Gouy Phase Shift Of Q-Gaussian Laser Beams In Collisional Plasma With Density Ramp

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Abstract: Gouy phase shift of q-Gaussian laser beams interacting nonlinearly with collisional plasmas with axially increasing density has been investigated theoretically. The Gouy phase shift also known as phase anomaly occurs at a fundamental level from position momentum uncertainty. Due to intensity gradient over the cross section of the laser beam redistribution of the carriers occurs on account of collisional heating of the plasma. The resulting index of refraction resembles to that of graded index fibers that stimulates the laser beam to get self-focused. The reduction in transverse dimensions of the laser beam in turn leads to spread in transverse momentum of its photons. This transverse momentum spread then modifies the axial phase of the laser beam. Following variational theory, a set of coupled differential equations for the evolution of beam width and axial phase of the laser beam has been obtained. The equations so obtained have been solved numerically so see the effect of laser and plasma parameters on the evolution of beam envelope.

Keywords: q-Gaussian, Density Ramp, Collisional Plasma, Gouy Phase Shift.

1.Introduction

The Gouy phase shift[1] or phase anomaly of an optical beam describes how the longitudinal phase of a focused electromagnetic beam differs from that of an infinite plane wave. Since its discovery, the anomalous behavior of the axial phase of the optical beams has been drawing attention of the researchers due to its relevance in a number of applications and physical problems. In wave optics it explains the phase shift obtained by the secondary wavelets emerging from primary wave front. In the working of lasers, it decides the resonant frequencies of various transverse modes in laser cavity. Applied physics problems also rely on Gouy phase shift. A potential example is optical trapping of particles where it produces lateral trapping force[2] and also provides a mechanism for the tracking of trapped particles[3, 4]. Moreover, a number of schemes for higher harmonic generation[5-8] of optical beams use the concept of longitudinal phase shift to meet the phase matching condition. Although the phase anomaly was discovered more than 100 years ago, curiosity about its origin and physical meaning is still at the vanguard of theoretical as well as experimental investigations. Various theories[9, 10] (ranging from classical to quantum) have been used to explain its origin. Classically the phase shift of an optical beam arises due to the contribution of an additional phase in the neighbourhood of the beam focal spot arising from the second order derivative of field amplitude with respect to transverse coordinates. However, in quantum mechanical terms the Gouy phase shift is considered to be originating as a consequence of modification of its transverse dimensions. The consequent change in the transverse momentum of the photon changes the longitudinal momentum as well that in turn modifies the longitudinal phase of the laser beam.

Erden and Ozaktas[11] investigated Gouy phase shift of Gaussian laser beams propagating through first order optical systems. Andresen et al[12-14] investigated similarity between spectral phase shift and the Gouy phase shift. Gordon and Barge [15] investigated the effect of Gouy phase shift on coherent phase control of chemical reactions. Literature review reveals the fact that in almost all the previous investigations on Gouy phase shift, the irradiance over the cross section of the laser beam has been considered to be ideally Gaussian. However, by investigating experimentally, Patel et al[16] have shown that although the laser operates in TEM00 mode, the irradiance over its cross section is not ideally Gaussian. Further, by fitting into the experimental data it was shown that[17] the actual irradiance over the beam's cross section can be modeled by a set of distribution functions known as q-Gaussian distribution given by Tsalli [18]. The difference in the behavior of irradiance over the laser beam wave front from ideal Gaussian is due to cavity imperfections that may be inherent or accidental in nature. Till date no experimental or theoretical investigation on Gouy phase shift of q-Gaussian laser beams in nonlinear media has been reported by any researcher. Thus, this paper aims to investigate for the first time Gouy phase shift of q-Gaussian laser beams in collisional plasma with axial density ramp.

2. Dynamics of Beam Envelope

The model equation for the propagation of intense laser beam through collisional plasma with axial density ramp is

$$2ik_0\frac{\partial A_0}{\partial z} = \nabla_\perp^2 A_0 + \frac{\omega_0^2(z)}{c^2}\left\{1-\left(1+\frac{1}{2}\beta A_0A_0^*\right)^{\frac{s}{2}-1}\right\}A_0 \quad (1)$$

where, $\omega_p^2(z) = \frac{4\pi e^2}{m}n(z)$ is the plasma frequency, n(z) being the axially increasing electron density of plasma, $\beta = \frac{e^2M}{(6K_0T_0m^2\omega_0^2)}$ is the constant associated with the strength of collisional nonlinearity. The nature of collisions is expressed by the parameter s. s = 0 indicates velocity independent collisions, s = 2 corresponds to the collisions between electrons and diatomic molecules and s = -3 indicates collisions between electrons and ions.

