The Implementation of Cosine Similarity Measures in Decision-Making Problems by Signless Laplacian Energy of an Intuitionistic Fuzzy Graph

Obbu Ramesh [1], S. Sharief Basha [2]

[#3] *Department of Mathematics, School of Advanced Sciences, Vellore Institute of Technology, Vellore 632 014, Tamilnadu, India*

[1] *rameshobbu@gmail.com*

[2] *shariefbasha.s@vit.ac.in*

Abstract**: In this paper, we train group decision-making problems created on intuitionistic fuzzy inclination associations. We suggest a new attitude to estimate the qualified reputation weights of authorities by calculating the uncertain evidence of intuitionistic fuzzy inclination associations and the normal similarity grade of one separable intuitionistic inclination association to the others. This new attitude takes both the 'impartial' and 'internal' evidence of authorities into deliberation. Then intuitionistic fuzzy inclination relations were assimilated by us the weights of authorities into the specific and progress a relative similarity method to originate the significances of substitutes and best of the substitutes. The contrast analysis with additional methods by mathematical illustrations demonstrates the realism and helpfulness of the projected approaches.

Keywords: cosine similarity measure, intuitionistic fuzzy preference relation, intuitionistic fuzzy graph, Signless Laplacian energy.

1. INTRODUCTION:

The Intuitionistic fuzzy sets (IFSs),[1]are divided into two types of functions such as association and non-association functions, have been practical in many region, such as medical diagnosis,[3]decisionmaking, [2,4]and pattern recognition.[21,19]Szimidt and Kacprzyk[11–13]investigated in group decision-making problems established on Intuitionistic fuzzy sets, the significance of arrangement in a group of authorities (personalities) as the intuitionistic fuzzy inclination relations which are described from individual inclinations.

They provided a new technique to collective the collective fuzzy intuitionistic inclination relation collected from individual intuitionistic fuzzy inclination relations, while the ranks may not be formed to substitutes.[14]We know that in Refs. 11–13 given the intuitionistic fuzzy inclination relations involve three types of matrices. Xu[16]combined one matrix from these three kinds of matrices and projected the perception of an 'intuitionistic predilection relation'. He developed intuitionistic inclination evidence that approaches to intuitionistic inclination relations group decision-making that is used to aggregate by an arithmetic averaging operator of the intuitionistic fuzzy graph and weighted arithmetic averaging operative of an intuitionistic fuzzy graph.

On top of decision-making problems among most of that literature, by authority's domain fields, etc., we recognized the authorities' social status and competence, which are predetermined the importance weights of authorities and usually viewed as the given parameters. In this article, the 'internal' weights of authorities are mentioned from such kind of given weights. Therefore it is reasonably motivating to expression what way to originate the weights of authorities, mentioned towards the impartial weights of authorities in this paper, as of their conforming intuitionistic inclination relations, which explain the authorities' inclination evidence on the subject of every pair of substitutes.

By using SignlessLaplacian energy and Cosine similarity measures technique we can evaluate impartial weights of authorities. We are arranging this paper is as follows. In Section 2, appearances that the formulation of Signless Laplacian energy of IFG which is laid out in Refs. 9, 10, distinctly, are equal, and also offerings an operative cosine similarity measure for IFGs by an adjustment of Laplacian energy measures into cosine similarity measures. Section 3 presents a method to use their 'membership' and 'non-membership' weights to find out the weights of experts and gives a rank for cosine similarity method. One application of decision-making problem is shown to represent the efficiency of the projected approaches by assessments with other approaches. Section 4 stretches the final conclusion.

2. INITIATIONS

2.1. SignlessLaplacian Energy of IFG

Definition 2.1.1:- [See 9]let G=(V, E,μ,γ)be an IFG, where V is the set of vertices and E is the set of edges and μ is a fuzzy association function defined on V×V and γ is a fuzzy non association function, we describe $\mu(v_i, v_j)$ μ_{ij} by and $\gamma(v_i, v_j)$ by γ_{ij} such that the condition

(i) $0 \le \mu_{ij} + \gamma_{ij} \le 1$ (ii) $0 \le \mu_{ij}, \gamma_{ij}, \pi_{ij} \le 1$, where $\pi_{ij} = 1 - \mu_{ij} - \gamma_{ij}$.

Therefore $(V \times V, \mu, \gamma)$ is understood to be an IFG.

Illustration2.1.2:

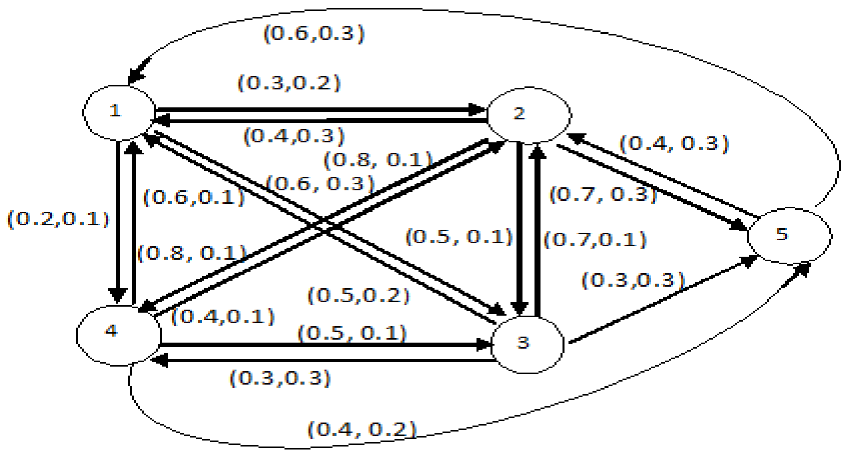

Fig 1:G_1, an intuitionistic fuzzy graph

Definition 2.1.3:-[See 10] An adjacency matrix of G= (V, E,μ,γ) is well-defined by A(G_1)= $[a_{ij}]$, where $a_{ij} = (\mu_{ij}, \gamma_{ij})$.

