dt} = \gamma I_a - (\delta + \mu) Q_a$$

In **Mathematical Model 7,** similarly, this model helps to create a number of new good models. It is possible to develop a number of new models from each mathematical model.

$$\frac{dS}{dt} = b - \frac{\beta_1 SP}{1+\alpha_1 P} - \frac{\beta_2 S(I_A + I_S)}{1+\alpha_2 (I_A + I_S)} + \psi E - \mu S$$
$$\frac{dE}{dt} = \frac{\beta_1 SP}{1+\alpha_1 P} + \frac{\beta_2 S(I_A + I_S)}{1+\alpha_2 (I_A + I_S)} - \psi E - \mu E - \omega E$$
$$\frac{dI_A}{dt} = (1-\delta)\omega E - (\mu + \sigma) I_A - \gamma_A I_A$$
$$\frac{dI_S}{dt} = \delta \omega E - (\mu + \sigma) I_S - \gamma_S I_S$$
$$\frac{dR}{dt} = \gamma_S I_S + \gamma_A I_A - \mu R$$
$$\frac{dP}{dt} = \eta_A I_A + \eta_S I_S - \mu_P P$$

4. RESULTS

In the previous section, we have discussed a number of COVID-19 models. However, in this section, we have to analyze the SIR model from the real-life data. The transmission of persons from the compartment of S(t) to the compartment of I(t), and from the compartment of I(t) to the compartment of R(t) can be described by a set of three system of nonlinear differential equations with only two parameter estimations. The simple SIR model assumes that a total population can be divided into three different compartments:

$$\frac{dS}{dt} = -\alpha_0 SI$$
$$\frac{dI}{dt} = \alpha_0 SI - \alpha_1 I$$
$$\frac{dR}{dt} = \alpha_1 I$$

where,

$S(t)$: fraction of susceptible

$I(t)$: fraction of infected persons

$R(t)$: fraction of recovered persons

α_0 : infection rate

α_1 : removal rate

Basic Reproduction Ratio $R_0 = \frac{\alpha_0}{\alpha_1}$

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Relative Removal Rate $\rho = \frac{\alpha_1}{\alpha_0}$

Average Infectious Period $= \frac{1}{\alpha_1}$

Initial Conditions: $S(0) \in (0,1], I(0) \in (0,1], R(0) = 0.$

$S(0) + I(0) = 1$. Let us define the domain: $\alpha_0 \in [0,2] \& \alpha_1 \in (0,1].$

The researchers took the parameter values from Table 1 and calculations of the SIR model. The researchers also provided the excel sheet for 14 days in Table 2 of COVID-19 pandemic in Tamilnadu. Figures 2-6 show the two parameter estimations for the whole SIR system predictions in the current situation.

Table 1. Parameter Values Used in Excel Sheet

dt	0.25
α_0	0.8
α_1	0.1
R_0	8
Peak	0.62719308
Days to Recover	10

Table 2. Compartment Values Used in Excel Sheet for 2 Weeks

Day	S(t)	I(t)	R(t)
0	0.999	0.001	0
0.25	0.9988002	0.0011748	2.5E-05
0.5	0.99856552	0.00138011	5.437E-05
0.75	0.9982899	0.00162123	8.88727E-05
1	0.9979662	0.00190439	0.000129403
1.25	0.9975861	0.00223689	0.000177013
1.5	0.9971398	0.00262726	0.000232935
1.75	0.99661585	0.00308553	0.000298617
2	0.99600084	0.00362341	0.000375755
2.25	0.99527905	0.00425461	0.00046634
2.5	0.99443215	0.00499515	0.000572706
2.75	0.99343868	0.00586373	0.000697584
3	0.99227363	0.00688219	0.000844178
3.25	0.99090783	0.00807594	0.001016232
3.5	0.98930732	0.00947454	0.001218131
3.75	0.98743268	0.01111233	0.001454994
4	0.98523814	0.01302906	0.001732803
4.25	0.9826708	0.01527067	0.002058529
4.5	0.97966959	0.01789012	0.002440296

