AN INCENTIVE EOQ INVENTORY MODEL WITH PRICE DISCOUNT

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ABSTRACT

This article looks at the problem of a supplier placing an order and providing a consumer a discount. This study's objective is to develop a decision-making procedure that will assist retailers in selecting between a regular order policy and a special order policy. The three possibilities in the created model are the optimal special order quantity is calculated with both regular and additional quantity benefits, while the optimal special order quantity is determined only with additional quantity benefits and the optimal order quantity is computed without taking advantage of price savings. This study finds the ideal solution, illustrates the theoretical conclusions using a variety of numerical examples, and then runs a sensitivity analysis of the ideal solution with respect to the key parameters.

Keywords: EOQ, Inventory, Special order quantity, Price discount

1. INTRODUCTION

In reality, if a supplier encounters significant market saturation, an unexpected overflow of inventory, or a change in the product's manufacturing run, he or she may offer customers a special price discount to acquire a special amount. Retailers are encouraged by the discount to place larger orders and give discounts to customers in order to boost demand and profits.

Heuristics for sourcing from several providers with different quantity discounts were examined by Burke et al. in 2008. The ideal order size was created by Leopoldo Eduardo

Cardenas-Barron et al. (2010) to benefit from a one-time discount deal with permitted backorders. EOQ model with deterioration factor and all-units discount was taken into consideration by Limansyah et al. in 2020. Inventory model with price-sensitive demand and quantity discount was developed by Lin and Ho (2011). Muniappan et al. (2018) described a manufacturing inventory model with partial backlog shortages for products that are deteriorating. Muniappan et al. in 2020 took into account an integrated economic order quantity model with inventory level and storage capacity restrictions. The coordination of a three-level supply chain with quantity discount was researched by Munson and Rosenblatt (2001).

In a periodic review inventory model with a backorder price discount, Ouyang et al. (2005) looked at how lead time and ordering costs could be reduced in a mutually exclusive manner. In the discounted EOQ model, Porteus (1989) created investing in new parameter values. With price-sensitive demand and volume discount, Qin et al. (2007) created coordination inventory model. An inventory model for degrading goods with a positive exponential function of price discount rate of demand was studied by Ravithammal et al. (2018). In a controllable lead time inventory model, Sarkar et al. (2015) created quality improvement and backorder price discounts. For inventory policies, Shah et al. (2018) looked into quantity discounts and price-sensitive stock-dependent demand. Modeling quantity discounts under general price-sensitive demand functions was encouraged by Weng et al. (1995). Yang et al. (2004) looked at a price-sensitive demand quantity discount pricing method for degrading goods. The bi-ramp type demand and price discount inventory model for degrading goods was created by Zhou et al. (2012).

2. NOTATIONS AND ASSUMPTIONS

The model uses the following notations and assumptions.

2.1 Notations

d	Demand rate/ units/ time
r_1	Buyer’s ordering cost / order /unit
h_1	Buyer’s unit carrying cost /unit, $h_1 = Ci$
i	Inventory carrying cost rate
C	Product unit cost/unit
u	Percentage of defecting items /unit
v	Percentage of scrap items /unit

d_c	Disposed cost / unit
S_c	Buyer's unit screening cost / unit
Q	Economic Order Quantity
Q'	Special Order Quantity
Z	Purchase additional units over Q to benefit from the discount
α	A predetermined discount offered by the supplier

2.2 Assumptions

- The planning horizon is one year, and demand is consistent and predictable.
- In case I, buyer did not use benefit of the price discount and continues to buy normal units.
- In case II, all units are eligible for the discount, but in case III, the additional units are the only ones eligible for the discount.

3. MODEL FORMULATION

The three cases addressed in this section are as follows. The buyer decides to continue buying standard units despite not taking advantage of the discounted cost, as shown in Case I. In case II, the customer places a special order quantity to receive both normal and additional units, whereas in case III, the buyer places a special purchase quantity to receive only additional units.