Illustration2.1.4:- The adjacency matrix of Fig.1 is

$$A\ (IG) = \begin{bmatrix} (0,0) & (0.3,0.2) & (0.6,0.3) & (0.2,0.1) & (0,0) \\ (0.4,0.3) & (0,0) & (0.5,0.1) & (0.8,0.1) & (0.7,0.3) \\ (0.6,0.1) & (0.7,0.1) & (0,0) & (0.3,0.3) & (0.3,0.3) \\ (0.8,0.1) & (0.4,0.1) & (0.5,0.1) & (0,0) & (0.4,0.2) \\ (0.6,0.3) & (0.4,0.3) & (0,0) & (0,0) & (0,0) \end{bmatrix}$$

Definition 2.1.5:- The adjacency matrix of an IFG Fig.1 can be represented as

A (I G) = $\left[(\mu_{ij}),(\gamma_{ij})\right]$, where

$$A\ (\mu_{ij}) = \begin{bmatrix} 0 & 0.3 & 0.6 & 0.2 & 0 \\ 0.4 & 0 & 0.5 & 0.8 & 0.7 \\ 0.6 & 0.7 & 0 & 0.3 & 0.3 \\ 0.8 & 0.4 & 0.5 & 0 & 0.4 \\ 0.6 & 0.4 & 0 & 0 & 0 \end{bmatrix} \text{ and } A\ (\gamma_{ij}) = \begin{bmatrix} 0 & 0.2 & 0.3 & 0.1 & 0 \\ 0.3 & 0 & 0.1 & 0.1 & 0.3 \\ 0.1 & 0.1 & 0 & 0.3 & 0.3 \\ 0.1 & 0.1 & 0.1 & 0 & 0.2 \\ 0.3 & 0.3 & 0 & 0 & 0 \end{bmatrix}$$

Definition 2.1.6:-The set (X, Y) is defined as Eigen values of A (IG)
Definition 2.1.7:- The Signless Laplacian matrix can be written as

$$SL[IG) = D(IFG) + A(IFG).$$

$$\text{i.e SL (IG)} = \left[\left(L(\mu_{ij})\right),\left(L(\gamma_{ij})\right)\right]$$

Illustration2.1.8:- The Membership and non-membership values of G_1 are

$$SL\ (\mu(G) = \begin{bmatrix} 2.4 & 0.3 & 0.6 & 0.2 & 0 \\ 0.4 & 1.8 & 0.5 & 0.8 & 0.7 \\ 0.6 & 0.7 & 1.6 & 0.3 & 0.3 \\ 0.8 & 0.4 & 0.5 & 1.3 & 0.4 \\ 0.6 & 0.4 & 0 & 0 & 1.4 \end{bmatrix} \text{ and}$$

$$SL\ (\gamma(G)) = \begin{bmatrix} 0.8 & 0.2 & 0.3 & 0.1 & 0 \\ 0.3 & 0.7 & 0.1 & 0.1 & 0.3 \\ 0.1 & 0.1 & 0.5 & 0.3 & 0.3 \\ 0.1 & 0.1 & 0.1 & 0.5 & 0.2 \\ 0.3 & 0.3 & 0 & 0 & 0.7 \end{bmatrix}$$

Definition 2.1.9:- Let SL [μ_{ij}] be SLM of graph IG = (V, E, μ,γ). Then $\phi(IG,\mu)$ =det $(\tilde{\mu}I_n - L(IG))$ is the IFG and the Signless Laplacian matrix's characteristic polynomial. The roots $\phi(IG,\mu)$ are Eigen values of IFG.

2.2. A Cosine similarity measures by Intuitionistic Fuzzy sets

In this portion, we measure cosine similarity and additionally the weighted cosine similarity amongst Intuitionistic fuzzy units for Intuitionistic fuzzy graphs.

A cosine similarity measure among IFSs A and B is projected as follows:

$$\text{Sim}_C(A,B) = \frac{1}{n}\sum_{i=1}^{n}\frac{\mu_A(x_i)\mu_B(x_i)+v_A(x_i)v_B(x_i)}{\sqrt{\mu_A^2(x_i)+v_A^2(x_i)}\sqrt{\mu_B^2(x_i)+v_B^2(x_i)}}$$

If we consider n = 1 then the cosine similarity measure among A and B as follows

(Property1): $0 \le \text{Sim}_C(A,B) \le 1$;

(Property2): $\mathrm{Sim}_C(A,B)=\mathrm{Sim}_C(\mathrm{B},\mathrm{A})$;

