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Mathematical Modeling On Population Growth In A Randomly Fluctuating Environmental Ecosystem.

Rakesh Yadav

Department of Mathematics, School of Chemical Engineering and Physical Sciences
Lovely Professional University, Jalandhar Punjab
Email: rakesh.21798@lpu.co.in

Abstract: ***In this paper, we are framing a mathematical model describing the population growth in a randomly fluctuating environmental ecosystem. we discuss the change in net growth rate by stochastic models and changes in the death rate using continuous model approximatingthe situation.***

Keyword:*Insect,grains,protein,populationgrowth,environmentalect.*

1.Introduction:

In sect feed upon grain in granaries results in quantitative and qualitative losses are loss in protein .feeding insects destroy the germ and endosperm thus decreasing the qualitative values of the grain. In general qualitative losses are loss in protein, insect fragment and excrement contamination, change in non-reducing sugar and loss in germinative value [1-6]. To estimate these losses and specify the factors responsible various experiments were conducted and data collected and found that insect population is major factor [7-9] .

Conducted the various experiments on kernel infestation percent and protein loss in stored grain However the increase and decrease in percent protein after insect damage variates. He found that protein loss depends upon the insect population and insect population depends upon the various environmental factors. Here we study the change in the insect population in a randomly fluctuating environment on a successfull colonized population[2, 10-12]. Let the population size is large than the critical size. The view of above assumption the relative change in the size of the population are small.

2.Formulation of the Model and Discussion: Let the population size be x which is continuous variable in the absence of fluctuating environment , the population is assumed to grow according to equations:

$$\frac{dx}{dt} = \frac{\overline{r}x\left[1 - \left(\frac{x}{R}\right)\right]^n}{n}, r > 0$$

Here we will discuss the case when $n = 1$ and leave the case when $n \neq 1$.The fluctuating environment may affect the growth in several different ways ,some of which we discuss below[3, 13]

2.1 Changes in net growth rate:

The fluctuating environment may introduce a stochastic form of r may be given by the equation

$\gamma = \overline{r} + \sigma F(t)(2)$

Where $F(t)$ is some noise .Since environmental changes are due to many factors ,and are fast compared to time scale of population growth[4, 12-14] .Here σ is a constant and $< r >= \overline{r}$ .The stochastic differential equation describing the growth of the population then becomes:

$\frac{dx}{dt} = \overline{r}x\left(1 - \frac{x}{R}\right) + \sigma x\left(1 - \frac{x}{R}\right)F(t)$ (3)

On comparing to equation (3) with the FokkaPllankequation :

$\frac{\partial P}{\partial t} = -\frac{\partial}{\partial x}\left[a(x)P + \frac{1}{2}\frac{\partial^2}{\partial x^2}b(x)P\right]$ (4)

We can write

$a(x) = \left(1 - \frac{x}{R}\right)\left(\overline{r} + \frac{\sigma^2}{2}\right)\left(1 - \frac{2x}{R}\right)$ (5)

$$b(x) = \sigma^2 x^2\left(1 - \frac{x}{R}\right)^2$$

The boundaries $x = 0, x = K$ are singular, approx. $x = 0, a(x) \approx \left(r + \frac{\sigma^2}{2}\right) x and b(x) = \sigma^2 x^2$

and by the classification of singular boundaries, the boundary x=0 is an inaccessible natural boundary and never be reached .