Case - I: Normal Order Quantity with no discount

The total cost for buyer contains as following cost

TC_b = Ordering cost+ Carrying cost +Screening cost + Disposed cost +Purchasing cost

i.e., $TC_b = \frac{r_1 d}{Q} + \frac{h_1 Q}{2} + \frac{S_c Q}{2} + \frac{uvd_c Q}{2} + Cd$

For optimality $\frac{\partial TC_b}{\partial Q} = 0$ and $\frac{\partial^2 \mathrm{TC_b}}{\partial \mathrm{Q}^2} > 0$ we get,

$$Q = \sqrt{\frac{2dr_1}{h_1 + S_c + uvd_c}}$$

Case -II: Special Order Quantity with discount for normal and additional units

The total cost for buyer is contains as following cost

TC_{b1} = Ordering cost + Carrying cost + Screening cost + Disposed cost + Purchasing cost

i.e., $$TC_{b1} = r_1 + r_1\left[\frac{d-Q-Z}{Q}\right] + \frac{h_1}{2d}[1-\alpha][Q+Z]^2 + \frac{h_1 Q}{2}\left[\frac{d-Q-Z}{d}\right] + \frac{S_c Q}{2} + \frac{S_c Q}{2}\left[\frac{d-Q-Z}{d}\right] + \frac{uvd_c Q}{2} + \frac{uvd_c Q}{2}\left[\frac{d-Q-Z}{d}\right] + C\,[1-\alpha][Q+Z] + C\,[d-Q-Z]$$

By accepting the offer, this difference shows the savings that will be made.

$$D(Z) = TC_b - TC_{b1}$$

$$= \frac{h_1}{2d}[\alpha-1]Z^2 + \left[\frac{r_1}{Q} - \frac{h_{1Q}}{2d} + \frac{h_{1\alpha Q}}{d} + \frac{S_c Q}{2d} + \frac{uvd_c Q}{2d} + C\alpha\right]Z + \frac{h_1\alpha Q^2}{2d} - \frac{S_c Q}{2} + \frac{S_c Q^2}{2d} - \frac{uvd_c Q}{2} + \frac{uvd_c Q^2}{2d} + C\alpha Q$$

$$D'(Z) = \frac{h_1}{d}[\alpha-1]Z + \frac{r_1}{Q} - \frac{h_{1Q}}{2d} + \frac{h_{1\alpha Q}}{d} + \frac{S_c Q}{2d} + \frac{uvd_c Q}{2d} + C\alpha$$

$$D''(Z) = \frac{h_1}{d}[\alpha-1]$$

For Optimality $D'(Z) = 0$ and $D''(Z) > 0$, we get additional units

$$Z = \left[\frac{\alpha}{1-\alpha}\right]\left[Q + \frac{Cd}{h_1}\right] + \frac{1}{[1-\alpha]}\left[\frac{2dr_1 + S_c Q^2 + uvd_c Q^2}{2h_1 Q} - \frac{Q}{2}\right]$$

Therefore, The special order quantity $Q' = Q + Z$

Case -III: Special Order Quantity with discount for additional units only

The total cost for buyer is contains as following cost

TC_{b2} = Ordering cost + Carrying cost + Screening cost + Disposed cost + Purchasing cost

i.e., $$TC_{b2} = r_1 + r_1\left[\frac{d-Q-Z}{Q}\right] + \frac{h_1}{d}[1-\alpha]QZ + h_1[1-\alpha]\left[\frac{Z}{2}\right]\left[\frac{Z}{d}\right] + \frac{h_1 Q}{2}\left[\frac{d-Z}{d}\right] + \frac{S_c Q}{2} + \frac{S_c Q}{2}\left[\frac{d-Q-Z}{d}\right] + \frac{uvd_c Q}{2} + \frac{uvd_c Q}{2}\left[\frac{d-Q-Z}{d}\right] + C\,[1-\alpha]Z + C\,[d-Z]$$

By accepting the offer, this difference shows the savings that will be made.

