Enhancement for the Position of Inverted Pendulum Using Linear Quadratic Regulator Based Fuzzy System

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Abstract
In this work, the problem of controlling the inverted pendulum has been investigated in detail. A different application that resembles inverted pendulum system has been studied. It considers the model, state space form representation with ricotta equation for optimal control applied to the nonlinear system with state and output variables. Later, to control the system several control system techniques has been applied in the literature survey. The challenging part is the fuzzy based method, to control position and for stabilization of the inverted pendulum with the application of appropriate rules of the overshoot and settling time is obtained within the desired value. The fuzzy based LQR is designed for a nonlinear system using MATLAB/SIMULINK. Also, a detailed study about the mathematical model of an inverted pendulum system is deliberated. The position of inverted pendulum is tuned using the same LQR controller. The simulation results show the performance of fuzzy LQR. By varying the pendulum parameters to different values, the robustness of the controller that has been used is checked.

Keywords: Linear quadratic regulator, Fuzzy system, inverted pendulum.

1. Introduction

In the year 1990, the IFAC (International Federation of Automatic Control) committee has developed [1] much real time design problems that are helpful in the comparative analysis of my control methods and mathematical tools. Out of the problems, the controlling of cascaded inverted pendulum is a benchmark problem which is highly unstable, and the difficulty increases with respect to number of links. There are many cases involved in conjunction with this system of cart-single inverted pendulum. The examples were shown in the fig.1. To understand more about such [1] system, consider a missile or rocket launcher dynamics with its centre of gravity located behind the centre of drag position to get a result of aerodynamic instability.

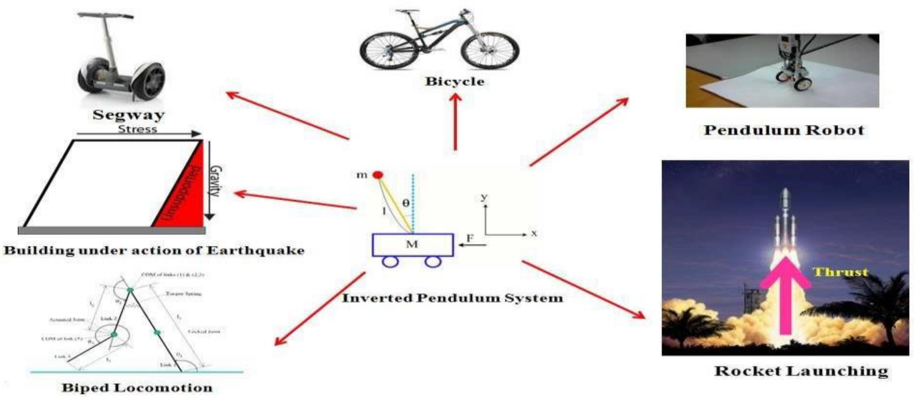


Fig.1. Different applications of inverted pendulum

So, overall, the stability of inverted pendulum stood as a target. In terms of a mechanical design it treated as a simple system comprising of D.C. motor, a pendulum looks like a pendant, a cart, and a driving mechanism. This can also be figured out as a SIMO system i.e. single input multi output system with voltage as input, the outputs are cart position and angle in which pendulum is positioned. There exists a bigger challenge once the working starts though the constructional view is very straightforward. The target follows with high instability, Non-linear case of operation, [2] Non- minimum phase system and under actuated. Many real time control systems concept works come is fuzzy control system. It provides rigorous analysis and perfect solution when the system is defined in a mathematical term. The fuzzy based controllers give the fast response for any nonlinear case system, provides high degree of precision and accuracy, less cost and reliable. To understand the operation of fuzzy based system, it is a clear cut aspect to know about the membership function which is a heart of fuzzy rule base model as it defines the [4] fuzziness of a control/process variable. It commonly expressed more in a graphic way and its tendency to demonstrate how easy and complete way a crisp variable belongs to a fuzzy set. To define it more clearly, researchers chose many different shapes that represents the variable which are based on their choices and observations. Membership functions are typically classified into four types: Trapezoidal, Triangular, Gaussian, generalized bell. Out of these shapes mentioned, the under the tree of inverted pendulum. The dynamic nature of inverted pendulum resembles the control systems that exist in robotic arms. [3] Also, the modelling of a human stand still, i.e. how the central nervous system registers the pose and changes of the human body. The modern control system in today's world most widely used are triangular and trapezoidal. [5] Reason, it is effortless to symbolize designer's proposal and requires low computational complexity as well as less time consuming.