Eq.(1) is the mathematical statement of interplay between diffraction and nonlinear refraction of the optical beam where, the diffraction phenomenon is modeled by first term on R.H.S and the nonlinear refraction represented by second term. Being nonlinear in nature conventional method of solving partial differential equations i.e., expansion in power series are not applicable eq.(1). In order to obtain physical insight into the propagation dynamics of the laser beam we use a semi analytical technique known as variational method[18]. According to this method eq.(1) is a variational problem for action principle based on Lagrangian

$$L = \int_0^\infty \pounds r dr \quad (2)$$

with

$$\pounds = i\left(A_0^*\frac{\partial A_0}{\partial z} - A_0\frac{\partial A_0^*}{\partial z}\right) + |\nabla_\perp A_0|^2 + \frac{\omega_0^2}{c^2}\int^{A_0A_0^*}\Phi(A_0A_0^*)d(A_0A_0^*)$$

The basic idea of this method is the selection of a trial function that characterizes the actual solution of the problem as close as possible. This trial function contains the parameters of interest. The variational method then recasts the original partial differential equation into Newton like ordinary differential equations for these parameters. In the present analysis we assume A0(r; z) takes the form of the function given by[19, 20]

$$A_0(r,z) = \frac{E_{00}}{f}\left(1+\frac{r^2}{qr_0^2f^2}\right)^{-\frac{q}{2}} e^{i\theta} \qquad \dots\dots\dots(3)$$

where, the parameter f(z) is currently undetermined and upon multiplication with initial beam width $r_0$it gives the waist size of the laser beam at particular location inside the medium. Also, upon dividing by axial intensity, f(z) also gives the estimate of instantaneous axial intensity of the laser beam. Hence, the parameter f(z) can be referred to as dimensionless beam width parameter. The phenomenological parameter q is related to the deviation of irradiance, over the cross section of the beam, from ideal Gaussian profile. The function f(z) is known as longitudinal phase of the laser beam which is also known as Gouy phase.

The corresponding Lagrange equations

$$\frac{d}{dz}\left(\frac{\partial L}{\partial\left(\frac{\partial f}{\partial z}\right)}\right) - \frac{\partial L}{\partial f} = 0 \qquad (4)$$

And

$$\frac{d}{dz}\left(\frac{\partial L}{\partial\left(\frac{\partial \theta}{\partial z}\right)}\right) - \frac{\partial L}{\partial \theta} = 0 \qquad (5)$$

gives following set of coupled equations describing the dynamical variations of beam width and axial phase of the laser beam with propagation distance.

$$\frac{d^2f}{d\xi^2} = \frac{\left(1-\frac{1}{q}\right)\left(1-\frac{2}{q}\right)}{\left(1+\frac{1}{q}\right)}\frac{1}{f^3} - \left(\frac{\omega_p(\xi)r_0}{c}\right)^2\left(1-\frac{1}{q}\right)\left(1-\frac{2}{q}\right)\left(\frac{s}{2}-1\right)$$
$$\times\frac{\beta E_{00}^2}{f^3}\int_0^\infty x\left(1+\frac{x}{q}\right)^{-2q-1}\left\{1+\frac{1}{2}\frac{\beta E_{00}^2}{f^2}\left(1+\frac{x}{q}\right)^{-q}\right\}^{-\frac{s}{2}-1} dx \qquad (6)$$

$$\frac{d\theta}{d\xi} = -\frac{1}{f^2\left(1+\frac{1}{q}\right)} + \left(\frac{s}{2}-1\right)\frac{1}{f^2}\left(\frac{\omega_{p0}r_0}{c}\right)^2\int_0^\infty\left\{1+\frac{1}{2}\frac{\beta E_{00}^2}{f^2}\left(1+\frac{x}{q}\right)^{-q}\right\}^{-\frac{s}{2}-1} dx \qquad (7)$$