$$D(Z) = TC_b - TC_{b2}$$

$$= \frac{h_1}{2d}[\alpha - 1]Z^2 + \left[\frac{r_1}{Q} - \frac{h_{1Q}}{2d} + \frac{h_{1\alpha Q}}{d} + \frac{S_c Q}{2d} + \frac{uvd_c Q}{2d} + C\alpha\right]Z - \frac{S_c Q}{2} + \frac{S_c Q^2}{2d} - \frac{uvd_c Q}{2} + \frac{uvd_c Q^2}{2d}$$

$$D'(Z) = \frac{h_1}{d}[\alpha - 1]Z + \frac{r_1}{Q} - \frac{h_{1Q}}{2d} + \frac{h_{1\alpha Q}}{d} + \frac{S_c Q}{2d} + \frac{uvd_c Q}{2d} + C\alpha$$

$$D''(Z) = \frac{h_1}{d}[\alpha - 1]$$

For Optimality $D'(Z) = 0$ and $D''(Z) > 0$, we get additional unit

$$Z = \left[\frac{\alpha}{1-\alpha}\right]\left[Q + \frac{Cd}{h_1}\right] + \frac{1}{[1-\alpha]}\left[\frac{2dr_1 + S_c Q^2 + uvd_c Q^2}{2h_1 Q} - \frac{Q}{2}\right]$$

Therefore, The special order quantity $Q' = Q + Z$

4. NUMERICAL EXAMPLE

Example 1:

Let $d = 7000$, $r_1 = 12000$,$i = 0.2$, $C = 3$,$S_c = 0.02$,$\alpha = 0.04$, $u = 0.2$,

$v = 0.3$, $d_c = 4$.

The Optimal Solution is$Q = 44198$, $TC_b = 2.4271 \text{ x } 10^5$, $Z = 16425$, $Q' = 60623$,

$TC_{b1} = 2.4068 \text{ x } 10^5$, $TC_{b2} = 2.4632 \text{ x } 10^5$.

Example 2:

Let $d = 12000$, $r_1 = 500$,$i = 0.1$, $C = 3$,$S_c = 0.05$, $\alpha = 0.01$, $u = 0.02$,

$v = 0.03$, $d_c = 2$.

The Optimal Solution is $Q = 5845$, $TC_b = 3.8049 \text{ x } 10^4$, $Z = 1271$, $Q' = 7116$,

$TC_{b1} = 3.7898 \text{ x } 10^4$, $TC_{b2} = 3.8078 \text{ x } 10^4$.

Sensitive Analysis

The sensitivity analysis typically takes each restriction separately while holding the additional limit constant. The results are displayed in Table 1.

