Fuzzy Ranking Approach for Randomized Block Designs using Median Value

K. Gnanapriya[1], M. Pachamuthu[2] and S. Kavitha[3]

[1]Research Scholar, Department of Statistics, Periyar University, Tamil Nadu, India.
[2]Assistant Professor, Department of Statistics, Periyar University, Tamil Nadu, India.
[3]Assistant Professor, Department of Statistics, Periyar University, Tamil Nadu, India.

Abstract

The Randomized Block Designs (RBD) is widely used in agricultural and industrial fields for many decades and uncertainty is usual to all real-life problems, to represent randomness and fuzziness. In such situations the statistical analysis of RBD using fuzzy observations are inevitable. In this paper, it proposes the statistical analysis of RBD through fuzzy ranking method using the location of median value in support of cardinality under Exponential Trapezoidal Fuzzy Numbers (ETZFNs). The numerical examples of the proposed approach would be more precise.

Keywords: *Decision Rule, Exponential Trapezoidal Fuzzy Numbers, Median Value, Randomized Block Designs, Ranking Method.*

AMS Mathematics Subject Classifications (2020): 62A86, 62K10.

1. Introduction

The Analysis of Variance (*ANOVA*) was first introduced by Prof. R.A. Fisher in 1920's [9] to deal with problems in agricultural and biological experiments. Now it is used in every field of science. A one-way classification is based on only one factor or one criterion. Two-way classification is based on two factors or two criteria. Experimentation is a necessary basis for inventions and innovations. Design of experiments is used to identify cause and effect of relationships and there are three basic principles of experimental designs are replication, randomization and local control. Completely Randomized Designs (*CRD*) is simplest of all the designs based on principle of randomization and replication. In some cases when the experimental materials are not homogenous. In *RBD* divide the experimental area into smaller homogenous blocks and the treatment are applied to this block at random and are replicated over all the blocks. The error of any treatment can be isolated and any number of treatments may be omitted from the analysis without complicating it. The data provided in this study are vague and need an extended version of the *RBD* to investigate these vague observations. In 1965 Lotfi A. Zadeh [16] introduces a fuzzy set theory. This theory provides appealing coordination bonds for integrating membership values that represent uncertain information. A fuzzy number is a generalization of a normal, real number, which does not point to a single value, but rather to a set of connected possible values, where each possible value has a membership function of its own weight between 0 and 1. Triangular Fuzzy Numbers (*TFNs*) are uniquely appropriated by triplet with membership functions which can be more sophisticatedly and efficiently method. Trapezoidal Fuzzy Numbers (*TZFNs*) are located on the left and right can be adjusted as desired by choosing the compact resistance values of a variable. *TZFNs* have lots of uses in the efficient adjustment of imprecise information. *ETZFNs* are an interesting method for fuzzy extension numbers. Ranking of fuzzy numbers is foremost essential in decision making problem. Many authors have discussed these methods. Some of them are; Jain [10] has first proposed ranking fuzzy numbers to make decision making in fuzzy situations. Dubois and Prade [6] have suggested that any fuzzy interval or fuzzy number can be viewed as a real number. Shaun-Hu Chen [4] has shown the representation of an exponential fuzzy number of rankings and distances then obtains some results by obtaining the functions of measurable product, addition and multiplication. Bodjanova [2] has introduced the concept of a fuzzy number A with respect to the distribution of their cardinality. Chen [3] has proposed a new algorithm of centroid points and standard deviations using ranking generalized trapezoidal fuzzy numbers. Sanjib Kumar Behera and Dhyan Singh [1] stated that the continuous application of manganese fertilizer in the soil as a fraction of a dose directly affected the randomized block designs. Shyi-Ming Chen [5] has proposed a new method for ranking generalized fuzzy numbers with different left heights and right heights. Salim Rezvani [12] has developed a new Method for Ranking in two Generalized Trapezoidal Fuzzy Numbers to location of median value in the support of cardinality. Rezvani [13] proposed the concept of ranking cardinality between exponential trapezoidal Fuzzy Numbers. Rezvani [14] introduced the concept of ranking exponential trapezoidal Fuzzy Numbers under TRD distance. Parthiban and Gajivarathan [11] have identified a two-way ANOVA process under different types of trapezoidal fuzzy numbers using two illustrations with trapezoidal fuzzy numbers. Gnanapriya et al. [7] have proposed identifying decision level through balanced incomplete block design using trapezoidal membership functions based on α-cut interval method and followed by author [8] they proposed partially balanced incomplete block design using trapezoidal membership functions based on α-cut interval method. In this paper, the approach to fuzzy raking analysis of RBD using median value with light tails using exponential trapezoidal fuzzy numbers is illustrated by numerical examples.