where,

$$x = \frac{r^2}{r_0^2f^2}$$
$$\xi = \frac{z}{k_0r_0^2}$$

Considering the plasma density to be increasing axially as $n(\xi) = n_0\big(1+tan(\xi d)\big)$ where,n_0 is the plasma density seen by the laser beam while entering into the plasma and d isthe measure of rate of increase of plasma density with distance, one can write the plasmafrequency as

$$\omega_p^2(\xi) = \omega_{p0}^2(1+tan(\xi d)) \qquad (8)$$

with

$$\omega_{p0}^2(z) = \frac{4\pi e^2}{m}n_0$$

Thus, it follows from eqs. (6) and (7) that the actual problem of solving a partial differential equation i.e., eq.1 has reduced to that of solving a set of ordinary differential equations. Although this reduced set of equations is also lacking from exact analytical solution, its approximate solution can be easily obtained with the help of simple numerical techniques. In solving eqs.(6) and (7) it has been assumed that initially the beam is collimated and is having perfect plane wave front while entering into the plasma region. Mathematically these two conditions impose the boundary conditions f = 1, $\theta = 0$ and on eqs.(6) and (7).

3. Results and Discussion:

Eq.(6) is identical to the equation of motion of a driven nonlinear oscillator. Thus, itpredicts that during the propagation of the laser beam through the plasma its beam width will evolve in an oscillatory manner i.e., the laser beam will undergo periodic focusing and defocusing. In the present analysis solution of eq.(7) in association with eq.(6) has been obtained with the help of Runge Kutta fourth order method for following set of parameters:

$$\omega_0 = 1.78 \times 10^{15}\frac{rad}{sec},$$

$$r_0 = 15\mu m \quad \left(\frac{\omega_p(\xi) r_0}{c}\right)^2 = 6, T_0 = 10^6 K, s = -3 \; and \; d = 0.025$$

and the corresponding variations of axial phase with distance have been shown in fig.(1).

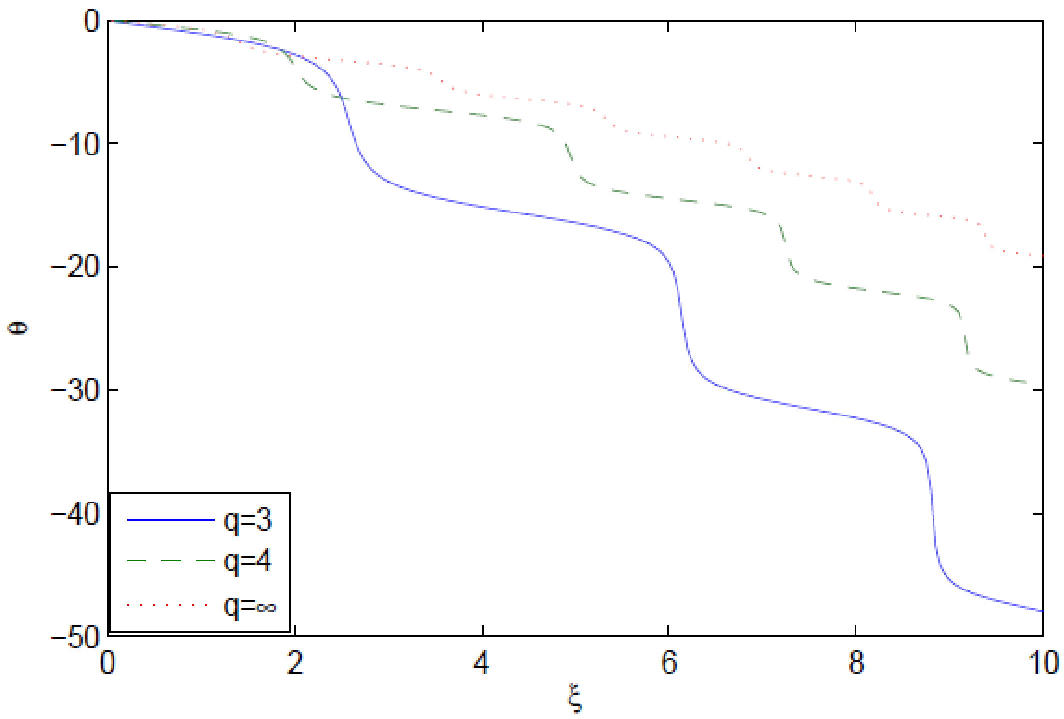

Fig.1. Effect of q on axial phase of laser beam.