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4.75	0.97616431	0.02094814	0.002887549
5	0.97207454	0.02451421	0.003411252
5.25	0.96730862	0.02866728	0.004024107
5.5	0.96176259	0.03349662	0.004740789
5.75	0.95531944	0.03910236	0.005578205
6	0.94784839	0.04559585	0.006555764
6.25	0.9392048	0.05309954	0.00769566
6.5	0.92923053	0.06174632	0.009023149
6.75	0.91775521	0.07167798	0.010566807
7	0.90459865	0.0830426	0.012358756
7.25	0.8895746	0.09599058	0.014434821
7.5	0.87249645	0.11066897	0.016834586
7.75	0.85318479	0.1272139	0.01960131
8	0.8314774	0.14574095	0.022781657
8.25	0.80724134	0.16633348	0.026425181
8.5	0.78038708	0.1890294	0.030583518
8.75	0.75088386	0.21380688	0.035309253
9	0.71877504	0.24057054	0.040654425
9.25	0.68419182	0.2691395	0.046668689
9.5	0.64736321	0.29923962	0.053397176
9.75	0.60861986	0.33050197	0.060878166
10	0.56838985	0.36246943	0.069140716
10.25	0.52718506	0.39461249	0.078202451
10.5	0.4855783	0.42635394	0.088067764
10.75	0.44417266	0.45710073	0.098726612
11	0.40356633	0.48627954	0.11015413
11.25	0.36431712	0.51337176	0.122311119
11.5	0.32691109	0.53794349	0.135145413
11.75	0.29173915	0.55966685	0.148594
12	0.25908381	0.57833052	0.162585671
12.25	0.22911659	0.59383947	0.177043934
12.5	0.2019049	0.60620518	0.191889921
12.75	0.17742574	0.61552921	0.207045051
13	0.15558359	0.62198313	0.222433281
13.25	0.13622952	0.62578762	0.237982859
13.5	0.11917937	0.62719308	0.25362755
13.75	0.10422967	0.62646295	0.269307377
14	0.09117047	0.62386058	0.28496895

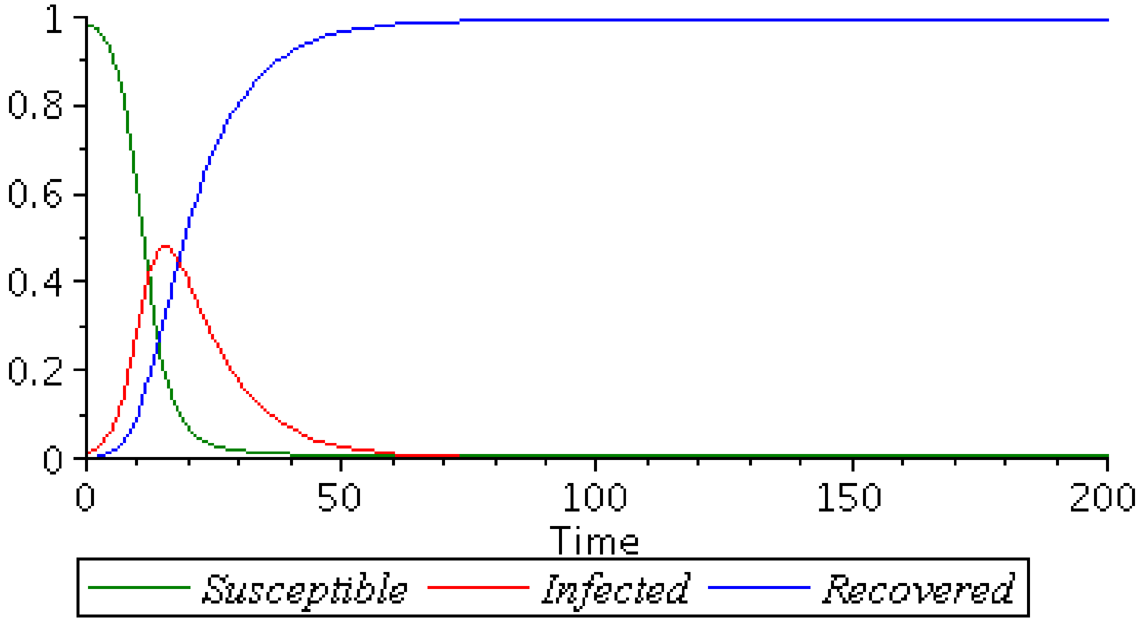

Fig 2. Parameter Estimation of $\alpha_0 = 0.5 \& \alpha_1 = 0.1$.