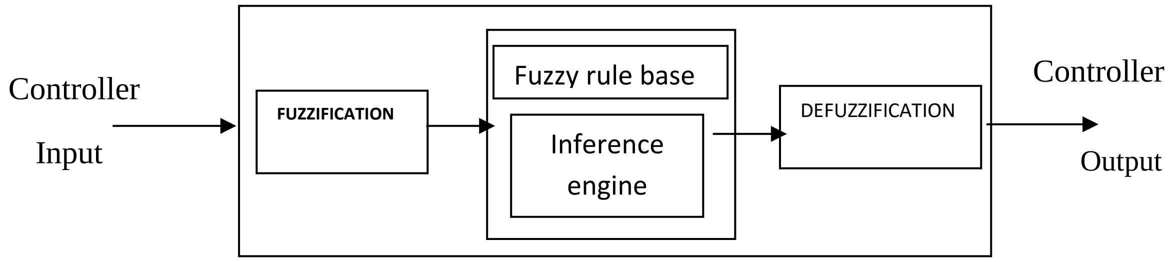


Fig. 2: Block diagram of fuzzy control system

In order to construct a [6] fuzzy system controller first it is necessary to create the membership values that decide which state variable has to represent the system non linearities as input signal to the controller, later, specify the rule table to make a [7] human decision process, for example, the IF-THEN procedure follows with the software code that functions with the computer program similar to a human brain. And finally determine the procedure for defuzzifying the result.

2. Literature Survey:

The LQR (Linear Quadratic Regulator) [8] controller mainly optimizes the cost function when the system is in nonlinear case. It is extremely used as state feedback gains as it is represented using state space form to study the system behaviour. Unlike, the traditional methods like PID controller based on transfer function of system. This proposed technique, [9] LQR controller is succeeded for inverted pendulum by using command function in MATLAB/SIMULINK. This LQR used in time varying system that also handles the perturbation and with addition to noise. It is a inclusive analysis with multi variable for design of feedback system. As learnt, that to optimize or to reduce the performance index to a decrease value. The minimization also should tend to perfectness in performance [10]. For as to design the linear quadratic regulator controller the performance index denotes with J and is written in equation which followed as:

Where Q= State weighting matrix (n*n), Symmetric positive semi definite (≥ 0),

And R= Control weighting matrix (m*m), Symmetric positive definite (>0).

Q and R are the crucial matrices in LQR optimization. It must be a selective who relates to the weighting of individual state variables and control inputs with base as vector against each other. This can be done by iterative process and physical concept must be known prior to apply. By trial and error, we can construct Q and R matrices. Also the gain matrix K has to be determined with the help of closed loop optimal control rule and followed with algebraic ricatti equation is defined as,