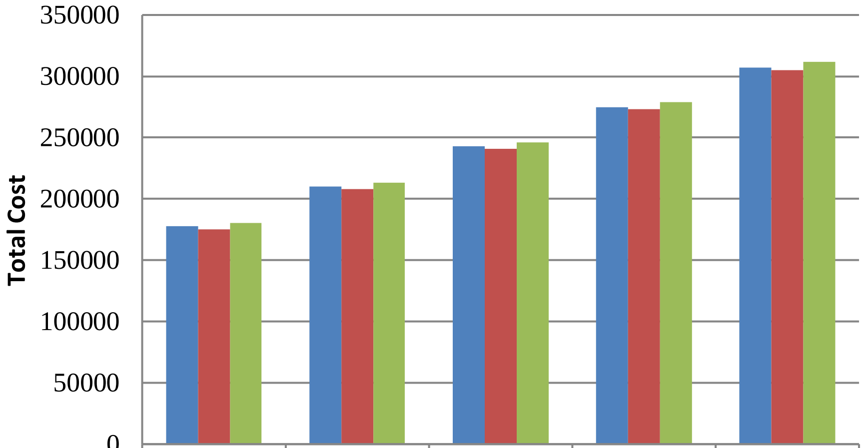
Table 1: Effects of Changes

Decision Variables	Cost / Unit	Q	TC_b	Z	Q'	TC_{b1}	TC_{b2}
d	50000	37354	1.7764 x 10^5	11973	49327	1.7538 x 10^5	1.8020 x 10^5
	60000	40920	2.1028 x 10^5	14205	55125	2.0815 x 10^5	2.1340 x 10^5
	70000	44198	2.4271 x 10^5	16425	60623	2.4068 x 10^5	2.4632 x 10^5
	80000	47250	2.7497 x 10^5	18635	65885	2.7303 x 10^5	2.7904 x 10^5
	90000	50116	3.0709 x 10^5	20838	70954	3.0523 x 10^5	3.1158 x 10^5
r_1	8000	36088	2.3671 x 10^5	16087	52175	2.3553 x 10^5	2.4009 x 10^5
	10000	40347	2.3986 x 10^5	16264	56612	2.3827 x 10^5	2.4339 x 10^5
	12000	44198	2.4271 x 10^5	16425	60623	2.4068 x 10^5	2.4632 x 10^5
	14000	47740	2.4533 x 10^5	16572	64312	2.4284 x 10^5	2.4896 x 10^5
	16000	51036	2.4777 x 10^5	16710	67746	2.4480 x 10^5	2.5137 x 10^5
i	0.1	54772	2.3410 x 10^5	31449	86221	2.2696 x 10^5	2.3379 x 10^5
	0.15	48644	2.3870 x 10^5	21471	70115	2.3502 x 10^5	2.4116 x 10^5
	0.2	44198	2.4271 x 10^5	16425	60623	2.4068 x 10^5	2.4632 x 10^5
	0.25	40784	2.4630 x 10^5	13366	54150	2.4521 x 10^5	2.5046 x 10^5
	0.3	38056	2.4958 x 10^5	11308	49364	2.4908 x 10^5	2.5402 x 10^5
C	2	50452	1.6724 x 10^5	1668	67138	1.6691 x 10^5	1.7124 x 10^5
	3	44198	2.4271 x 10^5	16425	60623	2.4068 x 10^5	2.4632 x 10^5
	4	39811	3.1742 x 10^5	16242	56053	3.1385 x 10^5	3.2058 x 10^5

	5	36515	3.9163×10^5	16105	52620	3.8663×10^5	3.9432×10^5
	6	33922	4.6546×10^5	15997	49918	4.5914×10^5	4.6768×10^5
S_c	0.01	44458	2.4245×10^5	16436	60893	2.4042×10^5	2.4610×10^5
	0.02	44198	2.4271×10^5	16425	60623	2.4068×10^5	2.4632×10^5
	0.03	43944	2.4296×10^5	16414	60358	2.4094×10^5	2.4655×10^5
	0.04	43693	2.4321×10^5	16404	60097	2.4120×10^5	2.4677×10^5
	0.05	43447	2.4345×10^5	16394	59841	2.4146×10^5	2.4700×10^5
α	0.04	44198	2.4271×10^5	16425	60623	2.4068×10^5	2.4632×10^5
	0.05	44198	2.4271×10^5	20747	64946	2.3792×10^5	2.4497×10^5
	0.06	44198	2.4271×10^5	25162	69360	2.3499×10^5	2.4345×10^5
	0.07	44198	2.4271×10^5	29671	73869	2.3188×10^5	2.4175×10^5
	0.08	44198	2.4271×10^5	34278	78467	2.2859×10^5	2.3987×10^5
u	0.05	49705	2.4237×10^5	16654	66359	2.3590×10^5	2.4229×10^5
	0.1	47647	2.4240×10^5	16569	64216	2.3751×10^5	2.4362×10^5
	0.15	45826	2.4254×10^5	16493	62318	2.3911×10^5	2.4496×10^5
	0.2	44198	2.4271×10^5	16425	60623	2.4068×10^5	2.4632×10^5
	0.25	42733	2.4290×10^5	16364	59097	2.4224×10^5	2.4768×10^5
v	0.2	46410	2.4249×10^5	16517	62927	2.3858×10^5	2.4451×10^5
	0.25	45263	2.4259×10^5	16469	61733	2.3963×10^5	2.4542×10^5
	0.3	44198	2.4271×10^5	16425	60623	2.4068×10^5	2.4632×10^5

	0.35	43205	2.4284 x 10^5	16384	59588	2.4172 x 10^5	2.4723 x 10^5
	0.4	42276	2.4298 x 10^5	16345	58621	2.4275 x 10^5	2.4813 x 10^5
d_c	1	49705	2.4231 x 10^5	16654	66359	2.3590 x 10^5	2.4229 x 10^5
	2	47647	2.4240 x 10^5	16569	64216	2.3751 x 10^5	2.4362 x 10^5
	3	45826	2.4254 x 10^5	16493	62318	2.3911 x 10^5	2.4496 x 10^5
	4	44198	2.4271 x 10^5	16425	60623	2.4068 x 10^5	2.4632 x 10^5
	5	42733	2.4290 x 10^5	16364	59097	2.4224 x 10^5	2.4768 x 10^5