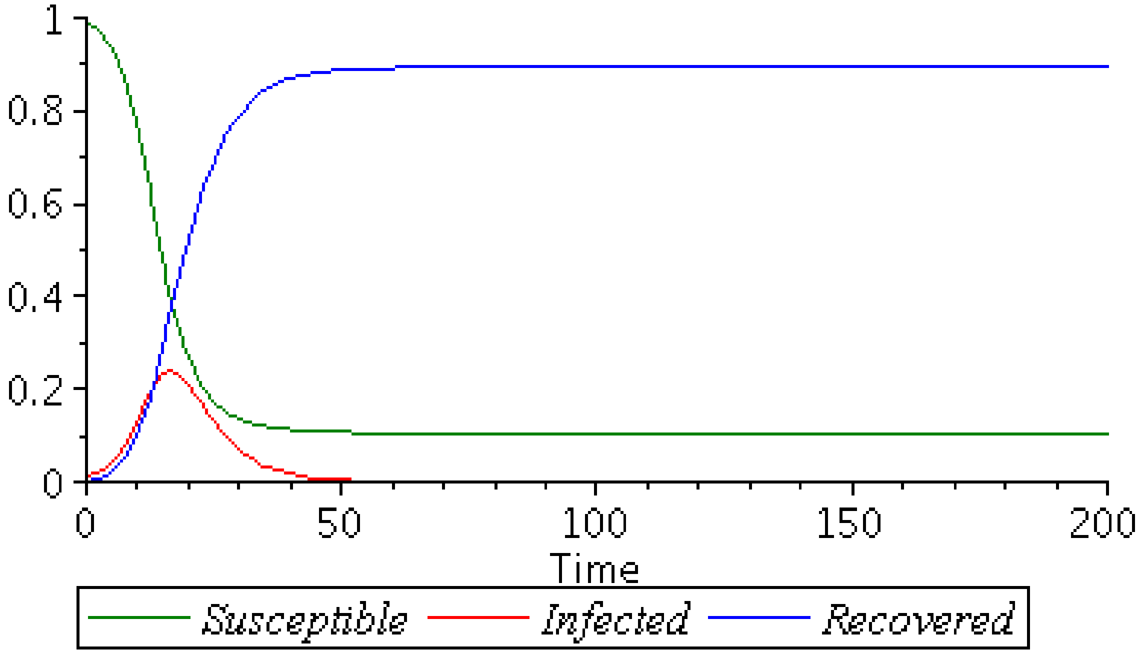

Fig 3. Parameter Estimation of $\alpha_0 = 0.5 \& \alpha_1 = 0.2$.