(Property3): $\mathrm{Sim}_C(A,B)$ = 1 if A = B,

i.e $\mu_A(x_i)=\mu_B(x_i)$ and $\nu_A(x_i)=\nu_B(x_i)$

Angle between the vectors to calculate the distance measures as follows:

$$d_i(A(x_i),B(x_i))=arc\cos(\mathrm{Sim}_C(A(x_i),B(x_i)))$$

Where, $Sim_C(A(x_i),B(x_i))=\dfrac{\mu_A(x_i)\mu_B(x_i)+\nu_A(x_i)\nu_B(x_i)}{\sqrt{\mu_A^2(x_i)+\nu_A^2(x_i)}\sqrt{\mu_B^2(x_i)+\nu_B^2(x_i)}}$

$$d_i(B(x_i),C(x_i))=arc\cos(\mathrm{Sim}_{C_i}(B(x_i),C(x_i)))$$

Where, $Sim_C(B(x_i),C(x_i))=\dfrac{\mu_B(x_i)\mu_C(x_i)+\nu_B(x_i)\nu_C(x_i)}{\sqrt{\mu_B^2(x_i)+\nu_B^2(x_i)}\sqrt{\mu_C^2(x_i)+\nu_C^2(x_i)}}\ for\ i=1,2,3,...,n.$

and

$$d_i(A(x_i),C(x_i))=arc\cos(\mathrm{Sim}_C(A(x_i),C(x_i)))$$

Where, $Sim_C(A(x_i),C(x_i))=\dfrac{\mu_A(x_i)\mu_C(x_i)+\nu_A(x_i)\nu_C(x_i)}{\sqrt{\mu_A^2(x_i)+\nu_A^2(x_i)}\sqrt{\mu_C^2(x_i)+\nu_C^2(x_i)}}\ for\ i=1,2,3,...,n.$

Compute weighted cosine similarity measure among A and B is projected as shown below:

$$W_C(A,B)=\sum_{i=1}^{n} w_i\,\frac{\mu_A(x_i)\mu_B(x_i)+\nu_A(x_i)\nu_A(x_i)}{\sqrt{\mu_A^2(x_i)+\nu_A^2(x_i)}\sqrt{\mu_B^2(x_i)+\nu_B^2(x_i)}}$$

Where, $w_i \in [0, 1]$, and $\sum_{i=1}^{n} w_i = 1$

If we consider w_i= 1/n, then it satisfies W_C(A, B) = Sim_C (A, B).

3. GDM by IFIR$_S$

In this portion, the Signless Laplacian Energy and cosine similarity measure of IFGs are functional to find the weights of experts and rank the substitutes for GDM issues by solving intuitionistic inclination relations.

3.1. Intuitionistic Inclination Relations

In the course of the GDM process, authority is commonly compulsory to offer his inclinations over the substitutes. The authority may offer his decisions in a confident way at the same time as occasionally he is not fairly assured of those decisions. Hence, it is suitable to prompt the authority's inclination standards with intuitionistic fuzzy values somewhat than the mathematical standards. Next, we deliberate how to get most evidence from the authorities' predilections above the substitutes to alter the given standing weights of authorities for most sensible GDM.

3.2. A new procedure to find out the weights of authorities

Let $X=\{x_1,x_2,...,x_n\}$ are substitutes set, $E=\{e_1,e_2,...,e_m\}$ be authorities set. The authority e_l provides their inclination evidence for all pair of substitutes, and forms an intuitionistic fuzzy inclination relation

$$D^{(l)}=\left(d_{ij}^{(l)}\right)_{n\times n}\left(l=1,2,...,m\right)$$

Where, $d_{ij}^{(l)}=\left(u_{ij}^{(l)},v_{ij}^{(l)}\right), 0\le u_{ij}^{(l)}+v_{ij}^{(l)}\le 1.$ (i, j = 1, 23,4, . . . , n)

The Signless Laplacian energy can measure the indeterminate indication of an IFG. Each intuitionistic fuzzy inclination relative $D^{(l)}$ is an IFG in adjacency matrix (*X* × *X)*, so we will quantity its indeterminate indication by Signless Laplacian energy measure. During the GDM method, we generally imagine the vagueness grade of intuitionistic inclination relation as little as likely for additional confidence of the obtained grades. We grow the succeeding Algorithm to weigh up the 'impartial' loads of the authorities.

Procedure I

Designed for the GDM real time problems centred on top of intuitionistic fuzzy inclination relations,

We consider $g=(\mathrm{g}_1,\mathrm{g}_2,\mathrm{g}_3,...,\mathrm{g}_m)$ be a particular allowance vector of authorities,

where $g_k>0, \sum_{i=1}^{m} g_i = 1$

Step I. Compute the Signless Laplacian energy $SLE\left(D^{(l)}\right)$ of $D^{(l)}$:

$$SLE\left(D^{(l)}\right)=\left|\tilde{\mu}_i-\frac{2\sum_{1\le i\le j\le n}\mu\left(u_i,u_j\right)}{n}\right| --------->(5)$$

Step II. Compute the weight g_l^a g_k^a, determined by $SLE\left(D^{(l)}\right)$, of the authority e_k:

$$g_i=\left(\left(g_\mu\right)_i,\left(g_\gamma\right)_i\right)=\left[\frac{SLE\left(\left(D_\mu\right)_i\right)}{\sum_{l=1}^{m}SLE\left(\left(D_\mu\right)_l\right)},\frac{SLE\left(\left(D_\gamma\right)_i\right)}{\sum_{l=1}^{m}SLE\left(\left(D_\gamma\right)_l\right)}\right] \ for\, i=1,2,3,...,m \quad ------->(6)$$