Fig 1: Effect of changes when r_1 increases



	50000	60000	70000	80000	90000
TCb	177640	210280	242710	274970	307090
TCb1	175380	208150	240680	273030	305230
TCb2	180200	213400	246320	279040	311580

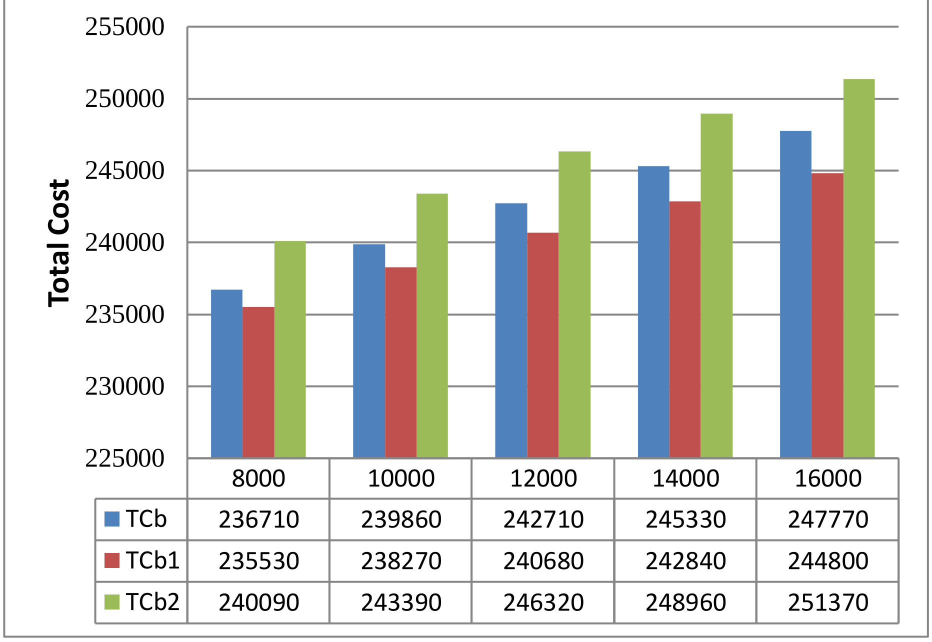


Fig 2: Effect of changes when r_1 increases

	8000	10000	12000	14000	16000
TCb	236710	239860	242710	245330	247770
TCb1	235530	238270	240680	242840	244800
TCb2	240090	243390	246320	248960	251370