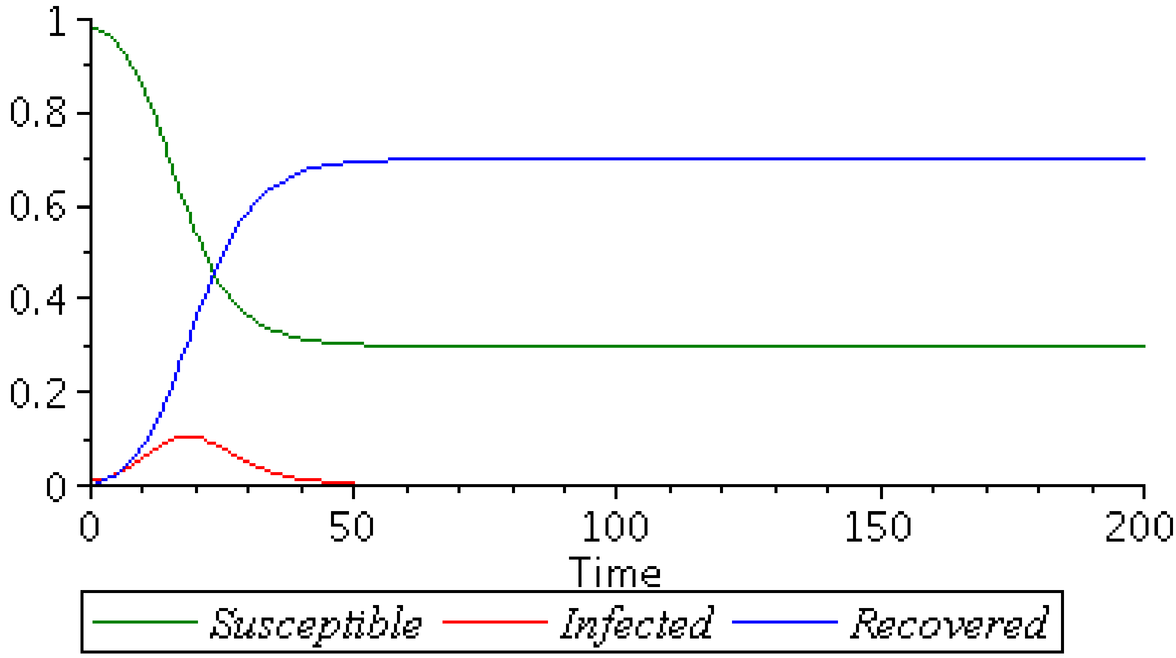

Fig 4. Parameter Estimation of $\alpha_0 = 0.5 \& \alpha_1 = 0.3$.

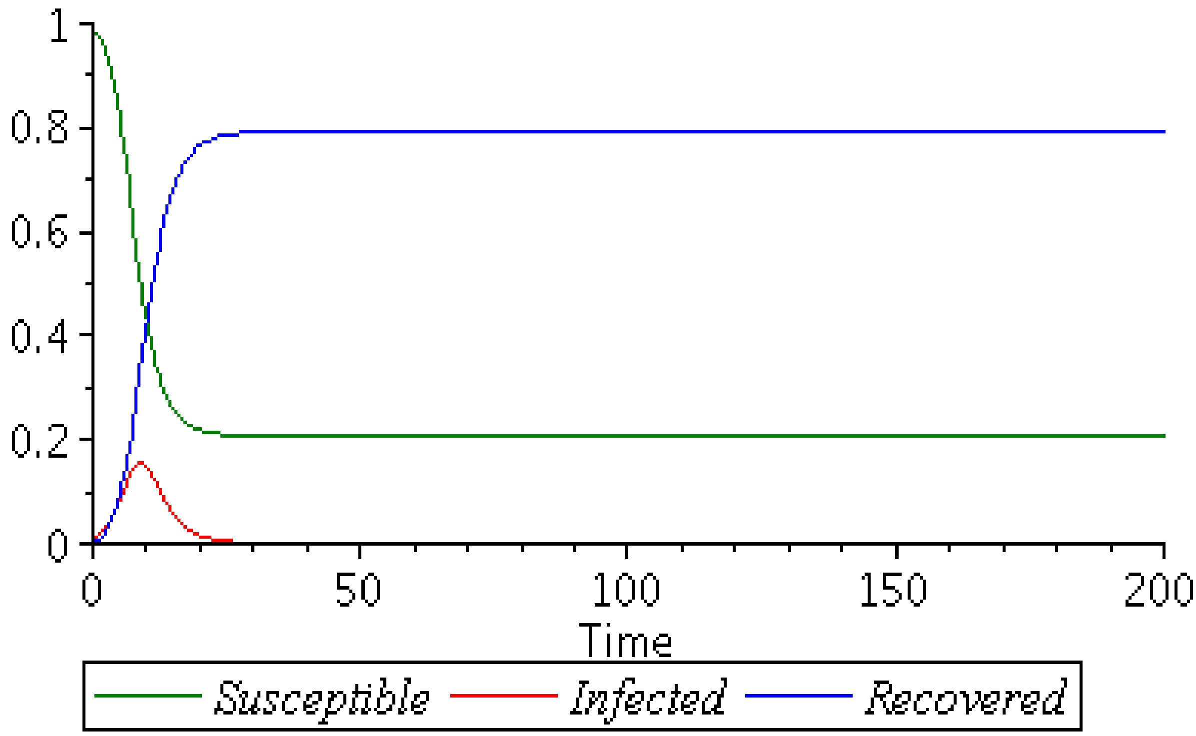

Fig 5. Parameter Estimation of $\alpha_0 = 1 \& \alpha_1 = 0.5$.