2. Preliminaries

2.1 Definition of Generalized fuzzy number

A generalized fuzzy number $\tilde{A}$ is described as any fuzzy subset of the real line $\mathbb{R}$, whose membership functions $\mu_{\tilde{A}}$ satisfies the following conditions;

(i) $\mu_{\tilde{A}}$ is a continuous mapping from $\mathbb{R}$ to the closed interval $[0,1]$.

(ii) $\mu_{\tilde{A}}(x) = 0, -\infty < x \le a$.

(iii) $\mu_{\tilde{A}}(x) = L(x)$ is strictly increasing on $[a,b]$.

(iv) $\mu_{\tilde{A}}(x) = w, b \le x \le c$.

(v) $\mu_{\tilde{A}}(x) = R(x)$ is strictly decreasing on $[c,d]$.

(vi) $\mu_{\tilde{A}}(x) = 0, d \le x < \infty$.

Where, $0 < w \leq 1$and a, b, c and d are real numbers. This type of generalized fuzzy number is a *TZFNS*, and it is denoted by $\tilde{A} = (a, b, c, d; w)_{LR}$. When $w = 1$, this type of generalized fuzzy number is called Normal Trapezoidal Fuzzy Numbers (*NTZFNs*) denoted by $\tilde{A} = (a, b, c, d; 1)_{LR}$. However, these fuzzy numbers always have a fix range as $[c, d]$.

2.2 Definition of *ETZFNs* Membership Functions

The *ETZFNs* if it's membership functions identifying by four parameters, $\tilde{A} = (a, b, c, d\,; w)_E \in$ □ as follows:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & ; \quad x < a \\ we^{-\left[(b-x)/(b-a)\right]} & ; \quad a \leq x \leq b \\ w & ; \quad b \leq x \leq c \\ we^{-\left[(x-c)/(d-c)\right]} & ; \quad c \leq x \leq d \\ 0 & ; \quad x > d \end{cases}$$

Where, $0 < w \leq 1, a, b$ are real numbers, and c, d are positive real numbers . Denote this type of generalized exponential fuzzy number as $\tilde{A} = (a, b, c, d; w)_E$. Especially,
3 when $w = 1$,denote it as $\tilde{A} = (a, b, c, d; 1)_E$ and define the representation of generalized exponential fuzzy number based on the integral value of grade mean h- level as follows. The generalized exponential fuzzy numbers is $\tilde{A} = (a, b, c, d)_E$. Now, two monotonic functions be

$$L(x) = we^{-\left[(b-x)/(b-a)\right]}; \; R(x) = we^{-\left[(x-c)/(d-c)\right]}$$

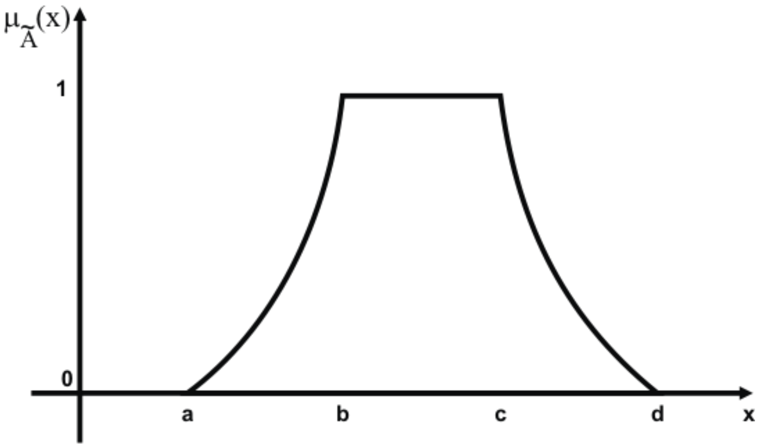

Figure 3: Exponential Trapezoidal Fuzzy Numbers

2.3 Definition of Scalar Cardinality

A fuzzy set $\tilde{A}$ defined on a finite universal set X is defined by its scalar cardinality $\left|\tilde{A}\right|$ by the formula is $\left|\tilde{A}\right| = \sum_{x\in X} \mu_{\tilde{A}}(x)$.

2.4 Definition of Fuzzy Cardinality

The fuzzy number of cardinality $\tilde{A}$ is the value of the integral. $card\ \tilde{A} = \int_a^b \mu_{\tilde{A}}(x)dx = \int_0^1 (b_\alpha - a_\alpha)\, d\alpha$.

