Application of Bipolar Fuzzy Soft Set Using Moora Methods

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Abstract: The main objective of this article is that the concept of a bipolar fuzzy soft set by using the Modified MOORA methods. We first proposed a new ranking procedure in MOORA methods and which applied in the bipolar fuzzy soft set. It is more effective for solving many decision making problems to find the ranking of alternatives. An illustrative example of the proposed approach is presented for the ranking of Pc and the comparative analysis of MOORA methods are also given.

Keywords: Soft Set, Fuzzy Soft Set, Bipolar fuzzy set, bipolar fuzzy soft set, MOORA Multimoora.

1. INTRODUCTION

Lotfi A. Zadeh introduced Fuzzy sets in 1965 as an extension of the classical sets. In 1999, [11] Molodtsov developed the soft set theory and is a generalization of fuzzy set theory. Maji et al [11] described the concept of fuzzy soft sets. In decision making problems, some results of the application of fuzzy soft sets have given by Roy and Maji [11]. Several types of fuzzy extensions are vague sets, intuitionstic fuzzy sets, interval-valued fuzzy sets, etc; one of the main extensions of fuzzy sets is bipolar fuzzy set whose membership degree range is different from all the extensions. In the year 2000, Lee [10] introduced bipolar fuzzy set and its membership value lies between $[-1,1]$. In the year 2006, the MOORA method was first initiated by Brauers and Zavadskas.[3]. Brauers et al first proposed the MULTI-MOORA method in 2010 [4].

In this article, we introduced the combined concept of soft set and bipolar fuzzy set by using the extension of MOORA methods. We also illustrate a decision making problem of bipolar fuzzy soft set. The design of the article presents: preliminaries and examples are in section 2. In section 3, comparison between the ordinary MOORA methods and our proposed method is tabulated. In the next section, numeric example is solved. At the end of the article, we present the conclusion.

2. PRELIMINARIES

Soft Set [13]

A pair (F, E) is called a soft set $(over\ U)$ if and only if F is a mapping of E into the set of all subsets of the set U.

Bipolar Fuzzy Set [1]

A bipolar fuzzy soft set A in a universe U is an object having the form, $A = \{(x, {}^{+}_{A}(x), {}^{-}_{A}(x)) : x \in U\}$ where $\mu_A^+ : U \to [0,1]$, $\mu_A^- : U \to [-1,0]$. so μ_A^+ denote for positive information and μ_A^-denote for negative information.

Bipolar Fuzzy Soft Set [1]

Let U be a universe, E a set of parameters and $A \subset E$. Define$F : A \to BF^U$, where $BF^U$the collection of all bipolar fuzzy subsets of U then(F, A) is said to be bipolar fuzzy soft set over a universe U. It is defined by

$$(F, A) = F(e_i)$$
$$F(e_i) = \{(c_i, \mu^+(c_i), \mu^-(c_i)) : \forall\, c_i \in U, \forall\, e_i \in A\}$$

Example

Let $U = \{s_1, s_2, s_3, s_4\}$ be the set of schools and $E = \{e_1 = CBSE, e_2 = Matriculation, e_3 = Government\}$, parameters and $A = \{e_1, e_3\} \subseteq E$. Then,

$$(F, A) = \begin{cases} F(e_1) = \begin{Bmatrix} (s_1, 0.7, -0.3), \\ (s_2, 0.2, -0.5), \\ (s_3, 0.1, -0.9), \\ (s_4, 0.6, -0.4) \end{Bmatrix} \\ F(e_3) = \begin{Bmatrix} (s_1, 0.4, -0.6), \\ (s_2, 0.8, -0.2), \\ (s_3, 0.6, -0.1), \\ (s_4, 0.2, -0.5) \end{Bmatrix} \end{cases}$$

3. COMPARISON BETWEEN MOORA METHODS AND MODIFIED MOORA METHODS

Three types of MOORA methods are (RS) Ratio system, (RPA) Reference Point Approach and (FMF) Full Multiplicative Form. At last, alternatives ranking are calculated by MULTI-MOORA depends on dominance theory.