$$J=\int_0^{\infty}\left(x^TQx+u^TRu\right)dt$$

$$U=-Kx$$

$$A^TP+PA-PBR^{-1}B^TP+Q=0$$

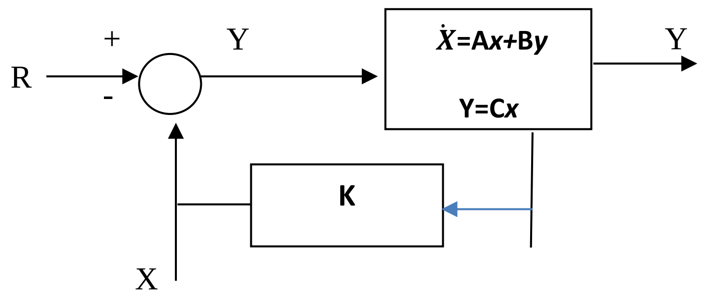
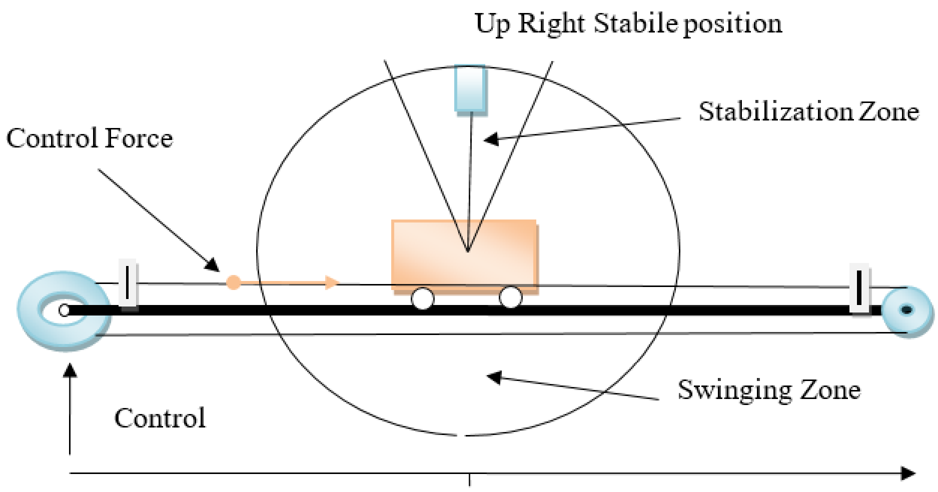


Fig.3. Optimum feedback controller of LQR

MATLAB command for LQR [11] is used to obtain the corresponding value of optimal feedback gain *K=lqr(A,B,Q,R)*, completing the LQR control strategy design.

Since 19th century the problem of controlling pendulum system has been described in many books in order to find solution for linear optimal control problem and to obtain stability for complex nonlinear systems. But the stabilization is still a challenging issue [12]. In general, to design the controller for nonlinear case of complex systems the approaches were sub divided into: System linearization and adapting the linear control laws to stabilize and then nonlinear controllers that directly based to NL systems.

The fig.4 shows the cart-pole inverted pendulum system. In this system, the main objective is to control the pendulum in its upright position and also simultaneously bring back the cart to centre of the rail. When the SIMULINK program is started, the position sensor takes the current position as zero while angle sensor takes the current value as $-\pi$ radians. The programmer should add (or subtract) suitable values to these readings o set the zero position of the sensor at the desired physical locations.



3. Proposed Method:

Modelling of Inverted Pendulum:

In earlier section, various applications for the problem have been discussed and the schematic of the control method is depicted. It is essential to obtain the algebraic ricatti equation for [13-14] optimal control and the optimal feedback gain K can be obtained by running m-file program in MATLAB. Based on this controllability and observability of the system assumed has been tested. So, optimal feedback control is applied for this system.

Considering the state space equations,

$$\dot{x}_1(t) = x_3(t)$$

$$\dot{x}_2(t) = x_4(t)$$

$$\dot{x}_3(t) = \frac{l\mu g \sin x_2 \cos x_2 - a\mu x_4^2 \sin x_2 - lf_p x_4 \cos x_2 - af_c x_3 + au}{\mu l \sin^2(x_2) + I}$$

$$\dot{x}_4(t) = \frac{\mu g \sin x_2 - f_p x_4 - l\mu x_4^2 \sin x_2 \cos x_2 - lf_c x_3 \cos x_2 + l \cos x_2 u}{\mu l \sin^2(x_2) + I}$$

Where u denotes the force

$$a = l^2 + I / (m_p + m_c)$$

$$\mu = (m_p + m_c)l$$

Where, $x_1(t)$ = cart position, $x_2(t)$ = pendulum angle,

$x_3(t)$ = cart velocity, $x_4(t)$ = pendulum angular velocity.