2.5 Definition of Fuzzy Median Value

The median value of a fuzzy number $\tilde{A}$ is the real number $m_{\tilde{A}}$ from the support of $\tilde{A}$. The fuzzy median value $m_{\tilde{A}}$ is the point that partitions of numbers under the membership functions of a fuzzy set into two equal regions satisfying the following equation is $\int_a^{m_{\tilde{A}}} \mu_{\tilde{A}}(x)\, dx = \int_{m_{\tilde{A}}}^{d} \mu_{\tilde{A}}(x)\, dx$. Here, a and d are the endpoints in the base variable of the fuzzy set $m_{\tilde{A}}$ such that $a < d$.

3. A Fuzzy Rank Analysis of *RBD* using *ETZFNs*

Generally, the crisp *ANOVA* model for *RBD* references found in [9].The fuzzy test of hypothesis of ranking analysis of *RBD* where the observed data are trapezoidal fuzzy numbers is proposed here. Using the theorem, we convert trapezoidal fuzzy numbers to exponential trapezoidal fuzzy numbers. Let analysis of *RBD* by assigning rank for each normalized exponential trapezoidal fuzzy numbers and established on the ranking grades the decisions are observed. In truth, ranking method of all fuzzy numbers using fuzzy analytic are crisp and here performed analysis of *RBD* test as usual and better decisions can be obtained.

3.1 Median Value for Ranking Method of *ETZFNs*

As per the rendition in different types of fuzzy numbers with cardinality method by S. Bodjanova[2] and S.Rezvani [12] delivered a median value and ranking function of exponential trapezoidal fuzzy numbers.

Proposition 3.2 If $\tilde{A} = (a,b,c,d)$ is a fuzzy number with light tails then

$$m_{\tilde{A}} = \frac{a+b}{2} + 0.5\left(\int_c^d \mu_{\tilde{A}}(x)dx - \int_a^b \mu_{\tilde{A}}(x)dx \right)$$

Theorem 3.3 If $\tilde{A}$ is an exponential trapezoidal fuzzy number with light tails then

$$m_{\tilde{A}} = \frac{w(b+c)}{2} + \frac{w}{2e}\left[(c-d)(1-e) - (b-a)(e-1)\right]$$

Proof.

$$m_{\tilde{A}} = \frac{w(b+c)}{2} + \frac{1}{2}\left[\int_c^d \mu_{\tilde{A}}(x)dx - \int_a^b \mu_{\tilde{A}}(x)dx \right]$$

$$m_{\tilde{A}} = \frac{w(b+c)}{2} + \frac{1}{2}\left[\int_c^d we^{-[(x-c)/(d-c)]}\,dx - \int_a^b we^{-[(b-x)/(b-a)]}\,dx\right]$$

$$m_{\tilde{A}} = \frac{w(b+c)}{2} + \frac{1}{2}\left[w(c+d)\left(\frac{1}{e}-1\right) - w(b-a)\left(1-\frac{1}{e}\right)\right]$$

$$m_{\tilde{A}} = \frac{w(b+c)}{2} + \frac{w}{2e}\left[(c+d)(1-e)-(b-a)(e-1)\right] \tag{1}$$

Now the results (1) are location of the median value $m_{\tilde{A}}$ in the support of $\tilde{A}$ and also identify the fuzziness of $m_{\tilde{A}}$ determined by its membership grade $\tilde{A}(m_{\tilde{A}})$. The proposed theorem to test hypothesis of fuzzy ranking analysis of *RBD* is illustrated with the following two numerical examples.

4. Applications 1

The paddy yields were collected through primary data at Salem district. The collected data is four varieties of paddy crops [ADT 45, White Ponni, PBT 5204, and TKM 13] and three types of fertilizers [N,P,K] are explored in analysis of *RBD*. The data of *RBD* studied were approximate. Hence fuzzy observations are needed to measure yield. Therefore, fuzzy observations are required to measure the fuzziness of this yield. For the purpose of the present data is based on *TZFNs* in kilograms per hectare.

Table 1: Table of *TZFNs* for Yield of Paddy Crops

Fertilizers	Varieties			
	ADT 45	White Ponni	PBT 5204	TKM 13
N	[59,62,65,68]	[22,24,25,27]	[22,24,25,27]	[43,47,50,54]
P	[43,47,50,54]	[31,34,37,40]	[12,15,17,20]	[12,15,17,20]
K	[5,8,10,13]	[31,34,37,40]	[22,24,25,27]	[12,15,17,20]

Test whether there is a significant difference in the yield varieties and fertilizers of paddy crops.

