General Class Of Improved Product Type Estimator

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Abstract: In survey sampling, auxiliary information provided by the auxiliary variables can be used to improve the precision of the estimators. Improved estimators can be developed by using auxiliary information at the estimation stage in order to estimate the population mean.In this paper, modified class of product type estimatorsfor estimation of population mean using auxiliary variable have been proposed. Relative bias andRelative mean square errors of the proposed estimators have been derived upto order O (n^{-1}) and O (n^{-2}) respectively and their efficiencies have been compared with the conventional product estimator and the estimators proposed by Robson (1957), Singh (1989), Dubey (1993) and Sharma et al., (2007).An empirical study has also been carried out through simulationin order to demonstratethe efficiencies of the proposed estimators using two population datasets P1 and P2.The empirical study showed that proposed product type estimator $t^*_{(3)} = \bar{y}_p + \frac{3}{n}\frac{s_{xy}}{\bar{x}\bar{X}}$ was found to be more efficient than the estimators proposed by Robson (1957), Singh (1989), Dubey (1993) and Sharma et al., (2007). The percent relative efficiency of the proposed estimator with respect to the conventional estimator and the estimators proposed by Robson (1957), Singh (1989), Dubey (1993) and Sharma et al., (2007) was found to lie in the range of 105.25 to 332.65.
Keywords: Auxiliary information, product type estimator, Relative mean square error and relative bias

1. Introduction

In survey sampling, auxiliary information provided by the auxiliary variables can be used to improve the exactitude of the estimators. Many techniques of using auxiliary information are available in the literature e.g., Cochran (1977), Murthy (1967), Singh and Chaudhary (1995), Bhushan et al., (2013), etc.It is a well-knownfact that product type estimators provide better estimates of population mean than a simple mean estimator if there is inverse relationship between the study and auxiliary (Robson, 1957) and the auxiliary variate satisfy the conditions: (i) if $-1 < \rho < -C_x/2C_y$ and both Y and X are positive or negative or (ii) if $C_x/2C_y < \rho \leq +1$ and either Y or X is negative (Singh and Chaudhary, 1995), where, $C_x = \frac{s_x}{\bar{X}}$ and $C_y = \frac{s_y}{\bar{Y}}$ are the coefficient of variation of X and Y respectively. Many researchers have worked to modify the conventional product estimator like Robson (1957), Murthy (1964), Ray and Sahai(1979), Chaudhuri and Arnab (1982), Rao (1987), Singh (1989), Kumar (2002), Sharma et al., (2007), Sharma and Bhatnagar (2008) etc.

2. Methodology

Consider the finite population of size N= (n_1, n_2, ...,n_N)andY denotes the study variable with population mean $\bar{Y}$whereasX represents auxiliary variable with population mean $\bar{X}$. $\bar{y}$ and

$\bar{x}$ denotes the sample means for variables Y and X respectively. Simple random sampling without replacement (SRSWOR) technique have been used to draw the samples of size n $(n < N)$. Further, s_y^2 and s_x^2 are unbiased estimators of population variances σ_y^2 and σ_x^2 respectively. Also, $s_{xy} = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})$ be an unbiased estimator of population covariance σ_{xy}.Further, $C_{ab} = \frac{1}{N-1}\sum_{i=1}^{N}\left(\frac{x_i-\bar{X}}{\bar{X}}\right)^a\left(\frac{y_i-\bar{Y}}{\bar{Y}}\right)^b$ where, a and b are non – negative integers. E (T), RB (T) and RM (T) denote expected value, relative bias (RB) and Relative mean square error (RM) of an estimator t, respectively. Also, $\theta = \left(\frac{C_{02}}{C_{20}}\right)^{1/2}$ and $\rho = \frac{C_{11}}{(C_{02})^{1/2}(C_{20})^{1/2}}$.In this paper, relative bias and relative mean square error have been derived upto first order O (n^{-1}) and second order $O(n^{-2})$ respectively.