Step III. Compute the cosine similarity measure ($\mathrm{D}^{(k)}$, $D^{(l)}$) among $\mathrm{D}^{(k)}$ and $\mathrm{D}^{(l)}$ for all $k\neq l$:

$$Sim_C(\mathrm{P}(x_i),\mathrm{Q}(x_i))=\frac{\mu_P(x_i)\mu_Q(x_i)+v_P(x_i)v_Q(x_i)}{\sqrt{\mu_P^2(x_i)+v_P^2(x_i)}\sqrt{\mu_Q^2(x_i)+v_Q^2(x_i)}}$$

$$\zeta_C(\mathrm{D}^{(k)},\mathrm{D}^{(l)})=\frac{1}{m}\sum_{i=1}^{m}\frac{\mu_{D^k}(x_i)\mu_{D^l}(x_i)+v_{D^k}(x_i)v_{D^l}(x_i)}{\sqrt{\mu_{D^k}^2(x_i)+v_{D^k}^2(x_i)}\sqrt{\mu_{D^l}^2(x_i)+v_{D^l}^2(x_i)}} \quad ------------->(7)$$

Then $Sim\left(D^{(l)}\right)$ of $\mathrm{D}^{(l)}$ is the average similarity grade to some other is premeditated by

$$\zeta_C(\mathrm{D}^{(k)})=\frac{1}{m-1}\sum_{l=1,\,l\neq k}^{m}\zeta_C(\mathrm{D}^{(k)},D^{(l)}) ------------------->(8)$$

Step IV:-Estimate the weight g_l^b resolute by $Sim\left(D^{(l)}\right)$ of the authority e_l:

$$g_l^b=\frac{\zeta_C\left(D^{(l)}\right)}{\sum_{i=1}^{m}\zeta_C\left(D^{(i)}\right)},(l=1,2,..,m)------------>(9)$$

Step V. Compute the 'impartial' weight y_k^2 of the authority e_l:

$$g_l^2=\eta g_l^a+(1-\eta)g_l^b,\eta\in[0,1],(l=1,2,..,m)------->(10)$$

Step VI. Assimilate the 'internal' weight g_l^1 and the 'impartial' weight g_l^2 into the weight g_l of the authority e_l:

$$g_l=\gamma g_l^a+(1-\gamma)g_l^2,\gamma\in[0,1],(l=1,2,..,m)------------->(11)$$

From Procedure I, the majority of specialists reflect both the 'internal' and 'impartial' evidence. We at that point acclimatize the different intuitionistic inclination family members into a common intuitionistic preference relative by succeeding hypothesis known by Ref. 10.

Theorem 3.[17]Let $D^{(l)}=\left(d_{ij}^{(l)}\right)_{n\times n}------------->(12)$ be an intuitionistic fuzzy inclination relations known by the authorities e_l and *g= (g₁, g₂, . . . ,gₙ)* be a weighting vector of authorities, where $d_{ij}^{(l)}=\left(u_{ij}^{(l)},v_{ij}^{(l)}\right),\mathrm{g}_l>0$, $\sum_{i=1}^{m}w_i=1$. Then the aggregation D = $\mathrm{D}=(\mathrm{d}_{ij})_{n\times n}$ of $D^{(l)}=\left(d_{ij}^{(l)}\right)_{n\times n}$ is as well an intuitionistic inclination relation, where $d_{ij}=(u_{ij},v_{ij});\ u_{ij}=\sum_{l=1}^{m}g_l u_{ij}^{(l)},\ v_{ij}=\sum_{l=1}^{m}g_l v_{ij}^{(l)}$

Procedure 1

Step I:-By using the formulation: $d_i^{(l)}=\dfrac{1}{m}\sum_{j=1}^{m}d_{ij}^{(l)}----------->(13)$ to obtain the averaged intuitionistic fuzzy values $d_i^{(l)}$ of substitute x_i in general the other substitutes.

Step II:-By using the formulation: $d_i=\sum_{l=1}^{m}y_l d_i^{(l)},i=1,2,3,...,n\to(14)$, we can find the values of $d_i(u_i,v_i)$.

Step III:-. Compute the score function: $\zeta_C(\mathrm{D}_i)=\dfrac{u_i-v_i}{\sqrt{(u_i^2+v_i^2)}} \quad\to(15)$

3.3. A comparative cosine similarity process to rank the substitutes

$\{(u_{ij},v_{ij})\ /i,j=1,2,3,4,\ldots,n\}$ is the i^{th} row vector of a cooperative intuitionistic fuzzy inclination relation D, represented by D^i, defines the two of a kind comparison inclination of the i^{th} substitute x_i over all the substitutes in X, and can be observed as an IFS_s in $\{x_i\}\times X$. Let the positive ideal and negative ideal substitute be $x+$ and $x-$ respectively. Consider the IFS_S $D^+=\{(1, 0), (1, 0), \ldots,(1, 0)\}$ and $D^-=\{(0, 1), (0, 1), \ldots, (0, 1)\}$ designate the pair wise assessment inclination of the ideals $x+$ and $x-$ for all substitutes in X, respectively.

Therefore, the finest substitute is developed to possess the grade of cosine similarity to x+ as large as probable and have the grade of cosine similarity' to x− as lesser as probable. Hence we rank the substitutes from the cooperative inclination relation by succeeding relative cosine similarity process: procedure II. Suppose $D^{(l)}\left(u_{ij},v_{ij}\right)$ and g is distinct as earlier.