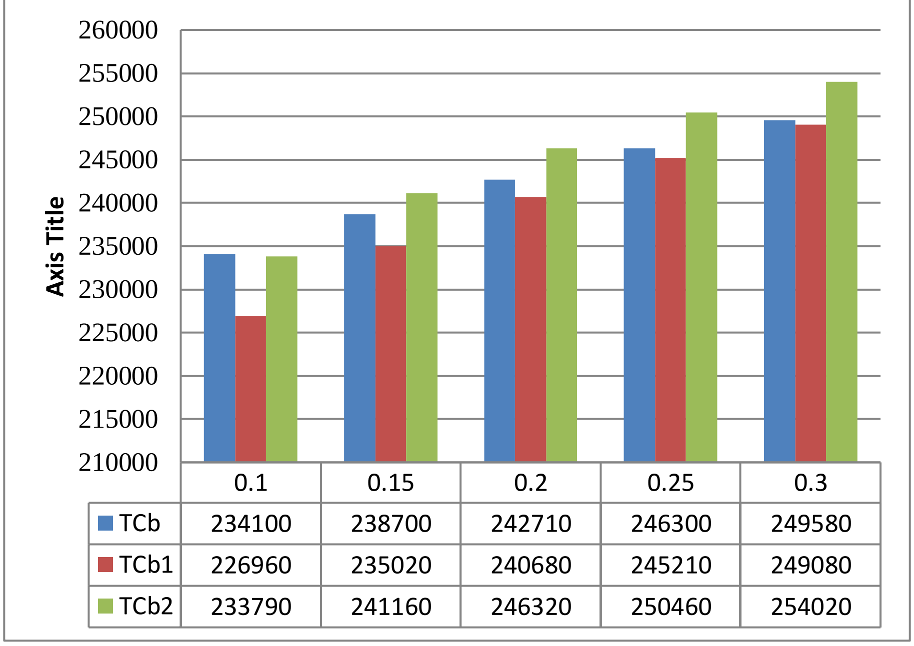
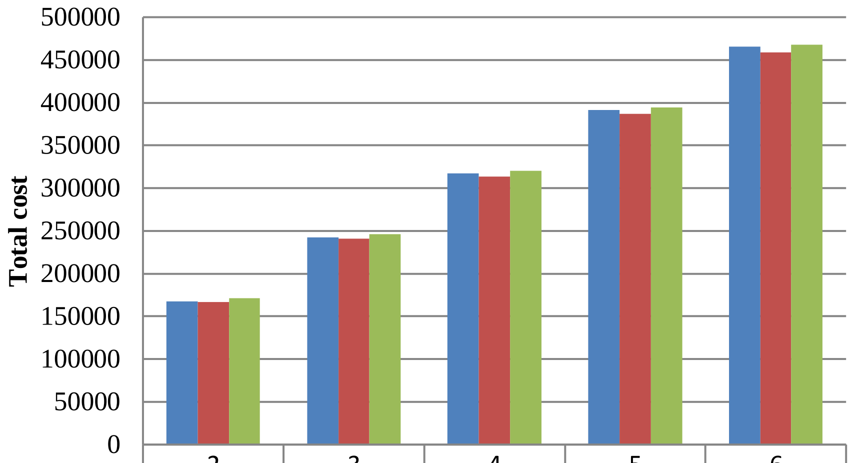


Fig 3: Effect of changes when i increases

	0.1	0.15	0.2	0.25	0.3
TCb	234100	238700	242710	246300	249580
TCb1	226960	235020	240680	245210	249080
TCb2	233790	241160	246320	250460	254020

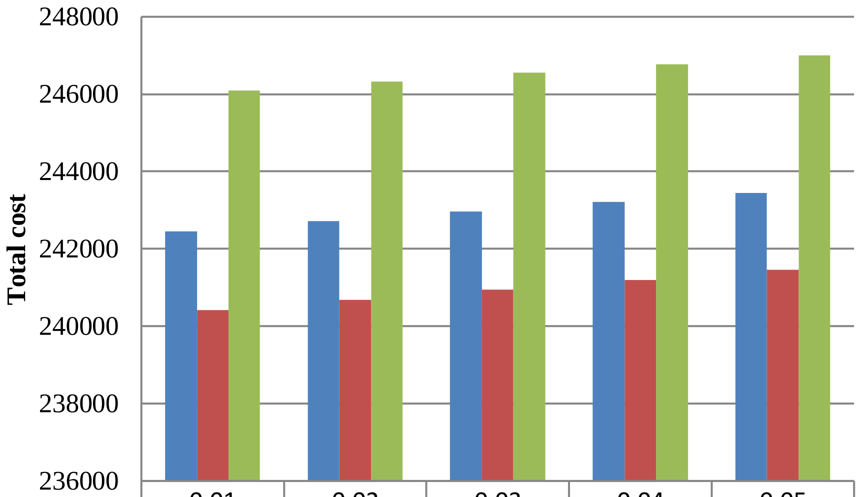
European Journal of Molecular & Clinical Medicine
ISSN 2515-8260 Volume 09, Issue 08, 2022