2.1 SOME EXISTING PRODUCT TYPE ESTIMATORS

Conventional product type estimator has been proposed by Robson (1957) for estimating the $\bar{Y}$ is as

$$\bar{y}_p = \frac{\bar{y}\bar{x}}{\bar{X}}. \quad (1)$$

The RB and RM of the estimator $\bar{y}_p$ are as

$$RB(\bar{y}_p) = \frac{C_{11}}{n} \quad (2)$$

$$RM(\bar{y}_p) = \frac{1}{n}(C_{20} + C_{02} + 2C_{11}) + \frac{1}{n^2}(C_{20}C_{02} + 2C_{11}^2 + 2C_{12} + 2C_{21}) \quad (3)$$

Since, the estimator $\bar{y}_p$ proposed by Robson (1957) was biased. He developed another unbiased product estimator by eliminating unbiased estimate of the bias of $\bar{y}_p$as follows:

$$t_R^p = \bar{y}_p - \frac{1}{n}\frac{s_{xy}}{\bar{X}}. \quad (4)$$

The relative variance of t_R^p is as

$$RV(t_R^p) = RM(\bar{y}_p) - \frac{1}{n^2}[C_{11}^2 + 2(C_{12} + C_{21})]. \quad (5)$$

Singh (1989) attempted to propose an unbiased product estimator of$\bar{Y}$ as

$t^* = \frac{\bar{y}\bar{x}}{X}\left[1 + \frac{s_{xy}}{n\bar{x}\bar{y}}\right]^{-1}$, whose RB and RM of the estimator t^* are given by

$$RB(t^*) = \frac{C_{11}^2}{n^2} \text{ and } RM(t^*) = RV(t_R^p). \quad (6)$$

Thus, the estimator t^* is unbiased and has RM equal to relative variance of t_R^p.

Dubey (1993) used modified the product estimator proposed by Robson (1957) by using the estimate of the $\bar{X}$ instead of $\bar{X}$and proposed an almost unbiased product type estimator as

$$t_D^p = \bar{y}_p - \frac{s_{xy}}{n\bar{x}}. \quad (7)$$

The RM of the estimator t_D^p is given as

$$RM(t_D^p) = RM(\bar{y}_p) + \frac{1}{n^2}[2C_{20}C_{11} + C_{11}^2 - 2C_{12} - 2C_{21}]. \quad (8)$$

Sharma et al., (2007) developed a general improved class of product type by taking the usual product estimator as linear combination of $\bar{y}$ alongwith$\frac{s_x^2}{\bar{X}^2}$ as:$t_s^p = \bar{y}\left[\frac{\bar{x}}{\bar{X}} + q\frac{s_x^2}{\bar{X}^2}\right]$ (9)

Where, q is the scalar specifying the estimator.

The RB and RM of the estimator $t_s^p$are as

$$RB(t_s^p) = RB(\bar{y}_p) + q\frac{C_{20}}{n}, \quad (10)$$

$$RM(t_s^p) = RM(\bar{y}_p) + \frac{q}{n^2}[4C_{20}C_{11} + 2C_{20}C_{02} + 2C_{30} + 2C_{21} + qC_{20}^2]. \quad (11)$$

2.2 PROPOSED IMPROVED CLASS OF PRODUCT TYPE ESTIMATORS AND COMPARISONS WITH RESPECT TO EXISTING PRODUCT ESTIMATORS

By keeping above in mind, an improved class of product type estimators for estimation of $\bar{Y}$have been proposed as $\mathbf{t}^*_{(\mathbf{q})} = \bar{\mathbf{y}}_\mathbf{p} + \frac{\mathbf{q}}{\mathbf{n}}\frac{\mathbf{s_{xy}}}{\bar{\mathbf{x}}\bar{\mathbf{X}}}$,where, q is scalar specifying the estimator. (12)

The RB and RM of the estimator $t^*_{(q)}$respectively as

$$\mathrm{RB}(t^*_{(q)}) = \mathrm{RB}(\bar{y}_p) + \frac{q}{n}C_{11}, \quad (13)$$

$$\mathrm{RM}(t^*_{(q)}) = \mathrm{RM}(\bar{y}_p) + \frac{q}{n^2}[qC_{11}^2 - 2C_{11}^2 - 4C_{20}C_{11} + 2C_{12} + 2C_{21}]. \quad (14)$$

The general class of improved product type estimator $t^*_{(q)}$reduces to conventional product type estimator. Thus,$\bar{y}_p$ is the particular member of general class of proposed product type estimator $\mathbf{t}^*_{(\mathbf{q})}$.

