Projective Flat Curvature Tensor Of Kähler Manifold

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Abstract: In this paper, we study the projective flat curvature tensor.

1. Introduction

In 1975, a linear connection is introduced by S. Golab [1] called quarter-symmetric connection. Golab [1] defined that condition $T(X,Y) = \omega(Y)\varphi X\ \omega(X)\varphi Y$.

Recently, some results have been obtained in weakly symmetric manifold in [2-5] (2017). Nijenhuis tensor becomes zero in a Kähler manifold [6-11]. In the same paper, they [6] obtained some results related to contra-variant vector field in Kähler manifold with semi-symmetric connection. Flatness of Kähler manifold under weak symmetric condition has been discussed in [1] (2014). In 2015 a new type of results has been studied [2] in an almost Hermitian manifold. The same authors [3] further studied a special type of Kähler manifold under weak symmetric condition. The study of conformal connection has been extended by Chaturvedi and Pandey [4] in an almost Hermitian manifold in 2016.

Let M be differentiable manifold of dimension 2k, for non-negative integer k. If the conditions

$$F^2(X) + X = 0,\ g(FX,FY) = g(X,Y), (\nabla_X F)Y = 0, \tag{1.1}$$

hold then M becomes Kähler manifold.

In this paper, we have considered a connection ∇^*

$$\nabla^*_X Y = \nabla_X Y + \omega(Y)FX \tag{1.2}$$

satisfying

$$(\nabla^*_X g)(Y,Z) = \alpha[\omega(Y)g(FX,Z) + \omega(Z)g(FX,Y) \tag{1.3}$$

$$T^*(X,Y) = \omega(Y)FX - \omega(X)FY \tag{1.4}$$

respectively for ω 1-form defined by $\omega(X) = g(X,\rho)$, where ρ is an associated vector field.

2. Preliminaries

Let M denotes a manifold of dimension 2k, for non negative integer, then Riemannian tensor is defined by

$R(A,B,P) = \nabla_A \nabla_B P - \nabla_B \nabla_A P - \nabla_{[A,B]} Z$
(2.1) and the Ricci tensor is contraction of R.

Now, equation (2.1) becomes for ∇^*

$R^*(X,Y,Z) = \nabla_X^* \nabla_Y^* Z - \nabla_Y^* \nabla_X^* Z - \nabla_{[X,Y]}^* Z$
(2.2)

Using (1.2) in (2.2), we get

$R^*(A,B,P) = R(A,B,P) + [(\nabla_A \omega)(P)FB - (\nabla_B \omega)(P)FA] + [(\nabla_A F)B - (\nabla_B F)A + \omega(FB)FA - \omega(FA)FB]\omega(P).$
(2.3)

If the associated vector field is unit parallel then $\nabla_X \rho = 0$, which imply

$(\nabla_P \omega)(Z) = 0.$
(2.4) Now, using (1.1) and (2.4) in (2.3), we get

$R^*(A,B,P) = R(A,B,P) + [\omega(FB)FA - \omega(FA)FB]\omega(P).$
(2.5)

Further, contracting (2.5), we get

$S^*(B,P) = S(B,P) + \omega(B)\omega(P)$
(2.6)

Again, contracting (2.6), we get

$r^* = r$
(2.7)

3. Projective tensor

The projective tensor is defined by

$W(A,P,K) = R(A,P,K) - \frac{1}{n-1}[S(P,K)A - S(A,K)P] = 0$.
(3.1)

For projective flat manifold, we get

$R(A,P,K) = \frac{1}{n-1}[S(P,K)A - S(A,K)P]$.
(3.2)

Now, the projective curvature tensor for quarter symmetric connection ∇^* defined in (1.2) is given by

$W^*(A,P,K) = R^*(A,P,K) - \frac{1}{n-1}[S^*(P,K)A - S^*(A,K)P]$.
(3.3)

Using (2.5), (2.6) and (3.1) in (3.3), we get

$$W^*(A,P,K) = W(A,P,K) + \left[\omega(FP)FA - \frac{1}{n-1}\omega(P)A - \omega(FA)FP + \frac{1}{n-1}\omega(A)P\right]\omega(K) \quad . \quad (3.4)$$

Hence for projective flat manifold we get

$$\omega(FP)FA - \omega(FA)FP = \frac{1}{n-1}[\omega(P)A - \omega(A)P]. \quad (3.5)$$

We can state here

Theorem 3.1: *For M being a Kähler manifold with ∇^* defined by (1.2) then for a associated parallel unit vector field ρ the manifold becomes projective flat if and only if (3.5) satisfy.*

Replacing A and P by FA and FP respectively in (3.5), we get

$$\omega(P)A - \omega(A)P = \frac{1}{n-1}[\omega(FP)FA - \omega(FA)FP] \quad . \quad (3.6)$$

Subtracting (3.6) from (3.5), we get

$$[\omega(FP)FA - \omega(FA)FP - \omega(P)A + \omega(A)P](n-2) = 0 \quad . \quad (3.7)$$

Let us defined

$$D(A,P) = \omega(FP)FA - \omega(P)A \quad . \quad (3.8)$$

With the help of (3.8), equation (3.7) yields

$$[D(A,P) - D(P,A)](n-2) = 0 \quad . \quad (3.9)$$

Equation (3.9) implies manifold is two dimensional for $D(A,P) \neq D(P,A)$.

From above, we conclude that

Theorem 3.2: *For M being a Kähler manifold with ∇^* defined by (1.2) then for a associated parallel unit vector field ρ the manifold becomes two dimensional projective flat if and only if $D(X,Y)$ defined in (3.8) is not symmetric.*

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