Table 1 Parameters considered for inverted pendulum

Parameter	**Value**
Acceleration due to gravity, g	9.81 m/ s^2
Pole mass, m_p	0.36kg
Cart mass, m_c	2.4 kg
Pole length, l	0.36 m
Moment of inertia of pole, I	0.099 kg- m^2
Cart friction coefficient, f_c	0.05 Ns/m
pendulum damping coefficient, f_p	0.005 Nm/rad
Maximum force on cart, u	20 N
Track Limit, $L/2$	$\pm$ 0.4 m

The nonlinear functions of the equation can be chosen as:

$$f_1(x(t)) = \frac{l\mu g \sin x_2 \cos x_2}{x_2(l\mu \sin^2 x_2 + I)} \quad ; f_2(x(t)) = \frac{-af_c}{(l\mu \sin^2 x_2 + I)}$$

$$f_3(x(t)) = \frac{-a\mu x_4 \sin x_2 - lf_p \cos x_2}{(l\mu \sin^2 x_2 + I)} \quad ; f_4(x(t)) = \frac{\mu g \sin x_2}{x_2(l\mu \sin^2 x_2 + I)}$$

$$f_5(x(t)) = \frac{-f_c l \cos x_2}{(l\mu \sin^2 x_2 + I)} \quad ; f_6(x(t)) = \frac{-f_p - \mu l \sin x_2 \cos x_2}{(l\mu \sin^2 x_2 + I)}$$

$$f_7(x(t)) = \frac{l \cos x_2}{(l\mu \sin^2 x_2 + I)} \quad ; f_8(x(t)) = \frac{a}{(l\mu \sin^2 x_2 + I)}$$

Inverted pendulum dynamics can be represented

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix}=\begin{bmatrix}0&0&1&0\\0&0&0&1\\0&f_1&f_2&f_3\\0&f_4&f_5&f_6\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}+\begin{bmatrix}0\\0\\f_8\\f_7\end{bmatrix}u$$

The operating region of nonlinear model can be selected as

$$x_1(t)\in[-0.4,0.4];x_2(t)\in[-0.2,0.2]$$
$$x_3(t)\in[-10,10];x_4(t)\in[-25,25]$$

For the given bounds on system states, therefore the minimum and maximum values of all the functions are

$$f_1(x)\in[90.543,104.24];\ f_2(x)\in[-0.1,-0.08863]$$
$$f_3(x)\in[-10.79,6.618];\ f_4(x)\in[91.922,104.24]$$
$$f_5(x)\in[-0.202,-0.17633];\ f_6(x)\in[-18.153,18.052]$$
$$f_7(x)\in[3.5266,4.04];\ f_8(x)\in[-0.0202,-0.0179]$$

Hence the nonlinear model for the inverted pendulum system can be given by the equation

$$\dot{x}=\sum_{j=1}^{2}\sigma_j(A_jx+B_ju)$$

The membership functions of the fuzzy rules are shown below

$$\text{ex:}\quad f_1(x(t))=\frac{l\mu g\sin x_2\cos x_2}{x_2(\mu l\sin^2 x_2+I)}=M_1f_{1\max}+M_2f_{1\min}$$
$$M_1+M_2=1$$

By solving the above equations, obtain membership functions

$$M_1=\frac{f_1(x(t))-f_{1\min}}{f_{1\max}-f_{1\min}};M_2=\frac{f_{1\max}-f_1(x(t))}{f_{1\max}-f_{1\min}}=1-M_1$$

Normalized membership functions is

$$\sigma_1(f(x))=\frac{M_1(f_1(x))M_1(f_2(x))....M_1(f_8(x))}{\sum_{i=1}^{2}M_i(f_1(x))M_i(f_2(x))....M_i(f_8(x))}$$

$$\sigma_2(f(x))=1-\sigma_1(f(x))$$

Gains are obtained by

$$K_1 = [-10.5984 \;\; 166.0063 \;\; -17.6850 \;\; 34.4504]$$

$$K_2 = [-12.1389 \;\; 178.4446 \;\; -20.1402 \;\; 11.3537]$$

$$u(t) = -\sum_{j=1}^{2} \sigma_j K_j x(t)$$

Simulation Results:

The following results are obtained with the help of MATLAB/SIMULINK. And with parameter variations such as length and mass of the pendulum these different results has been obtained.

