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EXPLORATORY APPROACH TOWARDS Q-FUZZY IDEALS

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ABSTRACT: *Undoubtedly L.A. Zadeh is the pioneer in the initiation of fuzzy sets and moreover numerical contributions are made by researchers on the notion of fuzzy sets. Fuzzy sets encompass vast areas of research in engineering, medical sciences, social sciences, graph theory, etc. In this paper we introduce the concept of Q-fuzzy ideals and explore the properties and theorems on it .*

KEYWORDS : Fuzzy groups, Fuzzy sets and logic, Theory of fuzzy sets, fuzzy lattices, fuzzy algebraic structures.

1. INTRODUCTION : The idea of a fuzzy subset of a set is formulated by Lofti Zadeh. In the current era this particular concept has diversified applications in several mathematical branches such as Group theory, Functional analysis, Probability theory, Topology and so on. Fuzzy sub rings and Fuzzy ideals were developed by Rajesh Kumar in 2009 [1]. Later on A. Solairaju and R. Nagarajan have carved out a new algebraic structure called Q-Fuzzy subgroups in their research work namely ‘ A New Structure and Construction of Q-Fuzzy Groups in 2009 [2]. Gopi Kanta Barthakur and Jugalkharghoria studied on Q- Fuzzy N- subgroup and Q- fuzzy ideals of an N-group in 2013. Bhimraj Basumatary and Gopi Kanta Barthakur work on Q-fuzy ideal and Q-fuzzy quotient near-rings in 2014 [4] stimulated various authors to develop these concepts. In 2019, B.Sailaja ,V.B.V.N. Prasad interpreted the ‘Exploring the axiom of excluded middle and axiom of contradiction in fuzzy sets’ in their research work [5]

In this manuscript we have discussed the definitions and properties of Q- Fuzzy subsets,Q-Fuzzy sub rings , Q-Fuzzy ideals .

2. PRELIMINARIES : The preliminaries section comprises some elementary definitions and properties of fuzzy sets.

Definition 2.1: Let X be a non empty set. A fuzzy subset Θ of X is a function $\Theta : X \rightarrow [0,1]$.

Example 2.1 : Let X = {V_1, V_2, V_3, V_4) be the four vaccines developed by the four countries to combat COVID.

The incidence of ‘SUSCEPTABILITY TO VIRUS’ is denoted by the fuzzy subset σ which indicates the zone of ’most susceptible’, ‘more susceptible’, ‘somewhat susceptible’, ‘not at all susceptible’. It can be stated as follows :

$\Theta = \{(V_1 , 0.8), (V_2 , 0.5), (V_3, 0.2) (V_4 , 0)\}$

Definition 2.2 : A fuzzy subset Θ of a ring R is called a fuzzy sub ring of R if for every x ,y Є R, the following conditions are satisfied :

(i) $\Theta(x - y) \geq \mathrm{Min}\ (\Theta(x), \Theta(y))$

(ii) $\Theta(x\ y) \geq \mathrm{Min}\ (\Theta(x), \Theta(y))$

Definition 2.3: Let Θ be any fuzzy sub ring (fuzzy ideal) of a ring R , t Є[0,1]. The sub ring (ideal)

$\Theta_t = \{x \in R / \Theta(0) \geq t\}$ is called a level sub ring (ideal) of Θ.

Definition 2.4 : A fuzzy subset Θ of a ring R is called a fuzzy ideal of R if for every x, y Є R the following conditions are satisfied :

(i) $\Theta(x - y) \geq \mathrm{Min}\ (\Theta(x), \Theta(y))$

(ii) $\Theta(x\ y) \geq \mathrm{Max}\ (\Theta(x), \Theta(y))$

Definition 2.5: Let σ and Θ be any two fuzzy ideals of a ring R. The product σ ∩ Θ of σ and Θ is defined by

$$(\sigma \cap \Theta)(x) = \sup_{x = \sum y_i z_i} (\mathrm{Min}\ (\mathrm{Min}\ (\sigma (y_i) , \Theta(z_i))))$$

3. Q-FUZZY SUBSETS AND Q-FUZZY IDEALS:

Definition 3.1 : Let X and Q be two nonempty sets. A function σ : X x Q→ [0,1]is called a Q-fuzzy subset of X.