Procedure II

Step I. Determine the co-operative intuitionistic inclination relation $\mathrm{D}=(d_{ij})_{n\times n}$ by

$$d_{ij}=\left(u_{ij},v_{ij}\right)=\left(\sum_{l=1}^{m}g_l u_{ij}^{(l)},\sum_{l=1}^{m}g_l v_{ij}^{(l)}\right)\forall i,j=1,2,,...,n------------->(16)$$

Step II. For all substitute x_i, find the cosine similarity measures $\zeta_C(\mathrm{D}^i, D^+)$ and $\zeta_C(\mathrm{D}^i, D^-)$ by formula (11). Then

$$\zeta_C(\mathrm{D}^i,\mathrm{D}^+)=\frac{1}{n}\sum_{j=1}^{n}\frac{u_{ij}.(1)+v_{ij}.(0)}{\sqrt{u_{ij}^2+v_{ij}^2}\sqrt{1^2+0^2}}=\frac{1}{n}\sum_{j=1}^{n}\frac{u_{ij}}{\sqrt{u_{ij}^2+v_{ij}^2}} \quad ----------->(17)$$

and

$$\zeta_C(\mathrm{D}^i,\mathrm{D}^-)=\frac{1}{n}\sum_{j=1}^{n}\frac{u_{ij}.(0)+v_{ij}.(1)}{\sqrt{u_{ij}^2+v_{ij}^2}\sqrt{1^2+0^2}}=\frac{1}{n}\sum_{j=1}^{n}\frac{v_{ij}}{\sqrt{u_{ij}^2+v_{ij}^2}} \quad ------->(18)$$

Step III. For every substitute x_i, determine its calculation value

$$\mathrm{h}(\mathrm{x}_i)=\frac{\zeta_C(\mathrm{D}^i,\mathrm{D}^+)}{\zeta_C(\mathrm{D}^i,\mathrm{D}^+)+\zeta_C(\mathrm{D}^i,\mathrm{D}^-)} \quad ---------->(19)$$

The larger value of $h(x_i)$, the superior of substitute x_i. Then we assimilated the rank of substitutes. The succeeding two cases are assumed to show how to attain assimilated weights by Procedure I and then Procedure II help show to rank the substitutes.

3.4. Cases

Now, we give two Illustrations by tolerating one possible Illustration utilized by Xu and Yager[10] and one more by Gong et al[5].During appraisal by procedures for the methodologies in Refs. 17 and 5, we endeavour to show our methodologies portrayal extra proof which has not presented past to.

Case 1.Attempt that we are having four alternatives x_i, and three specialists e_l (l= 1, 2, 3) in a GDM issue. Assume the weights for every authority are 0.5, 0.3, and 0.2, independently. Every specialist e_l (l= 1, 2, 3) analogize the four other options and fabricates the IFIR $D^{(l)}=\left(d_{ij}^{l}\right)_{4\times4}\left(l=1,2,3\right)$, independently, uncovered as follows

$$D^{(1)}=\begin{bmatrix}(0,0) & (0.1,0.6) & (0.2,0.4) & (0.7,0.3)\\ (0.5,0.2) & (0,0) & (0.3,0.6) & (0.4,0.2)\\ (0.3,0.5) & (0.4,0.3) & (0,0) & (0.6,0.2)\\ (0.2,0.7) & (0.5,0.4) & (0.2,0.8) & (0,0)\end{bmatrix}$$

$$D^{(2)}=\begin{bmatrix}(0,0) & (0.3,0.5) & (0.1,0.5) & (0.4,0.3)\\ (0.2,0.3) & (0,0) & (0.4,0.3) & (0.5,0.3)\\ (0.5,0.2) & (0.4,0.1) & (0,0) & (0.8,0.2)\\ (0.1,0.6) & (0.3,0.1) & (0.2,0.5) & (0,0)\end{bmatrix}$$

$$D^{(3)}=\begin{bmatrix}(0,0) & (0.3,0.1) & (0.3,0.5) & (0.1,0.4)\\ (0.1,0.5) & (0,0) & (0.5,0.3) & (0.4,0.5)\\ (0.5,0.3) & (0.3,0.2) & (0,0) & (0.3,0.6)\\ (0.4,0.3) & (0.6,0.4) & (0.3,0.3) & (0,0)\end{bmatrix}$$

Case 1 was actualized by Xu and Yager[17] for understanding examination in GDM fixated on intuitionistic fuzzy preference relations however for the corner to corner (diagonal) components of the nearness lattices are demonstrated as zeros as indicated by fuzzy diagrams which shows no association. Here we utilization the insights to begin the positioning request of the substitutes.

The three specialists are observed new weights 0.5, 0.3 and 0.2 as the objective weights. That means we are considering objective weight vector $y_1$is (0.5, 0.3, 0.2).