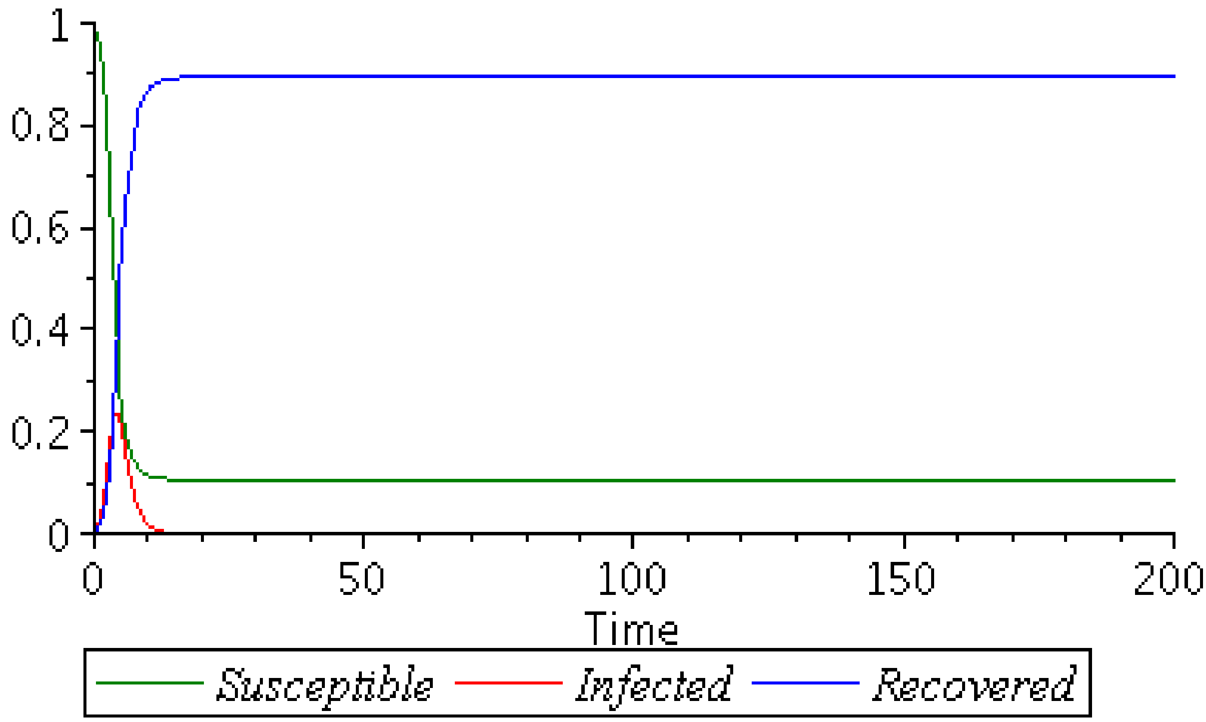

Fig 6. Parameter Estimation of $\alpha_0 = 2 \& \alpha_1 = 0.8$.

5. DISCUSSION

In view of the current mathematical models, the researchers gathered COVID-19 data from the particular domain for WHO or related website [14] (https://bit.ly/patientdb). After the collection of data, the researchers identified the suitable model (SIR, SEIR, SEIRS, etc.,) from other countries as ideas of Kar TK et al in 2019 [5]. Each suitable model had a number of parameters available. We checked if the parameters are valid for our current situation in Indian Council of Medical Research (ICMR), Government of India [4]. Suppose, these parameters are valid in our system and find the suitable model for parameter estimation. There are many parameter estimation models are available such as least square method, gradient-based algorithm, SIR Poisson Model, Markov-chain model, age structured SIR model, and iterative algorithm etc. The researchers derived the parameter values through Excel sheet preparation for least square method. The researchers carried out the graphical representation with the help of MATLAB [33]. However, the researchers in this study have given a number of mathematical models for COVID-19 pandemic's present situation. The same techniques were used for all models in order to predict the future infected ratio.