MOORA Method	Modified MOORA Method
3.1 *Ratio System* (RS) *of MOORA* Algorithm 1. Construct decision matrix 2. Compute normalized decision matrix by using the equation $x_{ij}^{\blacksquare} = \frac{x_{ij}}{\sqrt{\sum_{j=1}^{n} x_{ij}^2}}$ 3. Calculate weighted normalized decision matrix by using the equation $uij = wjxij\blacksquare$ Where w_j = the weight of j^{th} criterion. 4. Obtain final preference ($p_i^{\blacksquare}$) by using the equation	**3.1 *Ratio System* (RS) *of MOORA* Algorithm** 1. Construct decision matrix for the positive membership values. 2. Compute normalized decision matrix by using the equation $x_{ij}^{\blacksquare} = \frac{x_{ij}}{\sqrt{\sum_{j=1}^{n} x_{ij}^2}}$ 3. Calculate weighted normalized decision matrix by using the equation $u_{ij} = w_j x_{ij}^{\blacksquare}$ Where w_j = the weight of j^{th} criterion. 4. Obtain final preference ($p_i^{\blacksquare}$) by using the equation

$$p_i^{\blacksquare} = \sum_{j=1}^{g} u_{ij} - \sum_{j=g+1}^{n} u_{ij}$$ Here $j = 1\ to\ g$ indicates the Maximized criteria and $j = g+1\ to\ n$ Indicates minimized criteria. 5. Obtain the ranking of alternatives.	$$p_i^{\blacksquare} = \sum_{j=1}^{g} u_{ij} - \sum_{j=g+1}^{n} u_{ij}$$ Here $j = 1\ to\ g$ indicates the Maximized criteria and $j = g+1\ to\ n$ Indicates minimized criteria. 5. The above four steps are repeated for the negative membership values and its final preference values can be denoted by $p_i^{\blacksquare\blacksquare}$ 6. Obtain final score and ranking of alternatives by $$p = p_i^{\blacksquare} - p_i^{\blacksquare\blacksquare}.$$

MOORA Method	Modified MOORA Method
3.2 *The Reference Point Approach of MOORA*(RPA) **Algorithm** 1. Construct decision matrix. 2. Compute normalized decision matrix by using the equation $$x_{ij}^{\blacksquare} = \frac{x_{ij}}{\sqrt{\sum_{j=1}^{n} x_{ij}^2}}$$ 3. Calculate weighted normalized decision matrix by using the equation $$u_{ij} = w_j x_{ij}^{\blacksquare}$$ Where w_j = the weight of j^{th} criterion. 4. Find reference points $(r_j^{\blacksquare})$ for each criteria and it can be determined by choosing maximum value for maximization criteria whereas minimum value for minimization criteria. 5. Calculate distance between the reference points and alternatives are calculated by subtracting reference point value from u_{ij} values for minimization criteria and for maximization criteria, subtracting u_{ij} values from the reference point values. It can be denoted by $d_i^{\blacksquare}$. 6. Obtain the ranking of alternatives.	**3.2 *The Reference Point Approach of MOORA*(RPA)** **Algorithm** 1. Construct decision matrix for positive membership values. 2. Compute normalized decision matrix by using the equation $$x_{ij}^{\blacksquare} = \frac{x_{ij}}{\sqrt{\sum_{j=1}^{n} x_{ij}^2}}$$ 3. Calculate weighted normalized decision matrix by using the equation $$u_{ij} = w_j x_{ij}^{\blacksquare}$$ Where w_j = the weight of j^{th} criterion. 4. Find reference points $(d_j^{\blacksquare})$ for each criteria and it can be determined by choosing maximum value for maximization criteria whereas minimum value for minimization criteria. 5. Calculate distance between the reference points and alternatives are calculated by subtracting reference point value from u_{ij} values for minimization criteria and for maximization criteria, subtracting u_{ij} values from the reference point values. 6. The above five steps are repeated for the negative membership values also. 7. Obtain final score and ranking of alternatives by $d = d_i^{\blacksquare} - d_i^{\blacksquare\blacksquare}$.