Similarly, the boundary at x=k is a natural boundary and cannot reached in a finite time .This process describes a population which is far from extinction and which fluctuates about some average value (<k) due to fluctuations in the net growth rate[5]. The state probability density function given by equation

$P\left(\frac{x}{y}, \infty\right) = \frac{C}{b(x)}e^{2\int_0^x\left[\frac{a(\zeta)}{b(\zeta)}\right]d\zeta}$ (6)

Where C is determined by the condition $\int P(x/y, \infty)dx = 1$

Using equation (5) in equation (6) ,We arrive at

$P(x, \infty) = C_x\left(\frac{2\overline{r}}{\sigma^2} - 1\right)\left(1 - \frac{x}{R}\right).\left(\frac{2\overline{r}}{\sigma^2} + 1\right)$ (7)

Where C is the normalization constant. In equation (8) quantities $\left(\frac{2\overline{r}}{\sigma^2} - 1\right) and. \left(\frac{2\overline{r}}{\sigma^2} + 1\right)$ are replaced by $\left(\frac{2\overline{r}}{\sigma^2} - 2\right) and. \left(\frac{2\overline{r}}{\sigma^2} + 2\right)$respectively .If $\left(\frac{2\overline{r}}{\sigma^2}\right) < 1$ the density function (7) is U shaped indicating the tendency of the population to be either near zero or near R.

If $\left(\frac{2\overline{r}}{\sigma^2}\right) > 1$ then the density function (7) is monotonically increasing and graph is of J shaped, indicating accumulation of the population near R[6].The minimum of the U-shaped distribution is given by

$$a(x) = \frac{1}{2}\frac{db}{dx} i.ex = \left(1 - \frac{2\overline{r}}{\sigma^2}\right)\frac{R}{2}$$

To drive the time dependent probability density function, we introduce the variable

$$Z = \frac{1}{\sigma}\left[\frac{log\frac{x}{1-x}}{R}\right]$$

$dz = \left[\sigma x\left(\frac{1-x}{R}\right)\right]dx$ (8)

Equation (4) becomes $\frac{dz}{dt} = \left(\frac{\overline{r}}{\sigma}\right) + F(t)$ (9)

Which is the stochastic differential equation for an unrestricted weiner process; hence the Fokker Plank equation satisfied by the probability density function of is $z, g\left(\frac{z}{Z_0}, t\right)$

$\frac{\partial g}{\partial t} = -\frac{\partial}{\partial z}\left(\frac{\overline{r}}{\sigma}\right)g + \frac{1}{2}\frac{\partial^2 g}{\partial z^2}$(10)

With the boundary conditions $\lim_{z \to \pm\infty} g\left(\frac{z}{Z_0}, t\right) = 0$ (11)

Corresponding to the inaccessible boundaries $x = 0(z = -\infty) and x = R(z = +\infty)$ also

$g\left(\frac{z}{Z_0}, t\right) = \frac{1}{\sqrt{2\Pi t}}e^{\left[-\frac{1}{2t}\left(\frac{z - z_0.rt}{\sigma^2}\right)\right]}$(12)

Using equation (10) into (13) we get

$P\left(\frac{x}{y},t\right)=\frac{1}{\sigma\sqrt{2\Pi t}y^2x\left(1-\frac{x}{R}\right)}exp\left[-\frac{1}{2t}\left(\frac{1}{\sigma}log\frac{x}{y}-\frac{1}{\sigma}log\frac{1-x/R}{1-y/R}-\frac{rt^2}{\sigma}\right)\right]$ (13)

Which is the density function determining the behavior of the population using above probability function various moments of x can be calculated by numerical integration .When $R>-\infty$ i.e either the population is for from saturation or the supply of food in unlimited the integration can be evaluated analytically[7] .for this the equation (9) gives

$x=e^{\sigma_z}$ (14)$and <x>=$

$\int_0^R xP(x/y,t)\,dx$

$=\int_{-\infty}^{+\infty}e^{\sigma_z}\,g\left(\frac{z}{Z_0},t\right)dz$

$=exp\left(\frac{\sigma^2t}{2}\right)exp\left\{\sigma\left(Z_0+\frac{\overline{r}t}{\sigma}\right)\right\}$

$=ye^{\overline{r}t}\,exp\left(\frac{\sigma^2t}{2}\right)$ (15)

As compared to $x=ye^{\overline{r}t}$ (16)

For the deterministic case in the absence of random fluctuations .