Let us consider Table 1, using the above theorem 3.3 and we get each *ETZFNs* $m_{\tilde{A}}$ Which are tabulated below:

Table 2: Table for *RBD* using Rezvani Median Value of ETZFNs

Fertilizers	Varieties			
	ADT 45	White Ponni	PBT 5204	TKM 13
N	63.5	24.5	24.5	48.5
P	48.5	35.5	16	16
K	9	35.5	24.5	16

Based on this rank of *ETZFNs*, The *ANOVA* table values of using $m_{\tilde{A}}$ ranking grades:

(i) Total Sum of Squares (*TSS*) = 2987.

(ii) Blocks Sum of Squares (*BSS*) = 568.8569.

(iii) Treatments Sum of Squares (*TrSS*) = 730.5.

(iv) Error sum of Squares (*ESS*) = 1687.

All these results are summarized in Table 3 is drawn.

Table 3: *ANOVA* Table for *RBD* using *ETZFNs*

SV	df	SS	MSS	$\tilde{F}$ - Ratio
Blocks	3	731	243.6667	1.1539
Treatment	2	569	284.5	1.0119
Remainder	6	1687	281.1667	-
Total	11	2987	-	-

$\tilde{F}$ -Ratio for Varieties

$$\tilde{F}_B = \frac{MSS_B}{MSS_E} = 1.0119 \text{ at } F_t = 5.14. \text{ Here } \tilde{F}_B < F_t$$

The null hypothesis $\tilde{H}_0$ is accepted. The difference between varieties is no significant. Therefore, the four varieties do not differ significantly with respect to the paddy crops yield.

$\tilde{F}$ -Ratio for Treatments

$$\tilde{F}_{Trt} = \frac{MSS_{Trt}}{MSS_E} = 1.1539 \text{ at } F_t = 8.94. \text{ Here } \tilde{F}_{Trt} < F_t$$

The null hypothesis $\tilde{H}_0$ is accepted. The difference between varieties is no significant. Therefore, the three varieties do not differ significantly with respect to the paddy crops yield.

4.1 Applications 2

The following observed data are the petrol consumption rates through primary data at Salem district. There were 4 (A, B, C and D) different Makes of cars. The different speeds were (25, 35, 50, 60 and 70 mph). A problem was posed to estimate the petrol consumption rates of the different Makes of cars for suitable average speed and compare them. The proposed method of investigating *RBD* was taken to be *TZFNs* as follows [15].

Table 4: Table of *TZFNs* for Miles per gallon of petrol

Makes of cars	Speeds of the cars in miles per hour (mph)				
	25	35	50	60	70
A	[17,20,23,25]	[18,19,21,22]	[15,17,19,21]	[15,17,19,21]	[12,15,17,21]
B	[18,19,21,22]	[15,18,20,22]	[12,15,17,21]	[14,16,19,20]	[12,13,15,17]
C	[17,20,23,25]	[15,18,20,22]	[12,15,17,21]	[13,14,16,17]	[12,13,15,17]
D	[12,15,17,21]	[14,16,19,20]	[12,15,17,21]	[13,14,16,17]	[10,11,14,17]

Test whether there is a significant difference in Makes of car and Speeds in the petrol consumption rates.

Let us consider Table 4, using the above theorem 3.3 and we get each *ETZFNs* $m_{\tilde{A}}$ Which are tabulated below:

Table 5: Table for *RBD* using *ETZFNs*

Makes of cars	Speeds of the cars in miles per hour (mph)				
	25	**35**	**50**	**60**	**70**
A	23.3220	20	18	18	16.3644
B	20	18.6356	16.3644	16.7712	14.3644
C	23.3220	18.6356	16.3644	15	14.3644
D	16.3644	16.7712	16.3644	15	13.2288

Based on this rank of *ETZFNs*, The *ANOVA* table values of using $m_{\tilde{A}}$ ranking grades:

(i) Total Sum of Squares (*TSS*) = 140.9063.
(ii) Blocks Sum of Squares (*BSS*) = 89.0496.
(iii) Treatments Sum of Squares (*TrSS*) = 32.4964.
(iv) Error sum of Squares (*ESS*) = 19.3603

All these results are summarized in Table 6 is drawn.