Fig 4: Effect of changes when C increases



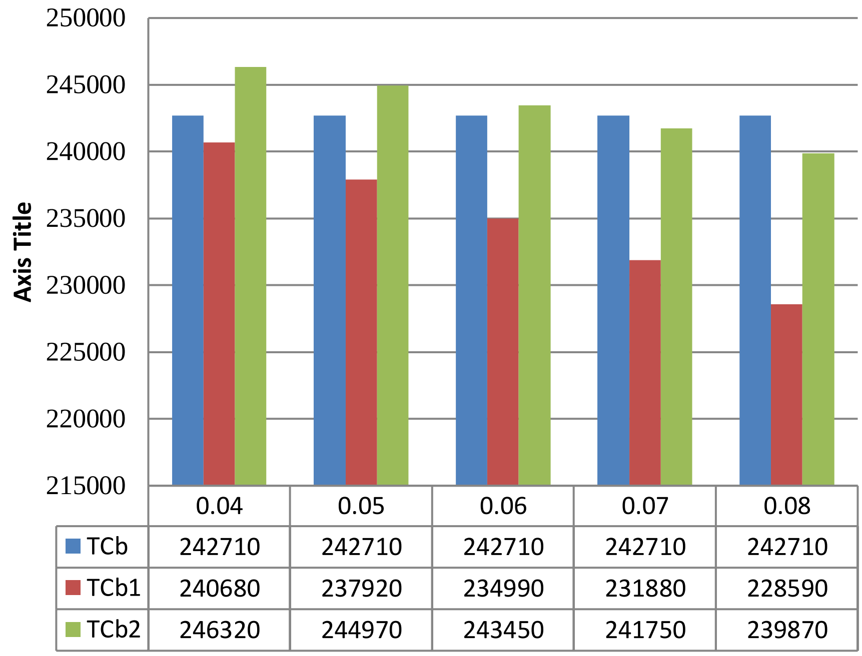
	2	3	4	5	6
TCb	167240	242710	317420	391630	465460
TCb1	166910	240680	313850	386630	459140
TCb2	171240	246320	320580	394320	467680

Fig 5: Effect of changes when S_c increases



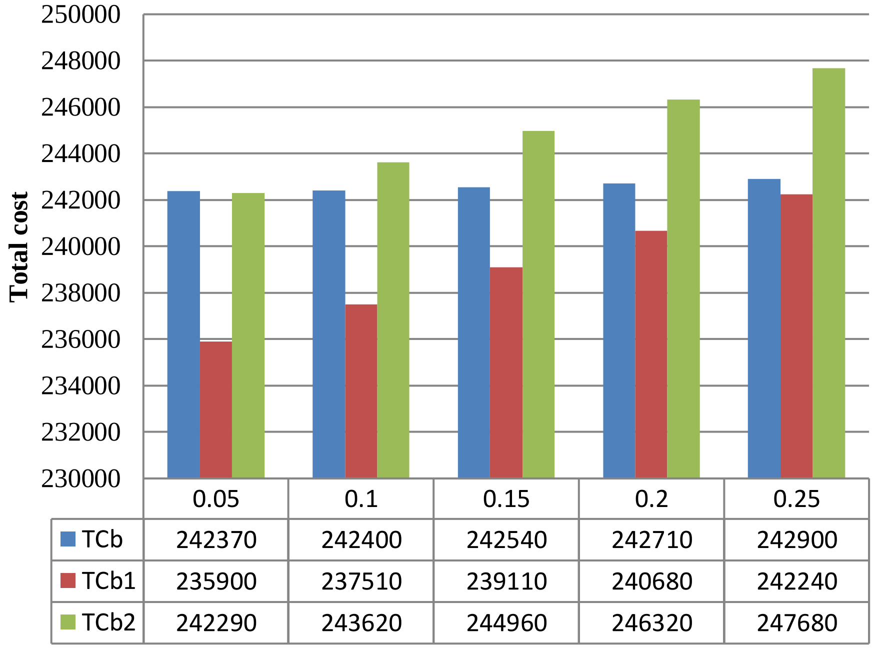
	0.01	0.02	0.03	0.04	0.05
TCb	242450	242710	242960	243210	243450
TCb1	240420	240680	240940	241200	241460
TCb2	246100	246320	246550	246770	247000