Now we have

$Var(x)=<x^2>-<x>^2=y^2e^{2\overline{r}t}[exp(\sigma^2t)-1]$

$=<x>^2\ [exp(\sigma^2t)-1]$ (17)

As compared to zero variance for the deterministic case. The coefficient of variation is given by

$\frac{[Var(x)]^{1/2}}{<x>}=[exp(\sigma^2t)-1]^{1/2}$ (18)

As t increases ,the coefficient of variation is increasing and already for moderate times the average cannot describe the growth of the population.

2.2 Change in the death rate: If the number of Zygotes (a cell formed by the union of two germ cells) produced is large compared to the adult population, and is therefore relatively less subject to random fluctuations[8] .then random variation will occur mostly in the death of adults .We now drive a continuous model approximating this situation.

Let the deterministic birth rate be λ_x when the population size is x .Let $\mu\Delta t$ be the probability for a given individual to die in time $\Delta t,$(independent of the age of the individual).If $x(t)$ is the number of individuals at time t , the number of individuals at time $(t+\Delta t)$ is a random variable which takes the value.

$x(t+\Delta t)=x(t)+\lambda_x\Delta t_{-i}(1)$

With probability $\frac{x(t)}{i}\mu^i(\Delta t)^i(1-\mu\Delta t)^{x-i}$ (2)

Up to first order in Δt the number $i$is Poisson distributed since

$<i>=\mu x\Delta t(3)$

$<i^2>=\mu x\Delta t(1-\mu\Delta t)+(\mu x\Delta t)^2$ (4)

Thus from equations (3) ,(4),(1) we get

$<x(t+\Delta t)-x(t)>=<\Delta x(t)>=(\lambda_x-\mu_x)\Delta t(5)$

$<[\Delta x(t)]^2>=\lambda_x(\Delta t)+2\lambda_x x\mu(\Delta t)^2+x\mu\Delta t(1-\mu\Delta t)+(x\mu\Delta t)^2(6)$

Therefore

$\lim_{\Delta t>0}\frac{1}{\Delta t}<\Delta x(t)>=\lambda_x-\mu_x$ (7)

$\lim_{\Delta t>0}\frac{1}{\Delta t}<\Delta x(t)>^2=\mu x$ (8)

If λ_x is chosen to be of the same form as $\lambda(n)$ of model

$\lambda(n) = \lambda\left[1 - \left(\frac{n}{R}\right)\alpha\right]$ with $\alpha = 1$ (9)

Using equation (7) and (8) the Fokker Plank equation (1.4) for the continuous model is given by

$$\frac{\partial P}{\partial t} = -\frac{\partial}{\partial n}\left[a(x)P + \frac{1}{2}\frac{\partial^2}{\partial x^2}b(x)P\right] \quad (10)$$

With $a(x) = rx\left(1 - \frac{x}{R}\right), = \lambda - \mu$, $R = K\left(1 - \frac{\mu}{\lambda}\right)$ (11)

$b(x) = \mu x$(12)

Here we may remark that if we had assumed the probability for a given individual to die in time Δt to be not constant ($\mu\Delta t$) but of the form $\left[1 + \left(\frac{x}{k}\right)\Delta t\right]$, R in equation (11) would have been relard to K in the same fashion in model (9) ,but $b(x)$ would have become $\mu x(1 + (x/K)^k)$.

Thus in deriving equation (12) ,we have incorporated the reduction in fertility due to the limitation of food, but neglected the increase in the probability of death.

Determination of Steady State Distribution:To determine the steady state distribution , we suppose that the variable x is confirmed to $x \geq 0$, with boundary condition $x = 0$ being singular $[b(0) = 0]. Near\ x = 0 , b(x) = \mu x\ and\ a(x) \approx rx$

And that the boundary condition is an exit boundary i.e whatever reaches the boundary $n = 0$ is trapped there forever , corresponding to the extinction of the population [9].

For large values of $x(n > R)$, $a(x) = (\frac{r}{R})\, x^2$ and $x = \infty$ is an entrance boundary.Thus eventually is bound to occur and the steady state probability density function is zero for all $x > 0$.