The ‘‘internal’’ loading vector g_1 is considered as (0.5, 0.3, 0.2). Then, we find the 'subjective' weighting vector of the specialists and summative the 'objective' and 'subjective' weighting vectors into the acclimatized weighting course of specialists through strategy I.
By using the formulation (5), we get the Signless Laplacian energies of $D^{(i)}$ (i= 1, 2, 3):

$$SLE\ (\mathrm{D}^1) = (4.5050,\ 5.2166) \quad , \quad SLE^+(\mathrm{D}^2) = (4.2000, 3.9000) \quad ,$$
$$SLE(\mathrm{D}^3) = (4.1000, 4.4435)$$

By equation (6), we acquire the weighting vector of the specialists controlled by the Signless Laplacian energies.

$$y_1^a = (0.3518,\ 0.3847)\ ,\ y_2^a = (0.3280,\ 0.2876), \quad y_3^a = (0.3202,\ 0.3277)$$

Using (7) formulation, we have

$$\zeta_C(\mathrm{D}^{(1)}, \mathrm{D}^{(2)}) = 0.9349,\ \zeta(\mathrm{D}^{(1)}, \mathrm{D}^{(3)}) = 0.8004,\ \zeta(\mathrm{D}^{(2)}, \mathrm{D}^{(3)}) = 0.8766.$$

Then, by (8) and (9)formulations, we obtain the be around cosine similarity grades ζ_C ($\mathrm{D}^{(i)}$) of $\mathrm{D}^{(i)}$ (i= 1, 2, 3) and the allowance course y^b of the authorities resolute by average ‘Cosine internal grades, correspondingly:

$$\zeta_C(\mathrm{D}^{(1)}) = 0.8676,\ \zeta_C(\mathrm{D}^{(2)}) = 0.9057,\ \zeta_C(\mathrm{D}^{(3)}) = 0.8385,$$
$$y_1^b = 0.3322, y_2^b = 0.3468, y_3^b = 0.3210.$$

Let us take η = 0.5, by formulation (10), we can find the ‘impartial’ weighting vectors

$$y_1^2 = (0.3420, 0.3585), y_2^2 = (0.3374, 0.3172), y_3^2 = (0.3206, 0.3244).$$

We can assimilate the ‘internal’ weighting vector g^1 and the ‘impartial’ weighting vector g^2 into the assimilated loading vector w by formulation (11): $g = \gamma g^a + (1 - \gamma)\ g^2$, $\gamma \in [0, 1]$, where γ is the ‘impartial’ and ‘internal’ weight indication. Initially we assume γ = 0.5 in formulation (21) and get the assimilated loading vector g:

$$y_1 = (0.3469, 0.3716);\ y_2 = (0.3327, 0.3024);\ y_3 = (0.3204,\ 0.3261)$$

Till now, we have got the assimilated weights of authorities for the actual GDM problem.
Primary, we explain Xu’s attitude to develop the consequence effect, which contains the succeeding step ladder:

Algorithm I

Step 1:-. Use the formulation: $d_i^{(l)} = \frac{1}{m}\sum_{j=1}^{m} d_{ij}^{(l)}, (1, 2, ,.., \mathrm{m})$, to turn out to be the average intuitionistic fuzzy value $d_i^{(l)}$ of the substitute x_i above all the extra substitutes:

$d_1^{(1)}$ = (0.3333, 0.4333) $\quad$ $d_2^{(1)}$ = (0.4000, 0.3333)
$d_3^{(1)}$ = (0.4333, 0.3333) $\quad$ $d_4^{(1)}$ = (0.3000, 0.6333)

$d_1^{(2)}$ = (0.2667, 0.4333) $\quad$ $d_2^{(2)}$ = (0.3667, 0.3000)
$d_3^{(2)}$ = (0.5667, 0.1667) $\quad$ $d_4^{(2)}$ = (0.2000, 0.4000)

$d_1^{(3)}$ = (0.2333, 0.3333) $\quad$ $d_2^{(3)}$ = (0.3333, 0.4333)
$d_3^{(3)}$ = (0.3667, 0.3667) $\quad$ $d_4^{(3)}$ = (0.4333, 0.3333)

Step II:- formula: $d_i = \sum_{k=1}^{n} y_k d_i^{(k)}$ to aggregate all $d_i^{(k)}$, related to m authorities, into a collective intuitionistic fuzzy value $d_i(u_i, v_i)$ of the substitute x_i for all the other substitutes:
d_1 = (0.2791, 0.4007), d_2 = (0.3676, 0.3559), d_3 = (0.4563, 0.2938), d_4 = (0.3094, 0.4650).

Step III. Compute $\zeta_C(\mathrm{d}_i) = \frac{u_i - v_i}{\sqrt{(u_i^2 + v_i^2)}}$ and we get

$$\zeta_C(\mathrm{d}_1) = -0.2478,\ \zeta_C(\mathrm{d}_2) = 0.0228,\ \zeta_C(\mathrm{d}_3) = 0.2293,\ \zeta_C(\mathrm{d}_4) = -0.2785.$$

Then, $\zeta_C(\mathrm{d}_3) > \zeta_C(\mathrm{d}_2) > \zeta_C(\mathrm{d}_1) > \zeta_C(\mathrm{d}_4)$ and

Hence, $x_3 > x_2 > x_1 > x_4$

Here the representation $\succ$ designates that one substitute is chosen to alternative.