MOORA Method	Modified MOORA Method

3.3 *Full Multiplicative Form of MOORA Algorithm*(FMF)	**3.3 *Full Multiplicative Form of MOORA Algorithm* (FMF)**
1. Construct the decision matrix. 2. Find multiplicative ranking index Z_i for each alternative by $Z_i = \frac{P_i}{Q_i}$, where $$P_i = \prod_{j=1}^{g} {x_{ij}}^{w_j}$$ for maximization criteria and for the minimization criteria, $$Q_i = \prod_{j=g+1}^{n} {x_{ij}}^{w_j}.$$ 3. Obtain alternatives rank.	1. Construct the decision matrix for positive membership values. 2. Find multiplicative ranking index $Z_i^{\blacksquare}$ for every alternative by $Z_i^{\blacksquare} = \frac{P_i}{Q_i}$, where $$P_i = \prod_{j=1}^{g} {x_{ij}}^{w_j}$$ for maximization criteria and for the minimization criteria, $$Q_i = \prod_{j=g+1}^{n} {x_{ij}}^{w_j}.$$ 3. The above two steps are repeated for the negative membership values also. 4. Obtain the ranking of alternatives by $$z = Z_i^{\blacksquare} - Z_i^{\blacksquare\blacksquare}.$$

4. NUMERICAL EXAMPLE

The idea of bipolar fuzzy soft set can be used numerously for evaluating many decision making problems. Let the universe$U = \{\boldsymbol{p_1}, \boldsymbol{p_2}, \boldsymbol{p_3}, \boldsymbol{p_4}, \boldsymbol{p_5}\}$ be five Pc, specifications of Pc's are $S = \{\boldsymbol{s_1} = processor\ speed, \boldsymbol{s_2} = RAM, \boldsymbol{s_3} = ROM, \boldsymbol{s_4} = hard\ disk, \boldsymbol{s_5} = model\}$ be the parameters. Out of these parameters, let$\boldsymbol{s_2}, \boldsymbol{s_3}, \boldsymbol{s_4}$be the beneficial criteria and $\boldsymbol{s_1}\ and\ \boldsymbol{s_5}$ be the non-beneficial criteria. Our aim is to calculate the ranking of Pc's by using bipolar fuzzy soft set using modified MOORA methods. The final ranking is calculated by MULTI-MOORA depends on dominance theory. Let

$$(F, A) = \left\{ \begin{array}{l} F(S_1) = \left\{ \begin{array}{l} (\boldsymbol{P_1}, 0.3, -0.6) \\ (\boldsymbol{P_2}, 0.5, -0.1) \\ (\boldsymbol{P_3}, 0.2, -0.4) \\ (\boldsymbol{P_4}, 0.8, -0.9) \\ (\boldsymbol{P_5}, 0.6, -0.1) \end{array} \right\} \\ F(S_2) = \left\{ \begin{array}{l} (\boldsymbol{P_1}, 0.5, -0.2) \\ (\boldsymbol{P_2}, 0.6, -0.8) \\ (\boldsymbol{P_3}, 0.1, -0.6) \\ (\boldsymbol{P_4}, 0.7, -0.4) \\ (\boldsymbol{P_5}, 0.3, -0.7) \end{array} \right\} \\ F(S_3) = \left\{ \begin{array}{l} (\boldsymbol{P_1}, 0.6, -0.3) \\ (\boldsymbol{P_2}, 0.5, -0.7) \\ (\boldsymbol{P_3}, 0.2, -0.7) \\ (\boldsymbol{P_4}, 0.6, -0.3) \\ (\boldsymbol{P_5}, 0.4, -0.2) \end{array} \right\} \\ F(S_4) = \left\{ \begin{array}{l} (\boldsymbol{P_1}, 0.8, -0.5) \\ (\boldsymbol{P_2}, 0.9, -0.9) \\ (\boldsymbol{P_3}, 0.5, -0.4) \\ (\boldsymbol{P_4}, 0.8, -0.1) \\ (\boldsymbol{P_5}, 0.6, -0.3) \end{array} \right\} \\ F(S_5) = \left\{ \begin{array}{l} (\boldsymbol{P_1}, 0.1, -0.6) \\ (\boldsymbol{P_2}, 0.2, -0.8) \\ (\boldsymbol{P_3}, 0.6, -0.5) \\ (\boldsymbol{P_4}, 0.4, -0.1) \\ (\boldsymbol{P_5}, 0.7, -0.4) \end{array} \right\} \end{array} \right\}$$