It can be seen that the axial phase of the laser beam decreases monotonically with distance, showing step like behavior. This is due to the periodical self focusing/defocusing of the laser beam. As the laser beam undergo self focusing, its intensity increases and hence, the laser phase fronts start experiencing larger refractive indices. This results in decreased phase velocity of the phase fronts that leads to decreased spacing between the phase fronts as shown in fig.2.

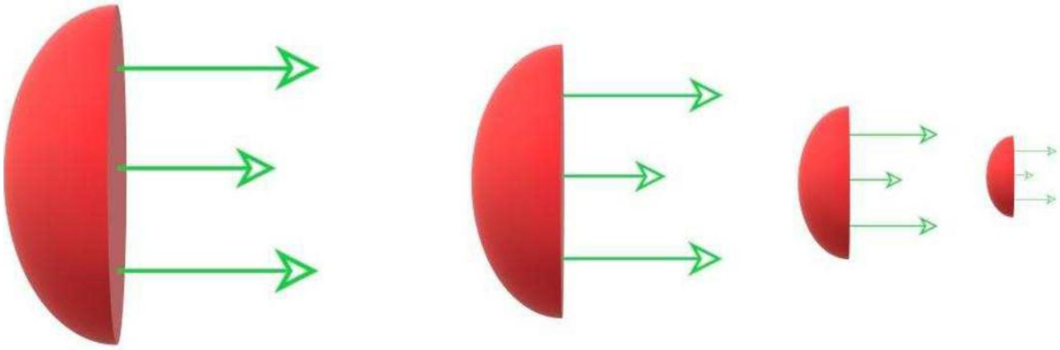

Fig.2. Effect of q on spacing between the wave fronts of the laser beam.

This fact can be explained in another way. The axial phase shift of the laser beam occurs due to the transverse momentum gained by the photons due to reduction in the volume of space available for their propagation. As the reduction in the transverse dimensions of the laser beam occurs due to self focusing, the photons gain additional transverse momentum (k_x, k_y) due to position momentum uncertainty $\Delta k_x \Delta x =$ constantand $\Delta k_y \Delta y =$ constant. As the over all momentum of the laser beam is conserved, theincrease in the transverse momentum reduces the longitudinal momentum. Thus during the propagation of laser beam its longitudinal momentum reduces as the beam keep on focusing. This results in monotonic decrease in its axial phase.

Step like behavior of the axial phase can also be seen from fig.1. These steps occur at the periodical positions of the minima of the beam width i.e., at the positions of focal spots of the beam. This indicates that there is slowest decrement in axial phase near its focal point. This is quite contrary to the behavior of the phase in graded index fibers where the phase decreases slowest at the positions of minimum intensity i.e., at the positions of maximum beam width. This difference in the behavior of phase in graded index fibers and that in plasmas can be explained qualitatively by the fact that plasmas a nonlinear medium behave like a linear wave guide. In linear wave guides the growth rate of the axial phase is inversely proportional to the beam width.

Further it can be seen that the size of each step keep on decreasing with distance. This is due to the fact that size of the step of the axial phase curve is determined by the frequency of oscillations of the beam width of the laser beam. As the magnitude of the refractive term in eq.(6) increases with distance due to the presence of density ramp, the frequency of oscillations of the beam with also increases with distance. This in turn reduces the size o the steps of the axial phase curve.

In order to see the effect of deviation parameter q on the evolution of axial phase of the laser beam eq.(7) has been solved for different values of q. It has been seen that with the increase in the value of deviation parameter q there is a reduction in the rate of change of axial phase of the laser beam with distance. This is due to the fact that there is one to one correspondence between the extent of focusing of the laser beam and the rate of decrease of its axial phase. As with increasing q the focusing of the laser beam gets reduced, hence, the rate of change of axial phase also reduces with increase of deviation parameter q.

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