Fig 6: Effect of changes when α increases



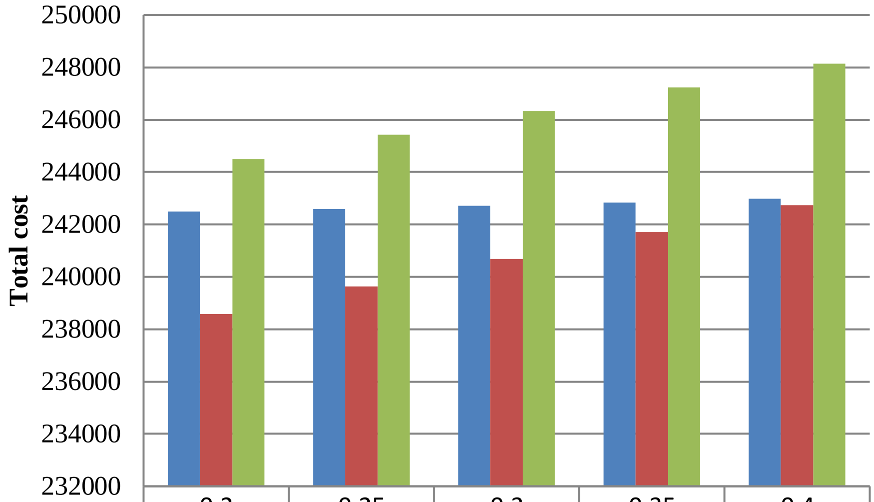
	0.04	0.05	0.06	0.07	0.08
TCb	242710	242710	242710	242710	242710
TCb1	240680	237920	234990	231880	228590
TCb2	246320	244970	243450	241750	239870

Fig 7: Effect of changes when u increases



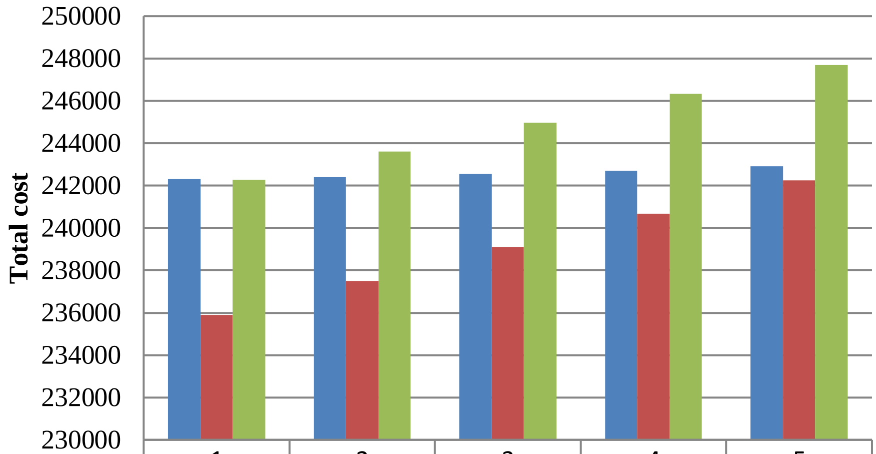
	0.05	0.1	0.15	0.2	0.25
TCb	242370	242400	242540	242710	242900
TCb1	235900	237510	239110	240680	242240
TCb2	242290	243620	244960	246320	247680

Fig 8: Effect of changes when v increases



	0.2	0.25	0.3	0.35	0.4
TCb	242490	242590	242710	242840	242980
TCb1	238580	239630	240680	241720	242750
TCb2	244510	245420	246320	247230	248130

Fig 9: Effect of changes when d_c increases



	1	2	3	4	5
TCb	242310	242400	242540	242710	242900
TCb1	235900	237510	239110	240680	242240
TCb2	242290	243620	244960	246320	247680

CONCLUSION

This research proposes a price-discounted incentive EOQ inventory model. Retailers can choose between a regular order policy and a special order policy with the help of the established model. In this study, the ideal solution is found, the theoretical results are shown with a variety of numerical examples, and finally the ideal solution is subjected to a sensitivity analysis with respect to the important factors. Finally, the outcome shows that it is more advantageous for the buyer to determine special order quantity with both ordinary and excess amount. Examining diverse types of demand, shortages, and a range of items are some possibilities for future studies.

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