Procedure II

By (16) in procedure II, we become the IFIR

$$d = \begin{bmatrix} (0,0) & (0.2306, 0.4068) & (0.1988, 0.4629) & (0.4080, 0.3326) \\ (0.2720, 0.3281) & (0,0) & (0.3884, 0.4115) & (0.4333, 0.3281) \\ (0.4306, 0.3441) & (0.3680, 0.2069) & (0,0) & (0.5704, 0.3305) \\ (0.2308, 0.5394) & (0.4655, 0.3093) & (0.2320, 0.5463) & (0,0) \end{bmatrix}$$

By using the formulation (17), we obtain

$\zeta_C(\mathrm{D}^1, \mathrm{D}^+)$ = 0.5543, $\zeta_C(\mathrm{D}^2, \mathrm{D}^+)$ = 0.7073,
$\zeta_C(\mathrm{D}^3, \mathrm{D}^+)$ = 0.8394, $\zeta_C(\mathrm{D}^4, \mathrm{D}^+)$ = 05390.

By formula (18), we obtain

$\zeta_C(\mathrm{D}^1, \mathrm{D}^-)$ = 0.8069, $\zeta_C(\mathrm{D}^2, \mathrm{D}^-)$ = 0.7002,
$\zeta_C(\mathrm{D}^3, \mathrm{D}^-)$ = 0.5385, $\zeta_C(\mathrm{D}^4, \mathrm{D}^-)$ = 0.7977.

By equation (19), we have

$$\mathrm{h}(x_1) = 0.4072, \mathrm{h}(x_2) = 0.5025,\ \mathrm{h}(x_3) = 0.6092, \mathrm{h}(x_4) = 0.4032.$$

Since $\mathrm{h}(x_3) > h(x_2) > h(x_1) > h(x_4)$

Hence $x_3 > x_2 > x_1 > x_4$

By the above method and Xu's methodology, [9] x_3 ranks the top most, x_4 ranks the last, and however, x_2 and x_1 have middle position command. The position commands of substitutes by both approaches are the same.

Case 2:- Application: Selection of Mobile company

Mobile phone is one of the most preferred digital devices in Consisting a large part of our daily lives in particular. There are many smart phones which have made our lifestyle convenient and easy, for example Apple, Samsung, Nokia and Motto etc. Mr. John wants to buy a smart phone for his multiple uses. There are various companies available for buying smart phones. But he requirements to the selection that company for purchasing "affordable" and "best quality" smart phone. Let us consider X= {a_1, a_2, a_3, a_4} be the set of four mobile companies and U ={u_1, u_2} be the set of parameters that specifies the 'affordable' and 'best quality' respectively. Mr. John compares the four companies a_i, where i=1,2, . . . 4. pairwise for the selection based on the parameters ''affordable'' and ''best quality'' and provides its inclination information in the form of IFIR $O=(o_{il})_{4x4}$, where $o_{il} = (\mu_{il}, v_{il})$ is the intuitionistic fuzzy element assigned by Mr. John expert with μ_{il} as the grade to which the company a_i is chosen over the company a_l concerning the given parameter and v_{il} as the grade to which the company a_i is not preferred over the company a_l concerning the given parameter. The IFPR $O=(o_{il})_{4x4}$ for the given restrictions is spoken in the below tabular appearance in Tables 1 and 2, respectively.

Table 1:- IFIR for parameter u_1

$$D^{(1)} = \begin{bmatrix} (0,0) & (0.2,0.8) & (0.8,0.1) & (0.4,0.5) \\ (0.6,0.2) & (0,0) & (0.4,0.5) & (0.4,0.3) \\ (0.3,0.5) & (0.4,0.3) & (0,0) & (0.7,0.2) \\ (0.6,0.4) & (0.7,0.1) & (0.4,0.2) & (0,0) \end{bmatrix}$$

Table 2:- IFIR for parameter u_2

$$D^{(2)} = \begin{bmatrix} (0,0) & (0.2,0.7) & (0.6,0.4) & (0.9,0.1) \\ (0.5,0.2) & (0,0) & (0.4,0.5) & (0.6,0.3) \\ (0.8,0.1) & (0.4,0.3) & (0,0) & (0.6,0.2) \\ (0.2,0.7) & (0.5,0.4) & (0.2,0.6) & (0,0) \end{bmatrix}$$

The initial loads 0.7, 0.3 of three authorities observed as ''internal'' loads. After that, we initiate the ''impartial'' loading vector of the authorities and summative the ''internal'' and ''impartial'' loading vectors into the assimilated weighting vector of the authorities by procedure I.

By using the formulation (5), we can find the Laplacian energies of $D^{(i)}$, (i= 1, 2):

SLE ($D^{(1)}$) = (3.2686, 2.3216); SLE ($D^{(2)}$) = (2.9919, 2.2808)

By the formulation (6), we get the weighting vector of the authorities determined by the Laplacian energies.

g_1^a = (0.5221, 0.5044) , g_2^a = (0.4779, 0.4956).

Using (7) formulation, we have

$\zeta_C(\mathrm{D}^{(1)}, \mathrm{D}^{(2)}) = 0.8774$,

Then, by (8) and (9) formulations, we acquire the averaged cosine similarity grades $\zeta_C(\mathrm{D}^{(i)})$ of $D^{(i)}$ (i takes the values 1, 2, 3) and allowance vector g^b of the authorities resolute by regular cosine similarity grades, respectively:

$\zeta_C(\mathrm{D}^{(1)})$ = 0.8774, $\zeta_C(\mathrm{D}^{(2)})$ = 0.8774.

g^b= (0.5, 0.5).

Let us take η = 0.5, by formulation (10), we get the 'impartial' weighting vector

$g_1^2 = (0.5111, 0.5022)$, $g_2^2 = (0.4890, 0.4978)$.