RS of MOORA

Table 1: Decision Matrix for Positive Information System

Alternatives	min	max	Max	max	min
	s_1	s_2	s_3	s_4	s_5
P_1	0.3	0.5	0.2	0.8	0.6
P_2	0.5	0.6	0.1	0.7	0.3
P_3	0.6	0.5	0.2	0.6	0.4
P_4	0.8	0.9	0.5	0.8	0.6
P_5	0.1	0.2	0.6	0.4	0.7

Table 2: Normalized Decision Matrix

Alternatives	s_1	s_2	s_3	s_4	s_5
P_1	0.258	0.385	0.239	0.529	0.497
P_2	0.430	0.461	0.119	0.463	0.248
P_3	0.516	0.385	0.239	0.397	0.331
P_4	0.688	0.692	0.597	0.529	0.497
P_5	0.086	0.154	0.717	0.264	0.579

Table 3: Weights of Criteria

Criteria	s_1	s_2	s_3	s_4	s_5
Weights	0.2	0.3	0.2	0.1	0.2

Table 4: Weighted Normalized Decision Matrix and Final Preference $p_i^{\blacksquare}$

Alternatives	s_1	s_2	s_3	s_4	s_5	$p_i^{\blacksquare}$
P_1	0.051	0.116	0.048	0.052	0.099	0.066
P_2	0.086	0.138	0.024	0.046	0.050	0.072
P_3	0.103	0.116	0.048	0.039	0.066	0.034
P_4	0.137	0.208	0.119	0.053	0.099	0.114
P_5	0.017	0.046	0.143	0.026	0.116	0.082

Table 5: Decision Matrix for Negative Information System

	Min	max	max	max	min
Alternatives	s_1	s_2	s_3	s_4	s_5
P_1	- 0.6	- 0.1	- 0.4	- 0.9	- 0.1
P_2	- 0.2	- 0.8	- 0.6	- 0.4	- 0.7
P_3	- 0.3	- 0.7	- 0.7	- 0.3	- 0.2
P_4	- 0.5	- 0.9	- 0.4	- 0.1	- 0.3
P_5	- 0.6	- 0.9	- 0.5	- 0.1	- 0.4

Table 6: Normalized Decision Matrix

Alternatives	s_1	s_2	s_3	s_4	s_5
P_1	- 0.572	- 0.062	- 0.336	- 0.870	- 0.112
P_2	- 0.191	- 0.497	-0.503	- 0.387	- 0.787
P_3	- 0.286	- 0.435	-0.587	- 0.290	- 0.225
P_4	- 0.477	- 0.559	-0.336	- 0.097	- 0.337
P_5	- 0.572	- 0.497	- 0.419	- 0.097	- 0.450