6. CONCLUSION AND RECOMMENDATION

These types of nonlinear ODE had discussed a number of numerical and analytical solutions which are available in literature. The researchers aspire to develop new models from the mathematical problems initially presented with initial and boundary value problems such as gives good results. Also, beginning researchers will be able to easily develop new models and will be able to solve numerically or analytically from the COVID-19 pandemic's present situation, with compare the system of equations are valid or not from current published data.

Finally the researchers compare their mathematical model from the current COVID-19 data and validated the outcome for the system. The researchers compared to get some results from the real-life data from COVID-19 Tamilnadu and they found out that the next two months complete lockdown is required. Through this order, it is possible to decrease the death rate and to increase the infection rate. The researchers calculated all the compartment populations from the reproduction number of the whole system. If the researchers change the reproduction number, the whole system will automatically be changed. The simple SIR model is helpful to predict future calculations and to present calculations from the current COVID-19 Tamilnadu data.

Availability of Data and Material

Data from WHO; & https ://ourworldindata.org/coronaviru s-sourc e-data, https ://bit.ly/patientdb

Competing Interests

The authors declare that there are no competing interests.

Authors' Contributions

For the writing of this paper, all authors equally contributed to the work. The researchers have also read and agreed the final copy of the manuscript.

7. REFERENCES

[1] Chakraborty T, Ghosh I. Real-time forecasts and **risk** assessment of novel coronavirus (COVID-19) cases: a data-driven analysis. Chaos Solitons Fractals 2020. doi: 10.1016/j.chaos.2020.109850.

[2] FanelliD , Piazza F . Analysis and forecast of COVID-19 spreading in China, Italy and France. Chaos Solitons Fractals 2020;134:109761.

[3] HellewellJ ,AbbottS , Gimma A , BosseNI , Jarvis CI , Russell TW , etal. Feasibility of controlling COVID-19 outbreaks by isolation of cases and contacts. Lancet Global Health 2020;8:e488–96.

[4] Indian council of medical research (ICMR), government of India. 2020. https: //icmr.nic.in.

[5] Kar TK, Nandi SK , Jana S , Mandal M . Stability and bifurcation analysis of an epidemic model with the effect of media. Chaos Solitons Fractals 2019;120:188–99.

[6] Kucharski AJ , Russell TW , Diamond C , Liu Y , Edmunds J , Funk S , et al. Early dynamics of transmission and control of COVID-19: a mathematical modelling study. Lancet Infect Dis 2020.

[7] Liu Y , Gayle AA , Wilder-Smith A , Rocklv J . The reproductive number of COVID-19 is higher compared to SARS coronavirus. J Travel Med 2020:1–4 .

[8] MizumotoK ,Chowell G . Transmission potential of the novel coronavirus (COVID-19) onboard the diamond princess cruises ship. Infect Dis Modell 2020;5:264–70 .

[9] Ndariou F, Area I, Nieto JJ, Torres DF. Mathematical modeling of COVID-19 transmission dynamics with a case study of Wuhan. Chaos Solitons Fractals 2020. doi: 10.1016/j.chaos.2020.109846 .

[10] Prem K , Liu Y , Russell TW , Kucharski AJ , Eggo RM , Davies N . The effect of control strategies to reduce social mixing on outcomes of the COVID-19 epidemic in Wuhan. China: a modelling study. The Lancet Public Health; 2020.