Example 3.1 : Let X ={a ,b, c}, Q ={d} then the Q-fuzzy subset is as follows :

σ = {<(a, d),(0.5)>, <(b, d),(0.3)>, <(c, d),(0.1)>} is a Q-fuzzy subset of X.

Definition 3.2 : A Q-fuzzy subset σ of a ring Ris called a Q-fuzzy subring of R, if for every x ,y Є R the following conditions are satisfied :

(i) $\sigma (x - y, q) \geq \mathrm{Min}\ (\sigma(x, q),\sigma(y, q))$

(ii) $\sigma (x\ y , q) \geq \mathrm{Min}\ (\sigma(x, q),\sigma(y, q))$

Definition 3.3 : A Q-fuzzy subset σ is called a Q-fuzzy ideal of R if for every x,y Є R the following conditions are satisfied :

(i) $\sigma (x - y , q) \geq \mathrm{Min}\ (\sigma(x, q),\sigma(y, q))$

(ii) $\sigma \ (x y , q) \geq \text{Max} (\sigma(x, q),\sigma(y, q))$

Definition 3.4 : Let σ be a Q- fuzzy ideal of a ring R, and for any t Є [0 ,1] we define the level ideal of σ as follows :

$$\sigma_t = \{x \in R, q \in Q / \sigma (0, q) \geq t \}$$

4 . PROPERTIES OF Q-FUZZY IDEALS :

Theorem 4.1 : If σ is any Q-fuzzy ideal of a ring R, then σ +σ = σ

Proof : Given that σ is any Q-fuzzy ideal of R.

Let x Є R

$$\text{Then } (\sigma +\sigma) (x , q) = \underset{x= a + b}{\text{Sup}} (\text{Min} (\sigma(a, q), \ \sigma(b , q)) \text{ where } a,b \in R$$

$$\geq \text{Min} (\sigma(x, q), \ \sigma(0 , q)) = \sigma(x ,q)$$

$$\therefore (\sigma +\sigma) (x ,q) \geq \sigma(x, q) \ \rightarrow 1$$

If x = a + b where a, b Є R then

$$\sigma (x, q) \geq \text{Min} (\sigma (a, q),\sigma (b, q))$$

$$\text{So, } \sigma (x ,q) \geq \text{Sup} (\text{Min} (\sigma(y, q),\sigma (z, q)) \ y, z \in R$$

$$\therefore \ \sigma (x, q) \geq (\sigma +\sigma) (x ,q) \ \rightarrow 2$$

From 1 and 2 we have that σ +σ = σ

Theorem 4.2 : If $\{ \sigma_n / n \in Z^+\}$ is a collection of Q-fuzzy ideals of a ring R such that

$$\sigma_1 \subseteq \sigma_2 \subseteq \ldots \subseteq \sigma_n \subseteq \ldots$$

then $\bigcup\limits_{n\in Z^+} \sigma_n$ is a Q-fuzzy ideal of R.

Proof : Given that If $\{ \sigma_n / n \in Z^+\}$ is a collection of Q-fuzzy ideals of a ring R such that

$$\sigma_1 \subseteq \sigma_2 \subseteq \ldots \subseteq \sigma_n \subseteq \ldots$$

then $\bigcup\limits_{n\in Z^+} \sigma_n$ is a Q-fuzzy ideal of R.

Let a, b Є R.

Now for any i , j $\in Z^+$ there exists k $\in Z^+$ such that we have

$$\sigma_i (a ,q) , \sigma_j (b, q) \leq \sigma_k (a ,q) , \sigma_k (b ,q)$$

$$\Rightarrow \text{Min} (\sigma_k (a ,q) , \sigma_k (b ,q) \leq \text{Min} (\sigma_k (a ,q) , \sigma_k (b ,q))$$

$$\leq \sigma_k (a - b ,q) \rightarrow 1$$

It is enough to prove the Q-fuzzy ideal properties. Now, consider

$$\text{Mi n}((\bigcup_{n\in Z^+} \sigma_n) (a ,q) , (\bigcup_{n\in Z^+} \sigma_n) (b , q)) = \text{Min} (\sup_{n\in Z^+} (\sigma_n (a ,q)) , \sup_{n\in Z^+} \sigma_n ((b ,q))$$