We can assimilate the 'internal' weighting direction g^l and the 'impartial' weighting vector g^2 into the assimilated loading vector g by formulation (11): $g = \gamma g^a + (1 - \gamma)g^2$, $\gamma \in [0, 1]$, where γ is the 'impartial' and 'internal' weight indication. Initially we consider $\gamma = 0.5$ in formulation (21) and we get the weighting vector g:

$$g_1 = (0.5166, 0.5033)\ g_2 = (0.4835, 0.4967).$$

Even now, we have attained the assimilated weights of authorities for the actual GDM problem.

At first, illustrate Xu's attitude to develop the decision result, which contains the succeeding steps:

Procedure I

Step I:-. By using the formulation: $d_i^{(l)} = \frac{1}{n}\sum_{j=1}^{n} d_{ij}^{(l)}, (i = 1, 2, 3, .., n)$ to become intuitionistic fuzzy value $d_i^{(l)}$ of the substitute x_i above every the previous substitutes:

$d_1^{(1)}$ = (0.4667, 0.4667) $d_2^{(1)}$ = (0.4667, 0.3333)

$d_3^{(1)}$ = (0.4667, 0.3333) $d_4^{(1)}$ = (0.5667, 0.2333)

$d_1^{(2)}$ = (0.5667, 0.4000) $d_2^{(2)}$ = (0.5000, 0.3333)

$d_3^{(2)}$ = (0.6000, 0.2000) $d_4^{(2)}$ = (0.3000, 0.5667)

Step II:-By using the formulation: $d_i = \sum_{l=1}^{m} g_l d_i^l$, we have

d_1 = (0.5151, 0.4336), d_2 = (0.4828, 0.3333),

d_3 = (0.5312, 0.2671), d_4 = (0.4378, 0.3989).

Step3. Using the formula: $\zeta_C(\mathrm{d}_i) = \frac{u_i - v_i}{\sqrt{(u_i^2 + v_i^2)}}$, we get

$$\zeta_C(\mathrm{d}_1) = 0.1210,\ \zeta_C(\mathrm{d}_2) = 0.2548,\ \zeta_C(\mathrm{d}_3) = 0.4441,\ \zeta_C(\mathrm{d}_4) = 0.0656.$$

Then, $\zeta_C(\mathrm{d}_3) > \zeta_C(\mathrm{d}_2) > \zeta_C(\mathrm{d}_1) > \zeta_C(\mathrm{d}_4)$ and

Hence, $x_3 > x_2 > x_1 > x_4$

Procedure II

At present we provide the result based ranking capability by our relative cosine similarity method. By using the equation (16) in procedure II, we develop the combined IFIR

$$D = \begin{bmatrix} (0,0) & (0.2, 0.7503) & (0.7034, 0.2490) & (0.6418, 0.3013) \\ (0.5517, 0.2) & (0,0) & (0.4, 0.5) & (0.4967, 0.3) \\ (0.5418, 0.3013) & (0.4, 0.3) & (0,0) & (0.6517, 0.2) \\ (0.4067, 0.5490) & (0.6034, 0.2490) & (0.3033, 0.3987) & (0,0) \end{bmatrix}$$

By using the equations (17) and (18), we attain

$\zeta_C(\mathrm{D}^1, \mathrm{D}^+)$ = 0.7003, $\zeta_C(\mathrm{D}^2, \mathrm{D}^+)$ = 0.8069,

$\zeta_C(\mathrm{D}^3, \mathrm{D}^+)$ = 0.8195, $\zeta_C(\mathrm{D}^4, \mathrm{D}^+)$ = 0.7196

$\zeta_C(\mathrm{D}^1, \mathrm{D}^-)$ = 0.5693, $\zeta_C(\mathrm{D}^2, \mathrm{D}^-)$ = 0.5462,

$\zeta_C(\mathrm{D}^3, \mathrm{D}^-)$ = 0.4280, $\zeta_C(\mathrm{D}^4, \mathrm{D}^-)$ = 0.6568.

Then formulation (19) gives the calculation standards of substitute's x_i (i= 1, 2, 3, 4):

$$\mathrm{h}(x_1) = 0.5516, \mathrm{h}(x_2) = 0.5963, f(x_3) = 0.6569, f(x_4) = 0.5228.$$

Therefore, $\mathrm{h}(x_3) > h(x_2) > h(x_1) > h(x_4)$,

Hence $x_3 > x_2 > x_1 > x_4$

By the above method and Xu's methodology, [9] x_3 ranks the top most, x_4 ranks the last, and however, x_2 and x_1 have middle position command. The position commands of substitutes by both approaches are the same.

4. CONCLUSIONS

In this paper, we've provided the cosine similarity measures and Signless Laplacian energy at the unspecified signal of intuitionistic fuzzy graph inclination family members and the normal cosine similarity grade of one of the unique intuitionistic fuzzy graph Inclination associated with the others, respectively. As of late, numerous entropy and similitude estimates plan have been spread to the GDM issues made on intuitionistic fuzzy proof. We take out evidence from the authorities' applied decisions i.e distinct intuitionistic inclination relations near the substitutes and convert it into the 'internal weights of the authorities by the Procedure I and also combined the distinct intuitionistic inclination relations into a combined IFIR by using an IFWA operator and proposition a relative similarity technique to originate the urgencies of substitutes from the combined IFIR by the Procedure II.

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