[11] MHDM R, Silva RG, Mariani VC, Coelho LS. Short-term forecasting COVID-19 cumulative confirmed cases: perspectives for Brazil. Chaos Solitons Fractals 2020. doi: 10.1016/j.chaos.2020.109853 .
[12] Adamu HA, Murtala M, Abdullahi MJ, Mahmud AU (2019) Mathematical modelling using improved SIR model with more realistic assumptions. Int J EngAppl Sci. https ://doi.org/10.31873 /IJEAS.6.1.22
[13] Aravind LR et al (2020) epidemic landscape and forecasting of SARSCoV-2 in India. Preprint at https ://www.medrx iv.org/content/10.1101/2020.04.14.20065 151v1
[14] Crowdsourced India COVID-19 tracker data. https ://bit.ly/patientdb
[15] Lopez L, Roda X (2020) A Modified SEIR model to predict COVID-19 outbreak in Spain and Italy: simulating control scenarios and multi scale epidemics. Preprint at https ://www.medrx iv.org/content/10.1101/2020.03.27.20045 005v3
[16] Ministry of Health and Family Welfare, Government of India (2020).District Wise list of reported Cases Our world in data : Coronavirus Source Data. https ://ourworldin data.org/coronaviru s-sourc e-data
[17] Peng L et al (2020) Epidemic analysis of COVID-19 in China by dynamic modelling. Preprint at https ://www.medrx iv.org/content/10.1101/2020.02.16.20023 465v1
[18] Sanders JM, Marguerite LM, Tomasz ZJ, James BC (2020) Pharmacologic treatments for Coronavirus Disease 2019—a review. JAMA 323(18):1824–1836. https ://doi.org/10.1001/jama.2020.6019
[19] M. A. Khan, A. Atangana, Modeling the dynamics of novel Corona-virus (2019-nCov) with fractional derivative, Alexandria Eng. J. (2020), https://doi.org/10.1016/j.aej.2020.02.033
[20] Y. Bai, L. Yao, T. Wei, et al. , Presumed asymptomatic carrier transmission of COVID-19. Journal of the American Medical Association, (2020). https://doi.org/10.1001/jama.2020.2565.
[21] Z. Bekiryazici, M. Merdan, and T. Kesemen, Modification of the random differential transformation method and its applications to compartmental models, Communications in Statistics - Theory and Methods, (2020), 1–21. DOI: 10.1080/03610926.2020.1713372.
[22] T. Chen, J. Rui, Q. Wang, Z. Zhao, J. Cui and L. Yin, A mathematical model for simulating the phasebased transmissibility of a novel Corona-virus, Infectious Diseases of Poverty (2020) 9–24.
[23] S. Choi, and M. Ki, Estimating the reproductive number and the outbreak size of COVID-19 in Korea, Epidemical Health, Volume: 42, Article ID: e2020011, (2020) 1–10, https://doi.org/10.4178/epih.e2020011.
[24] J. Danane, K. Allali and Z. Hammouch, Mathematical analysis of a fractional differential model of HBV infection with antibody immune response, Chaos Solitons Fractals 136 (2020), 109787, 1–9.
[25] Y. M. Hamada, Solution of a new model of fractional telegraph point reactor kinetics using differential transformation method, Appl. Math. Model. 78 (2020), 297–321.
[26] Q. Lin, S. Zhao, D. Gao, Y. Lou, S. Yang, S. S. Musa, M. H. Wang, Y. Cai, W. Wang, L. Yang and D. He, A conceptual model for the Corona-virus disease 2019 (COVID-19) outbreak in Wuhan, China with individual reaction and governmental action, International Journal of Infectious Diseases 93 (2020), 211–216.
[27] W. C. Roda, M. B. Varughese, D. Han and M. Y. Li, Why is it difficult to accurately predict the COVID-19epidemic? Infectious Disease Modelling 5 (2020) 271–281.

A. S. Shaikh, I. N. Shaikh and K. S. Nisar, A Mathematical model of COVID-19 using fractional derivative:Outbreak in India with dynamics of transmission and control. Preprints 2020, 2020040140 (doi:10.20944/preprints202004.0140.v1).

[28] P. Veeresha, D. G. Prakasha, N. S. Malagi, H. M. Baskonus and W. Gao, New dynamical behaviour of the Corona-virus (COVID-19) infection system with nonlocal operator from reservoirs to people, Research Square, Preprints 2020, 1–18.

[29] World Health Organization. "Coronavirus disease 2019". cited July 31, 2020. Available: https://www.who.int/health-topics/Corona-virus.

[30] DE Kirk. Optimal control theory: an introduction. Dover Publications, 2012.

[31] SM Lenhart and JT Workman. Optimal control applied to biological models, volume 15.CRC Press, 2007.

[32] X Wang. Solving optimal control problems with MATLAB: Indirect methods. Technical report, ISE Dept., NCSU, 2009.