To determine the value of $P\left(\frac{x}{y}\ , t\right)$ we transform the Fokker- Plank equation (10) by using some transformation $dz = \frac{dx}{\sqrt{\mu x}}$

$z = 2\left(\frac{x}{\mu}\right)^{1/2}$ (13)

$$P(x/y\ , t) = \frac{g(\frac{z}{z_0}, t)}{\sqrt{\mu x}} \quad (14)$$

In the light of equations (12) , (13) , (14) we arrive at ,

$\frac{\partial g}{\partial t} = -\frac{1}{2}\frac{\partial}{\partial z}(rz - \frac{rz^3\mu}{4R} - \frac{1}{z})g + \frac{1}{2}\frac{\partial^2 g}{\partial z^2}$(15)

Now on solving the equation by using simple transformation we get

$\zeta = z\left(\frac{r\mu}{4R}\right)^{1/4}$, $\alpha = 2\left(\frac{rR}{\mu}\right)^{1/2}$, $J = \frac{1}{4}\ t\ \left(\frac{r\mu}{R}\right)^{1/2}$(16)

So equation (16) reduces to $\frac{\partial g}{\partial t} = \frac{\partial}{\partial \zeta}\left[(\frac{1}{\zeta} - \alpha\zeta + \zeta^3)g\right] + \frac{1}{2}\frac{\partial^2 g}{\partial \zeta^2}$(17)

If we replace $1/\zeta\ by\ -1/\zeta$ then above equation becomes similar to the equation which has been studied in the theory of noise in the laser [4] .

Approximate solutions to equation (10) can be derived for tow regimes:

1.The Malthusion regime when the initial size y is small ($y < R$, or equivalently R $-----> \infty$)

2.The regime in which yy $\approx$ R for the Maelthus regime $\mathrm{a(x)} \approx \mathrm{rx}$ so that

$$P(x/y, t) = \frac{2r}{\mu} exp\left[\frac{2rx + ye^{rt}}{\mu e^{rt-1}}\right] I_1\left[\frac{4r}{\mu}\frac{\sqrt{xy}}{e^{rt/2} - e^{-rt/2}}\right] \quad (18)$$

According to the density equation , the Ith moment of $x(t)$ *is given by*

$\langle x^e(t)\rangle = \int_0^\infty x^e P(x/y,t)dx = ye^{rt}\left[\frac{\mu(e^{rt}-)}{2r}\right]^{e-1} exp\left[-\frac{2r}{\mu}\frac{y}{(1-e^{-rt})}\right]\Gamma(I+1)F\left[I+1:2;\ \frac{2ry}{\mu(1-e^{-rt}}\right]$(19)

In particular buy using the standard formula $F(a;a;z) = e^z$ (20)

$F(a+1;a;z) = (a+z/a)F(a;a;z)$(21)

We obtain $\langle x(t)\rangle = ye^{rt}$ (22)

$\langle x^2(t)\rangle - \langle x(t)\rangle^2 = \frac{\mu}{r}\langle x(t)\rangle(e^{rt}-)$ (23)

The average value given by equations (24) is identical to the Malthusian deterministic behavior

$b(x) = 0$. This is expected value since a(x) is linear in x .T be Probability of population growing without limit is $T(\frac{\infty}{y}) = \frac{\int_0^y \Pi(\eta)d\eta}{\int_0^\infty \Pi(\eta)d\eta}$ (24)

where $\Pi(\eta) = exp(-2\int_0^\eta \frac{a(y)}{b(y)}\,dy)$ (25)

substituting for a(y) and b(y) from equations (10) with $R = \infty$, $We\ get$

$T\left(\frac{\infty}{y}\right) = 1 - e^{-2ry/\mu}$ (26)

Thus for r >0 , the population will keep growing with probability$1 - e^{-2ry/\mu}$ and will become extinct with probability $1 - e^{-2ry/\mu}$, $r > 0$, t the fate of the population is extinction. So the probability of population having any size greater than zero.at infinite time is zero[10] .In the Malthusian regime ,the validity of the approximation is limited to short times and the limit t approaching to infinity have no meaning in this context.