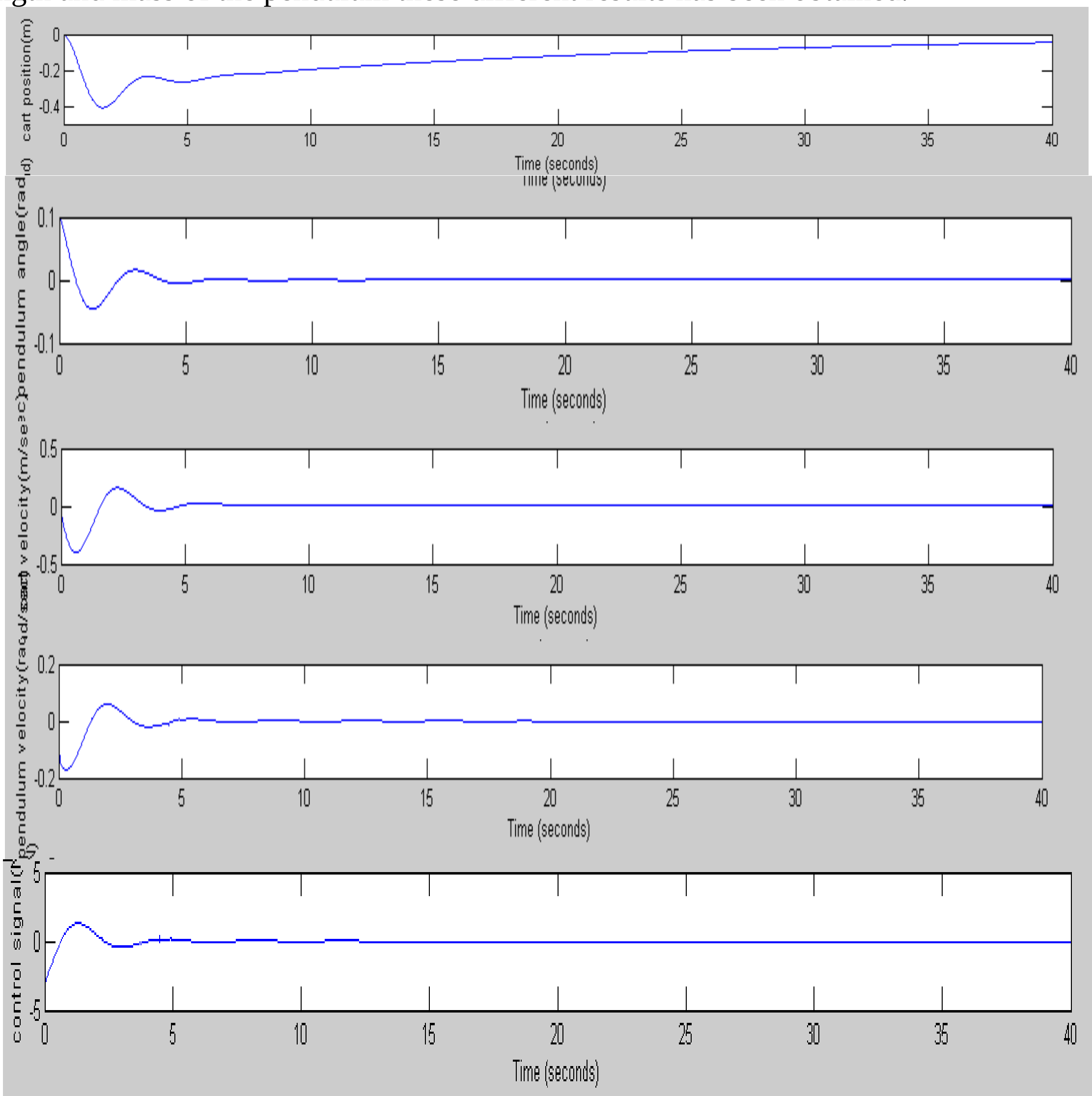


Fig 5: Results at length of pendulum l=0.36 and mass of pendulum m_p=0.36