Table 6: *ANOVA* Table for *RBD* with *ETZFNs* using Ranking Method

SV	df	SS	MSS	$\tilde{F}$ - Ratio
Speeds	4	89.0496	22.2624	13.7984
Makes of cars	3	32.4964	10.8321	6.7138
Remainder	12	19.3603	1.6134	-
Total	19	140.9063	-	-

$\tilde{F}$ -Ratio for Speeds

$$\tilde{F}_B = \frac{MSS_B}{MSS_E} = 13.7984 \text{ at } F_t = 3.26. \text{ Here } \tilde{F}_B > F_t$$

The null hypothesis $\tilde{H}_0$ is rejected. The difference between speeds is significant. Therefore, the five speeds differ significantly with respect to the miles per gallon of petrol.

$\tilde{F}$ -Ratio for Makes of cars

$$\tilde{F}_{Trt} = \frac{MSS_{Trt}}{MSS_E} = 6.7138 \text{ at } F_t = 3.49. \text{ Here } \tilde{F}_{Trt} > F_t$$

The null hypothesis $\tilde{H}_0$ is rejected. The difference between makes of cars is significant. Therefore, the four different makes of car is differ significantly with respect to the miles per gallon of petrol.

5. Conclusions

In practical cases, the data received for decision making are only approximately known. *ETZFNs* of membership functions are quite common in real-world phenomenon. In this paper, it is proposed an approach for the statistical analysis of *RBD* through fuzzy ranking method using the location of median value with light tails in support of cardinality under *ETZFNs*. The proposed numerical examples are very close to the real numbers. Generally, this method can be used in any kind of real-time calculations for exponential trapezoidal fuzzy numbers.

References

1. Behera S. K. and Singh D., "Impact of continuous fertilizer use on fractions of manganese in soil and their contribution to availability and its uptake by maize (Zea mays)-wheat (Triticum aestivum) cropping system," *Indian J. Agric. Sci.*, vol. 80, no. 4, pp. 316–320, 2010.

2. Bodjanova, S. (2005). Median value and median interval of a fuzzy number. *Information sciences*, *172*(1-2), 73-89.

3. Chen S. J. and Chen S. M. (2007). Fuzzy risk analysis based on the ranking of generalized trapezoidal fuzzy numbers. *Applied intelligence, 26*(1), 1-11.

4. Chen S. H. and Li G. C. (2000). Representation, ranking and distance of fuzzy number with exponential membership function using graded mean integration method.

5. Chen S. M, Munif A, Chen G. S, Liu H. C. and Kuo B. C. (2012). Fuzzy risk analysis based on ranking generalized fuzzy numbers with different left heights and right heights. *Expert Systems with Applications, 39*(7), 6320-6334.

6. Dubois D and Prade, H. (1987). The mean value of a fuzzy number. *Fuzzy sets and systems*, *24*(3), 279-300.

7. Gnanapriya K, Kavitha S and Pachamuthu M. (2020). A Statistical Analysis of Fuzzy Balanced Incomplete Block Designs with Intra-Block Analysis using Trapezoidal Method. *International Journal of Recent Technology and Engineering*, 8(6).

8. Gnanapriya K, Pachamuthu M and Kavitha S. (2020). A Statistical Analysis of Partially Balanced Incomplete Block Designs using Fuzzy Approach. *Journal of Xi'an University of Architecture & Technology*, Volume XII, Issue X, pg.no 79-91.

9. Gupta S. C. and Kapoor, V. K. (2000). Fundamentals of Mathematical Statistics (A Modern Approach), S. *Chand & Sons, New Delhi*, 10-1.

10. Jain R. (1976). Decision making in the presence of fuzzy variables, IEEE Transactions on Systems, Man and Cybernetics, 6 (10), 698-703.

11. Parthiban S. and Gajivaradhan, P. (2016). A comparative study of two factor ANOVA model under fuzzy environments using trapezoidal fuzzy numbers. *Int. J. of Fuzzy Mathematical Archive*, *10*(1), 1-25.

12. Rezvani S. (2013). Ranking exponential trapezoidal fuzzy numbers by median value. *Journal of Fuzzy Set Valued Analysis*, *2013*, 1-9.

13. Rezvani S. (2014). Ranking exponential trapezoidal fuzzy numbers with cardinality. *Comun Korean Math Soc*, *29*, 187-193.

14. Rezvani S, Aghdam M. M. and Ebrahimi M. (2014). A Approach for Ranking of Exponential Fuzzy Number With TRD distance. *Journal of Fuzzy Set Valued Analysis*, *2014*, 1-11.

15. Wu H. C. (2007). Analysis of variance for fuzzy data. *International Journal of Systems Science*, *38*(3), 235-246.

16. Zadeh L. A. (1965). Fuzzy sets. *Information and control*, *8*(3), 338-353.