For $q > 0$,consider $q = 1$,the estimator $\mathbf{t}^*_{(\mathbf{q})}$ will become

$$\mathbf{t}^*_{(\mathbf{1})} = \bar{y}_p + \frac{1}{n}\bar{y}_r\frac{s_{xy}}{\bar{x}\bar{y}}. \quad (15)$$

The RB and the RM of the estimator $\mathbf{t}^*_{(\mathbf{1})}$ are as

$$\mathrm{RB}(\mathbf{t}^*_{(\mathbf{1})}) = \mathrm{RB}(\bar{y}_p) + \frac{1}{n}C_{11}, \quad (16)$$

$$\mathrm{RM}(\mathbf{t}^*_{(\mathbf{1})}) = \mathrm{RM}(\bar{y}_p) + \frac{1}{n^2}[2C_{12} + 2C_{21} - C_{11}^2 - 4C_{20}C_{11}]. \quad (17)$$

The proposed estimator $\mathbf{t}^*_{(\mathbf{1})}$ has smaller bias than estimator $\bar{y}_p$, if $\mathbf{C_{11}} < 0$.

The RM of the estimator $\mathbf{t}^*_{(\mathbf{1})}$ is smaller than the estimator $\bar{y}_p$, if $2C_{12} + 2C_{21} - C_{11}^2 - 4C_{20}C_{11} < 0$ and vice versa (18)

Under bivariate normal distribution, the expression (18) reduces to $C_{11}^2 + C_{20}C_{11} > 0$ and vice versa.

Thus, the estimator $\mathbf{t}^*_{(\mathbf{1})}$will be more effective than $\bar{y}_p$, if $0 < \theta < -4\rho$.

Further, it is found that estimator $\mathbf{t}^*_{(\mathbf{1})}$is more effective than t_R^p, if $\rho < 0$ and$\theta > 0$.

The estimator $\mathbf{t}^*_{(\mathbf{1})}$performed better than estimator t_D^p, if $0 < \theta < -3$.

The estimator $\mathbf{t}^*_{(\mathbf{1})}$ has smaller bias than $t^p_{s(1)}$ and $t^p_{1(1)}$, if $C_{20} > C_{11}$.

The estimator $\mathbf{t}^*_{(\mathbf{1})}$ has smaller RM than $t^p_{s(1)}$, if

$$C_{11}^2 + C_{20}^2 + 8C_{20}C_{11} + 2C_{20}C_{02} + 2C_{12} - 2C_{30} > 0 \text{ and vice versa.} \quad (19)$$

under bivariate normal distribution, the expression (19) reduces to

$$C_{11}^2 + C_{20}^2 + 8C_{20}C_{11} + 2C_{20}C_{02} > 0 \text{ vice versa.} \quad (20)$$

Thus, the estimator $\mathbf{t}^*_{(\mathbf{1})}$ will be more efficient than $t^p_{s(1)}$, if

$\rho < \frac{3\theta^2+1}{8\theta}$ andthe value of θ should lie between $0.578 < \theta < 2.535$.

3. EMPIRICAL STUDY

Improved product type of estimators of population mean has been theoretically developed and their efficiencies have been tested by using simulation through datasets P_1 and P_2. The different values of the scalar q have been used. The value of q at which the proposed estimator was found to be more efficientthan other values of the scalar q have used for the numerical study.

Table 1.Descriptive statistics of the variable under study and auxiliary variable

Variable	P_1		P_2	
	Population Mean	Population Variance	Population Mean	Population Variance
Y	0.758	17.597	4.279	48.022
X	4.571	342.054	8.664	511.084
ρ_{XY}	-0.909		-0.981	

From table 1, it has been observed that the two variables (Y and X) was highly ($r = -0.909$) and negatively; the arithmetic mean of the variable Y and X were 0.758 and 4.571 whereas the variances were 17.597 and 342.054 respectively for dataset P1. Further, for dataset P2,the linear relationship between Y and X was found to be -0.981 which was high and negative and themean of the Y and X were 4.279 and 8.664 whereas the variances were 48.022 and 511.084 respectively.