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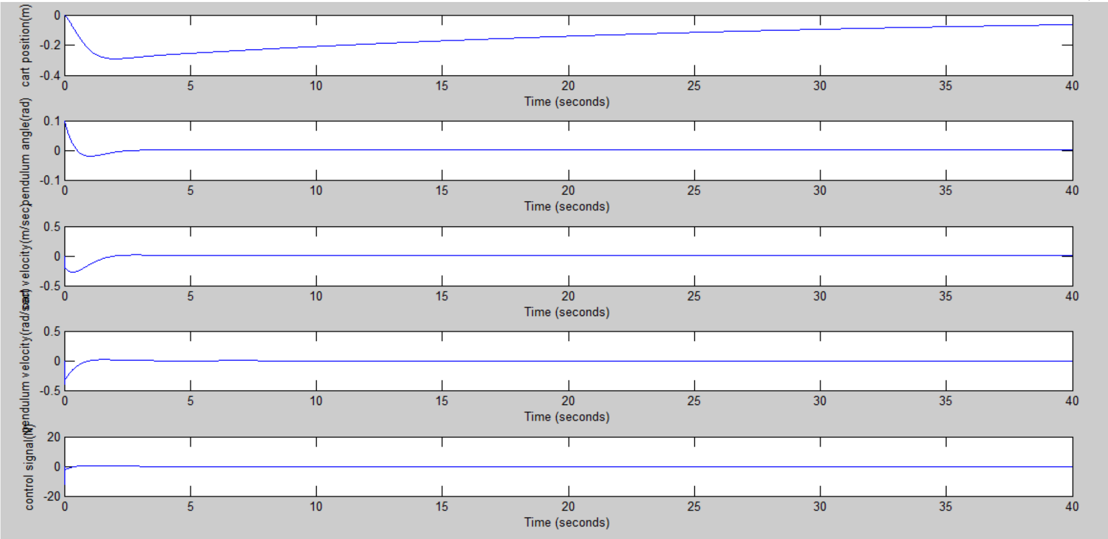