Table 7: Weights of Criteria

Criteria	s_1	s_2	s_3	s_4	s_5
Weights	0.15	0.2	0.3	0.15	0.2

Table 8: Weighted Normalized Decision Matrix and Final Preference

Alternatives	s_1	s_2	s_3	s_4	s_5	p
P_1	- 0.086	- 0.012	- 0.101	- 0.131	- 0.022	0.201
P_2	- 0.029	- 0.099	- 0.151	- 0.058	- 0.157	0.194
P_3	- 0.043	- 0.087	- 0.176	- 0.044	- 0.045	0.253
P_4	- 0.072	- 0.112	- 0.101	- 0.015	- 0.067	0.203
P_5	- 0.086	- 0.099	- 0.126	- 0.015	- 0.009	0.227

The ranking of alternatives, $P_4 > P_5 > P_1 > P_3 > P_2$.

RPA of MOORA

Table 9: Weighted Normalized Decision Matrix and Final Preference $d_i^{\blacksquare}$ for Positive Information System

Alternative	s_1	s_2	s_3	s_4	s_5	$d_i^{\blacksquare}$
P_1	0.034	0.092	0.095	0.001	0.049	0.095
P_2	0.069	0.070	0.119	0.007	0.000	0.119
P_3	0.086	0.092	0.095	0.014	0.016	0.095
P_4	0.129	0.000	0.204	0.000	0.049	0.129
P_5	0.000	0.162	0.000	0.027	0.066	0.162
	0.017	0.208	0.143	0.053	0.050	

Table 10: Weighted Normalized Decision Matrix and Final Preference for Negative Information System

Alternative	s_1	s_2	s_3	s_4	s_5	d
P_1	0.000	0.000	0.000	0.116	0.135	0.095
P_2	0.057	0.087	0.050	0.043	0.000	0.119
P_3	0.043	0.075	0.075	0.029	0.112	0.095
P_4	0.014	0.100	0.000	0.000	0.090	0.129
P_5	0.000	0.087	0.025	0.000	0.148	0.162
	- 0.086	- 0.012	- 0.101	- 0.015	- 0.157	

The ranking of the alternatives, $P_4 > P_1 > P_5 > P_2 > P_3$.

FMF of MOORA

Table 11: Multiplicative Ranking Index $Z_i^{\blacksquare}$ for Positive Information System

Alternative	s_1	s_2	s_3	s_4	s_5	$Z_i^{\blacksquare}$
P_1	0.786	0.812	0.725	0.978	0.903	0.811
P_2	0.871	0.858	0.631	0.945	0.786	0.747
P_3	0.903	0.812	0.725	0.950	0.833	0.743
P_4	0.956	0.969	0.871	0.978	0.903	0.956
P_5	0.631	0.617	0.903	0.912	0.931	0.865

Table 12: Multiplicative Ranking Index z for Negative Information System

Alternative	s_1	s_2	s_3	s_4	s_5	z
P_1	- 0.926	- 0.631	- 0.760	- 0.984	- 0.631	0.003
P_2	- 0.786	- 0.956	- 0.858	- 0.872	- 0.931	- 0.230
P_3	- 0.835	- 0.931	- 0.899	- 0.835	- 0.725	- 0.412
P_4	- 0.901	- 0.979	- 0.760	- 0.015	- 0.786	0.940
P_5	- 0.926	- 0.956	- 0.812	- 0.015	- 0.833	0.849

The ranking of the alternatives, $P_3 > P_4 > P_5 > P_1 > P_2$.

Table 13: MULTI-MOORA Ranking Order

RA OF MOORA	RPA OF MOORA	FMF OF MOORA	MM METHOD
P_4	P_4	P_3	P_4
P_5	P_1	P_4	P_1
P_1	P_5	P_5	P_5
P_3	P_2	P_1	P_3
P_2	P_3	P_2	P_2

5. CONCLUSION

In this article, we proposed the combination of bipolar fuzzy soft set and soft using Modified MULTI-MOORA method. Also, we give an application by choosing the best Pc among the rest using Modified MULTI-MOORA method. This method is very effective and useful for many researchers in decision making problems.

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