Table 2. RB of the proposed product estimator $\mathbf{t}^*_{(\mathbf{3})}$ and $\mathbf{t}^*_{(-\mathbf{1})}$ w.r.t. existing product estimators

Estimator	P_1			P_2		
	30	60	120	30	60	120
$\bar{y}_p$	-0.13498	-0.01894	-0.00699	-0.07878	-0.00397	-0.01107
t^p_R	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
t^*	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

t_D^p	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
$t_{s(3)}^p$	3.88137	0.40591	0.52082	2.78224	1.72514	1.44100
$t_{s(-1)}^p$	-1.47376	-0.16055	-0.18292	-0.82237	-0.56975	-0.46557
$t_{(3)}^*$	-0.53991	-0.07575	-0.02795	-0.31513	-0.01589	-0.04430

Table 2 revealed that the proposed product type estimator $t_{(3)}^*$was negatively biased and has more bias than the conventional estimator and less than the estimator proposed by Sharma et al., (2007) in both populations. The magnitude of the biases of the proposed estimator $t_{(3)}^*$ at samples of sizes 30, 60 and 120 were0.53991, 0.07575 and 0.02795 respectively in case of P_1 whereas the magnitude of the biases of the proposed estimator $t_{(3)}^*$at samples of sizes 30, 60 and 120 were0.31513, 0.01589 and 0.04430 respectively.

Table 3.RM and percent relative efficiency (PRE) of the proposed estimator $t_{(3)}^*$ w.r.t different product type estimators

Estimator	P_1			P_2		
	30	60	120	30	60	120
$\bar{y}_p$	1.15778	1.28830	0.36744	0.82175	0.64007	0.54379
t_R^p	1.18587	1.13633	0.36914	0.83452	0.64994	0.54378
t^*	1.18587	1.13633	0.36914	0.83452	0.64994	0.54378
t_D^p	1.18970	1.59871	0.37198	0.85945	0.65454	0.55458
$t_{s(3)}^p$	2.70947	2.64334	0.50991	5.86103	3.69052	2.69521
$t_{(3)}^*$	1.05132	0.79463	0.34913	0.63704	0.58311	0.48048
Percent relative efficiencies of proposed estimators						
$t_{(3)}^*$ w.r.t. $\bar{y}_p$	110.13	162.13	105.25	129.00	109.77	113.18
$t_{(3)}^*$ w.r.t. t_R^p and t^*	112.80	143.00	105.73	131.00	111.46	113.18
$t_{(3)}^*$ w.r.t. t_D^p	113.16	201.19	106.54	131.00	111.46	113.18
$t_{(3)}^*$ w.r.t. $t_{s(3)}^p$	257.72	332.65	146.05	134.91	112.25	115.42

*values in the parenthesis indicate the PRE of the proposed product estimator w.r.t. to different product estimators.

It has been observed from the table 3 that in both the populations P_1 and P_2, the proposed estimator
$t_{(3)}^*$ have small RM than the conventional estimator and the estimators proposed by Robson (1957), Singh (1989), Dubey (1993), Sharma et al., (2007). In case of population P_1,the values of the RM of the proposed estimator $t_{(3)}^*$ at samples of sizes 30, 60 and 120 were found to be 1.05132, 0.79463 and0.34913 respectively. The PRE of the proposed estimator lies between 105.25 to 162.13 with respect to conventional product type estimator. Further, the relative efficiency of the proposed estimator $t_{(3)}^*$ was found to lie between 105.73 to 143.00 with respect to t_R^p and t^*. With respect to t_D^p and$t_{s(3)}^p$, the PRE of the proposed estimator $t_{(3)}^*$ was lying between 106.54 to 201.19 and 146.05 to 332.65 respectively. In case of P_2, the values of the RM of the proposed estimator $t_{(3)}^*$ at samples of sizes 30, 60 and 120 were found to be 0.63704, 0.58311 and 0.48048 respectively. The PRE of the proposed estimator with respect to$\bar{y}_p$, t_D^p and $t_{s(3)}^p$ lies between 109.77 to 129.00, 111.46 to 131.00 and

112.25 to 134.91 respectively. The PRE of the proposed estimator $t^*_{(3)}$ was lying between 111.46 to 131.00 with respect to t_R^p and t^*.

4. CONCLUSIONS

In the present paper, modified general classes of product type estimator have been proposed for estimation of population mean by making the use of auxiliary information using simple random sampling. The RB and RM of the proposed class have been theoretically worked out. The empirical study showed that proposed product type estimator $\mathbf{t}^*_{(3)} = \bar{\mathbf{y}}_{\mathbf{p}} + \frac{\mathbf{3}}{\mathbf{n}}\frac{\mathbf{s_{xy}}}{\bar{\mathbf{x}}\bar{\mathbf{X}}}$ was found to be more efficient than the estimators proposed by Robson (1957), Singh (1989), Dubey (1993) and Sharma et al., (2007).

Acknowledgement: I acknowledge the financial support by Dept. of Science and Technology – INSPIRE Fellowship (IF140961).

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