In the regime $y \approx R$; $\frac{x-y}{R} \ll 1$ and we can take $a(x) \approx r[R-x]$, $r > 0$ (27)

Let us define the parameter by $\eta = \frac{2rR}{\mu}$

For $\eta > 1$ *the boundary* $x = 0$ *is an entrance boundary while for* $0 < \eta < 1$, it is a regular boundary which is reflecting.

For all $\eta > 0$,

$x = \infty$ is a natural in accessible boundary. The steady s tate probability density exists and from review of equations (6) ,(12) ,(28)is given by

$P(x/y.\infty) = cx^{\eta-1}exp\left\{-\eta\left(\frac{x}{R}\right)\right\}$ where c I a normalization constant and is given by

$c = \frac{\left(\frac{r}{R}\right)^\eta}{\Gamma(\eta)}$(28)

The most probable steady -state size of the population is

$x_0 = R - \frac{\mu}{2r} = R\left(1 - \frac{1}{\eta}\right)$ (29)

Which is positive and below R for$\eta > 1$. *As* η increase , this size tends to R. The moment of the population size at steady state is

$\langle x^e\rangle = \left(\frac{R}{\eta}\right)^e \frac{\Gamma(1+\eta)}{\Gamma(\eta)}$, $\langle x\rangle = R > x_0$ (30)

The time dependent probability density is

$P(x/y,t) = \frac{2r}{\mu}\left(\frac{x}{y}\right)^{(n-1)/2} \frac{e^{\eta rt/2}}{e^{rt/2}-e^{-rt/2}}\ exp\left[-\frac{2rx+ye^{-rt}}{\mu(1-e^{-rt})}\right] I_{n-1}\left[\frac{4r}{\mu}\frac{(xy)^{1/2}}{(e^{rt/2}-e^{-rt/2})}\right]$ (31)

From equation (30) and (33) ,we conclude that for all t >0 the probability of having a large population $(x \gg R)$is very small for all $\eta > 0$, $\Gamma I_v(Z) \approx \frac{e^z}{(2\Pi z)^{1/2}}$ for z ---->∞ While the probability of having population of small size is low when $\eta > 1$ and very high for $\eta < 1$>.Therefor the case $\eta > 1$ describes a population which fluctuates around as average far from zero,and only in this case the approximation leading equation (28) a good approximation to the original process (11) using equation (21) the lth moment is given by

$$\langle x^l \rangle = \left(\frac{\mu}{2r}\right)^l exp\left[-\frac{2ry}{\mu(e^{rt}-1)}\frac{\Gamma(l+n)}{\Gamma(\eta)}F\left(l+\eta;\ \eta; \frac{2ry}{\mu(e^{rt}-1)}\right)\right] \quad (32)$$

For l = 1,2 using the equations (22) ,(23) we get from the above equation

$$\langle x \rangle = R(1-e^{-rt}) + ye^{-rt} = R-(R-y)e^{-rt} \quad (33)$$

As expected, the average given by this equation indicated the deterministic behavior is obtained by solving the deterministic equation

$$\frac{dx}{dt} = r(R-x)$$

$$\langle x^2 \rangle - \langle x \rangle^2 = \frac{\mu}{2r}[\langle x \rangle + ye^{-rt}](1-e^{-rt}) \quad (34)$$

In this variance we get the comparison to the zero value in the deterministic approach

3.Conclusion:

We conclude from this model that present mathematical model lays emphasis on the protein loss depends upon the insect population and insect population depends upon the various environmental factors. We also study the change in the insect population in a randomly fluctuating environment on a success full colonized population .we discuss the change in net growth rate by stochastic models and changes in the death rate using and the model approximating the situationdescribes a population which fluctuates around as average far from zero.

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