$$= \sup_{i,j\in Z-} (\text{Min} (\sigma_i (a , q) , \sigma_j (b, q))$$

$$\leq \sup_{k\in Z^+} (\text{Min} ((\sigma_k (a , q) , \sigma_k (b ,q))$$

$$\leq \sup_{k\in Z^+} (\sigma_k (a - b , q)) = (\bigcup_{n\in Z^+} \sigma_n) (a - b , q)$$

Therefore $(\bigcup_{n\in Z^+} \sigma_n) (a - b , q) \geq \text{Min} ((\bigcup_{n\in Z^+} \sigma_n) (a ,q) , (\bigcup_{n\in Z^+} \sigma_n) (b , q))$

Finally, $(\bigcup_{n\in Z^+} \sigma_n) (a b ,q) = \sup_{n\in Z^+} (\sigma_n (a b , q))$

$$\geq \sigma_i (a b , q)$$

$\geq \text{Max} (\sigma_i (a, q) , \sigma_i (b, q))$ for every i $\in Z^+$

So that $(\bigcup_{n\in Z^+} \sigma_n) (a b ,q) \geq \text{Max} ((\bigcup_{n\in Z^+} \sigma_n) (a ,q), (\bigcup_{n\in Z^+} \sigma_n) (b , q))$

Hence $\bigcup_{n\in Z^+} \sigma_n$ is a fuzzy ideal of R.

Theorem 4 .3 : Let f be a homomorphism from a ring R onto a ring R^1. If σ^1 and Θ^1 are any two fuzzy ideals of R^1 , then the following holds :

$$f^{-1} (\sigma^1) .f^{-1} (\Theta^1) \subseteq f^{-1} (\sigma^1 \Theta^1)$$

Proof: Given that f is a homomorphism from a ring R onto a ring R^1. γ

Let σ^1 and Θ^1 are any two fuzzy ideals of R^1.

Let x $\in$ R and let $\varepsilon > 0$ be given .

Set $\gamma = (f^{-1}(\sigma^1) . f^{-1}(\Theta^1))(x, q) \rightarrow 1$

$\delta = f^{-1}(\sigma^1 \Theta^1)(x, q)$ 1

From 1 $\gamma = \operatorname{Sup}_{x = x_1 x_2} (\operatorname{Min}(f^{-1}(\sigma^1)(x_1, q), f^{-1}(\Theta^1)(x_2, q)))$

$\Rightarrow \gamma - \varepsilon < \operatorname{Sup}_{x = x_1 x_2} (\operatorname{Min}(f^{-1}(\sigma^1)(x_1, q), f^{-1}(\Theta^1)(x_2, q)))$ where $x_1, x_2 \in R$

$\Rightarrow \gamma - \varepsilon = \operatorname{Sup}_{x = x_1 x_2} (\operatorname{Min}(\sigma^1(f(x_1, q), \Theta^1(f(x_2, q)))$

$\Rightarrow \gamma - \varepsilon < \operatorname{Min}(\sigma^1(f(x_1, q), \Theta^1(f(x_2, q)))$ for some $x_1, x_2 \in R$ such that $x = x_1 x_2$

$\Rightarrow \gamma - \varepsilon \leq (\sigma^1 \Theta^1)(f((x_1, q), (x_2, q))) = (f^{-1}(\sigma^1 \Theta^1))(x, q) = \delta$

$\Rightarrow \gamma - \varepsilon \leq \delta$

$\Rightarrow \gamma \leq \delta$ since $\varepsilon > 0$ is arbitrary

Hence $f^{-1}(\sigma^1) . f^{-1}(\Theta^1) \subseteq f^{-1}(\sigma^1 \Theta^1)$.

CONCLUSION : Before approaching towards Q-Fuzzy ideals we have discussed about fuzzy subsets and Q-fuzzy subsets. Thereafter we discussed the properties and theorems on Q-fuzzy ideals.

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