Fig 6: Results at pendulum length l = 0.432 and mass of pendulum m_p=0.36

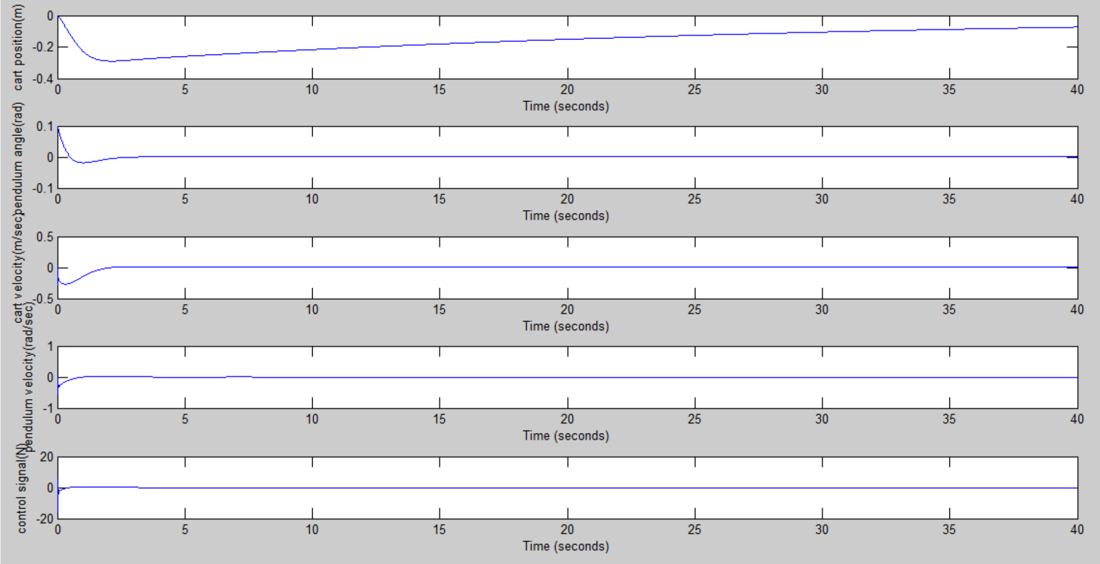

Fig 7: Result of Inverted pendulum at pendulum length l=0.36 mass of pendulum m_p=0.72

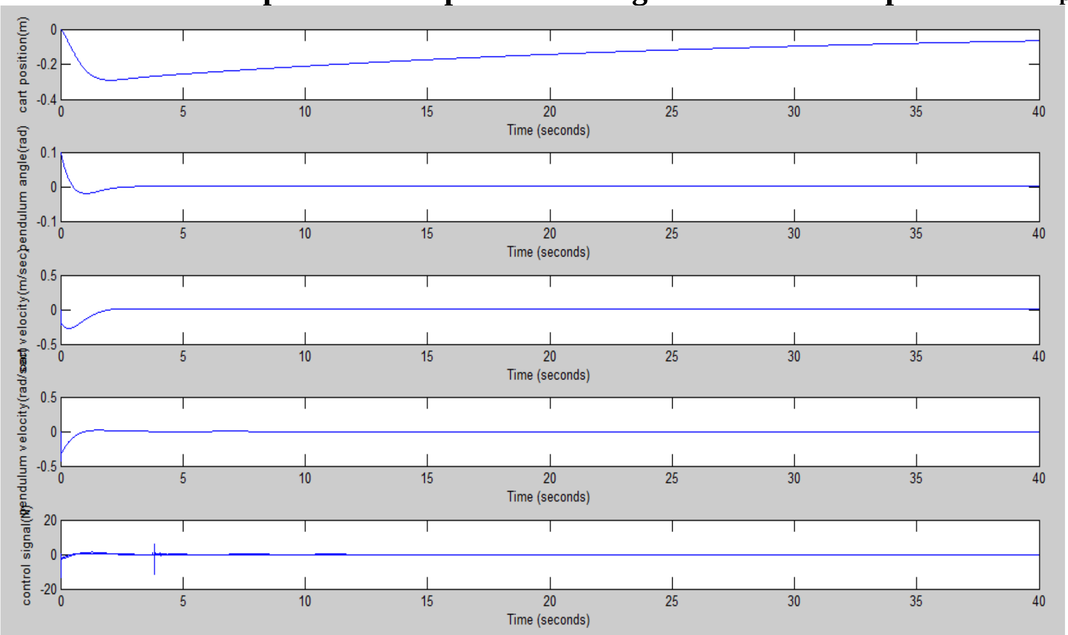

Fig 8: Results at pendulum length l=0.432 and mass of pendulum m_p=0.72

4. Conclusion:

The problem of controlling a nonlinear system has been studied and fuzzy controller is designed for nonlinear systems. Fuzzy model of the system is obtained based on the sector nonlinear method and obtained controller model is able to describe the system dynamics accurately within some required operation range. Fuzzy logic intelligence technique is used to control position and for stabilization of the inverted pendulum with the application of appropriate rules of the overshoot and settling time is obtained within the desired value. The control problem has been investigated for class of large scale nonlinear system with sub systems which can exchange information through networks by fuzzification. For each sub system, the interconnected fuzzy systems have been obtained. For stabilizing the system a state feedback controller is designed based of fuzzy mode. The controller for the above nonlinear system is constructed in such a way that the closed loop system is stable with an exponential decay rate. The robustness of the controller is checked at different values of pendulum parameters.

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